FYSH555 spring 2014

Exercise 3, return by Mon Feb 3rd at 10.00., discussed Mon Feb 3rd, at 16.15 in FYS5

1. In the lecture we skipped over the proper color projector algebra. We have an amplitude with 4 external colored legs $A_{ij,lk}$. We project out the color singlet and octet parts using the projectors defined as

$$P_{\underline{1}kl}^{ij} = \frac{1}{N_c}\delta_{ij}\delta_{kl} \tag{1}$$
$$P_{\underline{8}kl}^{ij} = 2t^a_{ji}t^a_{lk} \tag{2}$$
$$\tag{3}$$

Show that these are properly normalized projectors, i.e. $P_{\underline{1}kl}^{ij}P_{\underline{1}mn}^{lk} = P_{\underline{1}mn}^{ij}$, $P_{\underline{8}kl}^{ij}P_{\underline{8}mn}^{lk} = P_{\underline{8}mn}^{ij}$ and $P_{\underline{1}kl}^{ij}P_{\underline{8}mn}^{lk} = 0$. Compute the normalization $P_{\underline{1}kl}^{ij}P_{\underline{1}ji}^{lk}$ and $P_{\underline{8}kl}^{ij}P_{\underline{8}ji}^{lk}$. Now we want to decompose the color structure in the imaginary and real part of the one loop amplitude into color components as

$$\mathrm{Im}A \sim (t^bt^a)_{ji}(t^bt^a)_{lk} = \mathrm{I}_{\underline{1}}P_{\underline{1}kl}^{ij} + \mathrm{I}_{\underline{8}}P_{\underline{8}kl}^{ij}$$

and

$$\mathrm{Re}A \sim (t^bt^a)_{ji}[t^b,t^a]_{lk} = \mathrm{R}_{\underline{1}}P_{\underline{1}kl}^{ij} + \mathrm{R}_{\underline{8}}P_{\underline{8}kl}^{ij}.$$

Calculate $\mathrm{I}_{\underline{1}}$, $\mathrm{R}_{\underline{1}}$ and $\mathrm{R}_{\underline{8}}$. It is a bit more difficult to compute $\mathrm{I}_{\underline{8}}$, can you do it?

2. Go through the derivation of the Lipatov vertex in the lectures: calculate explicitly the 5 diagrams using eikonal vertices for the connections to the quark lines and the multi-Regge kinematics to get the effective vertex $g^{\mu\nu}C^\rho$ where $C = (C^+, C^-, \mathbf{C}_T) = (k_1^+ + \mathbf{k}_{T1}^2/k_2^-, k_2^- + \mathbf{k}_{T2}^2/k_1^+, -\mathbf{k}_{T1} - \mathbf{k}_{T2})$.

3. The Lipatov vertex is $C^\rho = (C^+, C^-, \mathbf{C}_T) = (k_1^+ + \mathbf{k}_{T1}^2/k_2^-, k_2^- + \mathbf{k}_{T2}^2/k_1^+, -\mathbf{k}_{T1} - \mathbf{k}_{T2})$. The outgoing gluon has momentum $p = k_1 - k_2$

   (a) Using the approximations in multi-Regge kinematics, express C^ρ in terms of p^+, p^- and $\mathbf{k}_{T1}, \mathbf{k}_{T2}$.

   (b) Remembering that $p^2 = 0$ show that $p^\rho C_\rho(k_1,k_2) = 0$

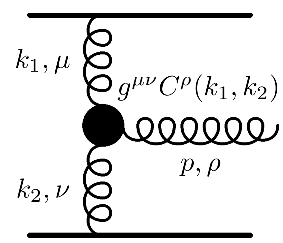
   (c) Calculate $C^\rho(k_1,k_2)C_\rho(k_1,k_2)$

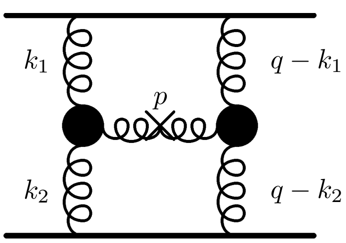
   (d) Calculate the transverse momentum integrand appearing in the real part of the α_s^3 amplitude:
$$\frac{C^\rho(k_1,k_2)C_\rho(q-k_1,q-k_2)}{\mathbf{k}_{T1}^2\mathbf{k}_{T2}^2(\mathbf{q}_T-\mathbf{k}_{T1})^2(\mathbf{q}_T-\mathbf{k}_{T2})^2}$$

4. Show how a Laplace transform deconvolutes the nested rapidity integrals in the BFKL ladder. I.e. if

$$f(y) = \int_0^y \mathrm{d}y_1 \int_0^{y_1} \mathrm{d}y_2 \cdots \int_0^{y_{n-1}} \mathrm{d}y_n e^{(y-y_1)(\varepsilon(k_1)+\varepsilon(q-k_1))} e^{(y_1-y_2)(\varepsilon(k_2)+\varepsilon(q-k_2))} \times \cdots e^{(y_{n-1}-y_n)(\varepsilon(k_n)+\varepsilon(q-k_n))} e^{y_n(\varepsilon(k_{n+1})+\varepsilon(q-k_{n+1}))}$$

calculate the Laplace transform $f(\omega) = \int_0^\infty \mathrm{d}y e^{-\omega y} f(y)$. Hint: take the rapidity differences $y_n - y_{n+1}$ as integration variables.





Left: Lipatov vertex. Right: Real contribution to the imaginary part of the α_s^3 amplitude