A correction to: Sobolev inequalities on sets with irregular boundaries

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The proof of Theorem 2.3 in the original paper that appeared in Z. Anal. Anwendungen vol 19 (2000), no. 2, 369–380, was slightly erroneous. Here we provide a new proof:

Theorem 2.3 *Suppose that Ω is an s-John domain and $b \geq 1-n$. Then there is a constant $C = C(n,p,q,\Omega) > 0$ such that*

$$\Big(\int_\Omega |u-\bar{u}_a|^q \rho^a \, dx\Big)^{1/q} \leq C\Big(\int_\Omega |\nabla u|^p \rho^b dx\Big)^{1/p}$$

for each $u \in C^1(\Omega)$; here the Sobolev exponent is

$$q = \frac{p(n+a)}{s(n+b-1)-p+1}.$$

Proof. We will verify the assumptions of Theorem 2.1. First we notice that the inequalities $s \geq 1$ and $b \geq 1-n$ imply

$$p\Big(\frac{a+n}{q} + 1 - \frac{n}{p}\Big) = s(n+b-1) + 1 - n \geq b,$$

so that (2.4) is true. For fixed $y \in \Omega \setminus B_0$, the s-John core γ on $[0,\ell]$ gives us the desired worm: Let m be the integer such that

$$3\ell < 2^m \rho(y) \leq 6\ell.$$

Since

$$\rho(y) \leq \rho(x_0) + |y - x_0| \leq 3|y - x_0| \leq 3\ell,$$

we have $m \geq 1$. Set

$$\xi_k = 2^{k-m}\ell.$$

Then $(\gamma, \{\xi_k\})$ is a worm with parameters m, $\{\ell_k\}$, $\{R_k\}$, $\{r_k\}$. Observe that we may assume that the part of γ lying inside the ball $B(y, 2^{1-m}\ell) \subset \Omega$ is a line segment. Thus we have that

$$\rho(y) \leq 6\, 2^{-m}\ell \leq 3R_1 \leq 3R_k$$

so that (2.5) holds.

Moreover,

$$\begin{aligned} &\ell_k \leq \xi_k, \\ &\xi_k \geq R_k, \\ &r_k \geq c_0 \xi_{k-1}^s = c_0(\xi_k/2)^s. \end{aligned}$$

Since

$$(n+a)/q = (s(n+b-1)+1-p)/p$$

we have by choosing $\tau_k = \min(2^{k-m}, \xi_k)^{(n+a)/q} \in (0,1]$ that

$$\tau_k \leq \xi_k^{(n+a)/q} \leq C\tau_k\,,$$

where the constant C does not depend on y since for the s-John core γ it holds that

$$c_0\ell^s \leq \mathrm{diam}(\Omega).$$

Thus we have

$$\boldsymbol{\mu}(B(y, 3R_k))^{1/q} \leq C\xi_k^{(n+a)/q} \leq C\tau_k$$

and furthermore,

$$\begin{aligned} r_k^{-(n+b-1)/p} \ell_k^{(p-1)/p} &\leq C\big(\xi_k\big)^{-s(n+b-1)/p} \xi_k^{(p-1)/p} \\ &= C\xi_k^{-(n+a)/q} \\ &\leq C\tau_k^{-1}\,. \end{aligned}$$

Hence the claim follows from Theorem 2.1.