OPERATOR SPLITTING METHODS FOR PRICING AMERICAN OPTIONS UNDER JUMP-DIFFUSION MODELS AND STOCHASTIC DISCRETE DIVIDENDS

Santtu Salmi\textsuperscript{1} and Jari Toivanen\textsuperscript{1,2}

\textsuperscript{1}Department of Mathematical Information Technology
P.O. Box 35 (Agora), FI-40014 University of Jyväskylä, Finland
e-mail: \{santtu.salmi,jari.toivanen\}@jyu.fi

\textsuperscript{2}Institute for Computational and Mathematical Engineering
Stanford, CA 94305, USA
e-mail: toivanen@stanford.edu

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\textbf{Abstract.} Pricing American options under finite activity jump-diffusion models like Kou’s model with discrete dividends is considered. The American option pricing problem is formulated as a linear complementarity problem (LCP). The derivatives in the partial integro-differential operator are discretizing using finite differences. The discretization of the jump integral leads to a full matrix. The early exercise constraint is treated using an operator splitting method leading at each time step to two simpler substeps. For time discretization, implicit and implicit-explicit (IMEX) methods are considered. Numerical results are presented under Kou’s model without dividend and with fixed and stochastic discrete dividend. The results demonstrate the high speed and good accuracy of the proposed numerical methods.
1 INTRODUCTION

Since the pioneering work by Black and Scholes [4] and Merton [20] in 1973, the limitations of their original model have become evident. The volatility in the market prices of options is clearly not constant, but exhibits a smile like shape with respect to the strike price. Also, occasionally there is a sudden jump in market prices, which a purely Brownian motion type process does not exhibit. Already in 1976 Merton proposed the addition of jumps into the model of the underlying asset [21].

The addition of jumps leads to an option pricing model that naturally generates implied volatilities with a smile like shape [3, 9]. Besides Merton’s model, other jump models in the literature include the CGMY model [7] and Kou’s model [18]. The CGMY model allows infinite jump activity, while the other mentioned models are finite activity jump-diffusion models. Here we consider finite activity jump-diffusion models.

A partial integro-differential equation (PIDE) can be derived for the price of a European option. Similarly the price of an American option is given by a partial integro-differential inequality which can be formulated as a linear complementarity problem (LCP). Efficient semi-analytic FFT based implementations can be used to obtain the prices of European options under jump-diffusion [6, 11]. However, prices of American options generally have to be obtained by discretizing the continuous problem. Here we adopt the finite difference approach for this purpose. Already in 1977, Brennan and Schwartz proposed a finite difference method for American options under the Black-Scholes model [5]; see also [1, 16].

The discretization of the integral term in a jump-diffusion model leads to a full matrix. Due to this, special discretizations and algorithms have to be developed in order to obtain the option prices fast; see [25] for a survey on efficient methods. Efficient methods circumvent inverting the full matrix, but a matrix vector multiplication with a full matrix is still required. A straightforward implementation of the full matrix vector multiplication would require \( O(n^2) \) operations, where \( n \) is the number of spatial grid points. The jump distribution in Kou’s model has the memoryless property, which enables the use of efficient recursion formulas to calculate the matrix vector multiplication with only \( O(n) \) operations [8, 27]. In the case of Merton’s model, the integral can be transformed into convolution form and FFT can be used for the multiplication requiring \( O(n \log n) \) operations [2, 14].

To avoid inverting a full matrix, we consider two efficient methods for temporal discretization: an implicit discretization combined with a stationary iterative method, and an implicit-explicit (IMEX) discretization employing three levels in time [19]. For the early exercise constraint, we employ the operator splitting method [15, 17]. The operator splitting method is particularly useful when combined with the IMEX discretization, since it leads to a direct method for pricing American options; see [25] for a comparison. Alternative approaches for treating the early exercise under jump-diffusion models in the literature are a penalty method [14], a fixed point iteration [24], and a front-tracking method [28], for example.

Dividend payment on the underlying asset is usually assumed to be paid continuously over the lifetime of the option. This approximation is sufficiently accurate when the time to maturity of the option is long, but for shorter maturities it is inaccurate. This is especially true for American options, since the early exercise region is sensitive to the effects of dividend payment. In practice, dividend payment is a discrete event and financial practitioners have an expectation on the dividend amount already before the declaration date. In this paper, we include a discrete dividend into the jump-diffusion model. We consider the case of fixed discrete dividend and stochastic discrete dividend, and compare the obtained numerical results.
2 JUMP-DIFFUSION MODEL FOR STOCK PRICE

A jump-diffusion model for the price \( x \) of a stock reads

\[
dx(t) = \mu x dt + \sigma x dw(t) + x J dN(t),
\]

where \( \mu \) is a growth rate, \( \sigma \) is a volatility, \( w(t) \) is a Wiener process, \( J \) is an independent identically distributed random variable with a predefined distribution, and \( N(t) \) is a Poisson process.

The value \( v \) of a European option can be obtained by solving a partial integro differential equation (PIDE)

\[
v_t = \frac{1}{2} \sigma^2 x^2 v_{xx} + (r - \lambda \kappa) x v_x - (r + \lambda) v + \lambda \left( \int_0^\infty v(xy, \tau) f(y) dy \right) =: Lv
\]

for all \((x, \tau) \in [0, \infty) \times (0, T]\). Here, \( \tau = T - t \) is the time to the maturity, \( r \) is the risk free interest rate, \( f \) is the jump density function, \( \lambda \) is the jump arrival rate, and \( \kappa \) is the expected relative jump size.

The value at maturity \( v(x, 0) \) is given by a payoff function \( g \). For a put option with the strike price \( K \), it is

\[
g(x) = \max\{K - x, 0\}.
\]

The value \( v \) for an American option can obtained by solving the linear complementarity problem (LCP)

\[
\begin{align*}
(v_t - Lv) &\geq 0, \\
(v - g) &\geq 0,
\end{align*}
\]

The constraint \( v \geq g \) is called the early exercise constraint. The boundary between the regions in which \( v = g \) and \( v_t = Lv \) is called the early exercise boundary.

Under Kou’s model \[18\], jump sizes follow a log-double-exponential distribution with the density function

\[
f(y) = \begin{cases} 
q \alpha_2 y^{\alpha_2 - 1}, & y < 1, \\
p \alpha_1 y^{-\alpha_1 - 1}, & y \geq 1,
\end{cases}
\]

where \( p, q, \alpha_1 > 1 \), and \( \alpha_2 \) are positive constants such that \( p + q = 1 \). Another popular model is Merton’s jump-diffusion model \[21\] in which the jumps have a log-normal distribution. Both these models have finite activity jumps. We will consider only the finite activity case in the following and especially Kou’s model.

3 DIVIDEND

3.1 Discrete dividend

Many stocks pay dividends. Let \( \tau_d \) be a dividend date when a fixed amount \( D \) is paid to the stockholder. When the dividend is paid the value of the stock drops by \( D \). From an arbitrage argument, it follows that option prices have to be continuous across the dividend date, that is,

\[
v(x, \tau_d^+) = v(x - D, \tau_d^-),
\]

where \( \tau_d^- \) is just below \( \tau_d \) and \( \tau_d^+ \) is just above \( \tau_d \). The jump condition \(6\) can be incorporated easily into the following finite difference methods using an interpolation at the dividend date.
3.2 Stochastic discrete dividend

The dividend amount is declared at the declaration date. Before this date, the market participants have their expectations on the dividend \( D \). This expectation can be described by a probability density function \( s(D) \) which tells how likely different dividend amounts \( D \) are. The function \( s \) is such that
\[
\int_{0}^{\infty} s(D)dD = 1 \quad \text{and} \quad s(D) = 0, \quad \forall D < 0.
\] (7)

The continuity of the option prices implies
\[
v(x, \tau^+) = \int_{0}^{\infty} v(x - D, \tau^-)s(D)dD.
\] (8)

The jump condition (8) can be also incorporated into the following finite difference methods. In this case, an approximation of the integral will give the grid point values.

As an example of the probability density function \( s \), we consider a truncated normal distribution given by
\[
s(D) = \begin{cases} 
0, & D < 0, \\
\rho e^{-\frac{1}{2} \frac{(D-\mu)^2}{2}}, & D \geq 0,
\end{cases}
\] (9)

where \( \mu \) and \( \delta \) are positive constants, and \( \rho \) is a normalization constant such that \( \int_{0}^{\infty} s(D)dD = 1 \).

4 SPACE DISCRETIZATION

The spatial derivatives are approximated using finite differences. First, the infinite space domain is truncated to \([0, X]\) with a sufficiently large \( X \). A grid with \( n \) grid points
\[
0 = x_1 < x_2 < \cdots < x_n = X
\] (10)
is used. The spatial derivatives of the PIDE are approximated with nonuniform grid generalized central-differences
\[
v_x(x_i, \tau) \approx \frac{v_{i+1}(\tau) - v_{i-1}(\tau)}{\Delta x_{i-1} + \Delta x_i}
\] (11)
and
\[
v_{xx}(x_i, \tau) \approx \frac{2[\Delta x_{i-1}v_{i+1}(\tau) - (\Delta x_{i-1} + \Delta x_i)v_i(\tau) + \Delta x_iv_{i-1}(\tau)]}{\Delta x_{i-1}\Delta x_i(\Delta x_{i-1} + \Delta x_i)},
\] (12)
where \( v_i(\tau) = v(x_i, \tau) \) and \( \Delta x_i = x_{i+1} - x_i \).

4.1 Quadrature for integral term in Kou’s model

The discretization of the integral
\[
\lambda \int_{0}^{X} v(xy, \tau)f(y)dy
\] (13)
leads to a full matrix which we denote by \( J \). In the following, we describe an approximation for the integral. By making the change of variable \( y = z/x \), we obtain
\[
I = \int_{\mathbb{R}_+} v(xy, \tau)f(y)dy = \int_{\mathbb{R}_+} v(z, \tau)f(z/x)/x dz.
\] (14)
We decompose the integral as \( I = I^- + I^+ \), where

\[
I^- = \int_0^x v(z, \tau) f(z/x) dz = q \alpha_2 x^{-\alpha_2} \int_0^x v(z, \tau) z^{\alpha_2-1} dz
\]  

(15)

and

\[
I^+ = \int_x^\infty v(z, \tau) f(z/x) dz = p \alpha_1 x^{\alpha_1} \int_x^\infty v(z, \tau) z^{-\alpha_1-1} dz.
\]  

(16)

At each grid point \( x_i, i = 1, \ldots, n - 1 \), we need to approximate \( I^-_i = q \alpha_2 x^{-\alpha_2} \int_0^{x_i} v(z, \tau) z^{\alpha_2-1} dz = \sum_{j=0}^{i-1} I^-_{i,j} \),

(17)

where

\[
I^-_{i,j} = q \alpha_2 x_i^{-\alpha_2} \int_{x_j}^{x_{j+1}} v(z, \tau) z^{\alpha_2-1} dz.
\]  

(18)

The linear interpolation for \( v \) between grid points leads to the approximation

\[
I^-_i \approx A^-_i = \sum_{j=0}^{i-1} A^-_{i,j},
\]  

(19)

where

\[
A^-_{i,j} = q \alpha_2 x_i^{-\alpha_2} \int_{x_j}^{x_{j+1}} \left( \frac{x_{j+1} - z}{\Delta x_j} v(x_j, \tau) + \frac{z - x_j}{\Delta x_j} v(x_{j+1}, \tau) \right) z^{\alpha_2-1} dz.
\]  

(20)

By performing the integration, we obtain

\[
A^-_{i,j} = \frac{q x_i^{-\alpha_2}}{(\alpha_2 + 1)\Delta x_j} \left[ (x_{j+1}^{\alpha_2+1} - (x_{j+1} + \alpha_2 \Delta x_j)x_j^{\alpha_2}) v(x_j, \tau) \right.

+ \left. (x_j^{\alpha_2+1} - (x_{j-1} - \alpha_2 \Delta x_j)x_{j+1}^{\alpha_2}) v(x_{j+1}, \tau) \right]
\]  

(21)

for \( j = 1, \ldots, i-1 \) and

\[
A^-_{i,0} = \frac{q x_i^{-\alpha_2}}{\alpha_2 + 1} \left[ x_1^{\alpha_2} v(0, \tau) + \alpha_2 x_1^{\alpha_2} v(x_1, \tau) \right].
\]  

(22)

The integral \( I^+ \) is approximated in the same way.

For European options, we obtain a semi-discrete set of equations

\[
v_\tau = Dv + Jv - (r + \lambda)v,
\]  

(23)

where \( D \) is a tridiagonal matrix resulting from the differential operator and \( J \) is a full matrix resulting from the integral operator. For American options, we obtain a semi-discrete LCP

\[
\begin{cases}
(v_\tau - Dv - Jv + (r + \lambda)v) \geq 0, & v \geq g, \\
(v_\tau - Dv - Jv + (r + \lambda)v)^T(v - g) = 0.
\end{cases}
\]  

(24)
5 TIME DISCRETIZATION

5.1 Implicit time discretization

The Rannacher time-stepping scheme \cite{23} takes one or few first times with the implicit Euler method and the rest of steps with the Crank-Nicolson method; see \cite{12} also. The purpose of implicit Euler steps is to damp the highly oscillatory error in the numerical solution which is not damped effectively by the Crank-Nicolson steps. This scheme is second-order accurate. For European options, this leads to

\[ B^{(k)} v^{(k)} = b^{(k)}, \quad k = 1, \ldots, m, \]  
(25)

where

\[ B^{(k)} = I - \theta_k \Delta \tau (D + J - (r + \lambda)I) \]  
(26)

and

\[ b^{(k)} = [I + (1 - \theta_k) \Delta \tau (D + J - (r + \lambda)I)] v^{(k-1)}, \]  
(27)

where the parameter \( \theta_k \) is

\[ \theta_k = \begin{cases} 1, & k = 1, \\ \frac{1}{2}, & k = 2, \ldots, m. \end{cases} \]  
(28)

For American options, this leads to

\[ \begin{cases} (B^{(k)} v^{(k)} - b^{(k)}) \geq 0, \\ (B^{(k)} v^{(k)} - b^{(k)})^T (v^{(k)} - g) = 0, \end{cases} \]  
(29)

for \( k = 1, \ldots, m \). A drawback of implicit time steps is that at each time step, a system needs to be solved with the full matrix \( B^{(k)} \). While this can be performed efficiently, it would be even more efficient if it can be avoided.

5.2 Implicit Explicit Time Discretization

We propose to treat differential part \( Dv \) implicitly and the rest \( Jv - (r + \lambda)v \) explicitly. Such an implicit explicit (IMEX) time discretization has been considered in \cite{19,25}. An earlier first-order IMEX method was used in \cite{10}. An approximation for the time derivative reads

\[ v_{\tau} \approx \frac{1}{2\Delta \tau} (v^{(k)} - v^{(k-2)}). \]  
(30)

The differential part is approximated as

\[ Dv \approx \frac{1}{2} (Dv^{(k)} + Dv^{(k-2)}) \]  
(31)

and the rest is approximated as

\[ Jv - (r + \lambda)v \approx Jv^{(k-1)} - (r + \lambda)v^{(k-1)}. \]  
(32)

For European options, the above leads to

\[ C^{(k)} v^{(k)} = c^{(k)}, \quad k = 1, \ldots, m, \]  
(33)
where for \( k > 1 \)
\[
C^{(k)} = \frac{1}{2}I - \frac{1}{2} \Delta \tau D
\]
and
\[
c^{(k)} = \Delta \tau (J - (r + \lambda)I)v^{(k-1)} + \left[ \frac{1}{2}I + \frac{1}{2} \Delta \tau D \right] v^{(k-2)}.
\]
For \( k = 1 \), we have
\[
C^{(1)} = I - \Delta \tau D
\]
and
\[
c^{(1)} = [I + \Delta \tau (J - (r + \lambda)I)] v^{(0)}.
\]
For American options, this leads to
\[
\begin{cases}
(C^{(k)}v^{(k)} - c^{(k)}) \geq 0, \\
(C^{(k)}v^{(k)} - c^{(k)})^T (v^{(k)} - g) = 0,
\end{cases}
\]
for \( k = 1, \ldots, m \). Now it is necessary to solve problems with the tridiagonal matrices \( C^{(k)} \). Thus, only multiplications need to be performed with the full matrix \( J \).

6 OPERATOR SPLITTING METHOD FOR AMERICAN OPTIONS

The operator splitting method \([15]\) for LCPs \((29)\) decouples the early exercise constraint \( v^k \geq g \) and the PIDE into two separate steps. Formulating the LCP \((29)\) with a Lagrange multiplier \( \lambda^{(k)} \) leads to
\[
\begin{cases}
B^{(k)}v^{(k)} - b^{(k)} = \lambda^{(k)}, \\
\lambda^{(k)} \geq 0, \\
v^{(k)} \geq g, \\
(\lambda^{(k)})^T (v^{(k)} - g) = 0.
\end{cases}
\]

The operator splitting method yields the following two steps:
\[
B^{(k)}\tilde{v}^{(k)} - b^{(k)} = \lambda^{(k-1)}
\]
and
\[
\begin{cases}
v^{(k)} - \tilde{v}^{(k)} = \Delta \tau \left( \lambda^{(k)} - \lambda^{(k-1)} \right), \\
\lambda^{(k)} \geq 0, \\
v^{(k)} \geq g, \\
(\lambda^{(k)})^T (v^{(k)} - g) = 0.
\end{cases}
\]
The first step requires solving a linear problem instead of an LCP. The second step can be performed efficiently and easily componentwise. Under realistic assumptions, the method can be shown to maintain the same order-order accuracy as the Crank-Nicolson method \([17]\). For the IMEX time discretization, the operator splitting method is obtained by replacing \( B^{(k)} \) and \( b^{(k)} \) by \( C^{(k)} \) and \( c^{(k)} \) in \((40)\).

With the implicit time discretization, the operator splitting method leads to solving a system of linear equations with a full matrix. They can be solved efficiently by a stationary iterative method \([2, 26, 27]\)
\[
v^{l+1} = T^{-1} (b^{(k)} + \lambda^{(k-1)} + \theta_k \Delta \tau Jv^l), \quad l = 0, 1, \ldots,
\]
where \( T = I - \theta_k \Delta \tau (D - (r + \lambda)I) \) is a tridiagonal matrix. Thus, one iteration requires solving a tridiagonal system and a multiplication by the full matrix \( J \). In practice, only a few iterations are required for sufficient accuracy.
7 NUMERICAL EXAMPLES

We present numerical results pricing American put options under Kou’s jump-diffusion model. We use the model parameters

\[
\sigma = 0.15, \quad r = 0.05, \quad T = 0.25, \quad K = 100,
\]

\[
\lambda = 0.1, \quad \alpha_1 = 3.0465, \quad \alpha_2 = 3.0775, \quad \text{and} \quad p = 0.3445,
\]

which were used in [14, 19, 24, 25, 27, 28]. The corresponding jump distribution \( f \) is shown in Figure 1. The first example is without dividend. In the second example, 50 cent dividend is paid one month before the expiry of the option, that is, \( D = 0.50 \) and \( \tau_d = \frac{1}{12} \). The third example has a stochastic dividend with the distribution \( s \) in (9) with the parameters \( \mu = 0.5 \) and \( \delta = 0.4 \). The function \( s \) is shown in Figure 2. We set the truncation boundary to be at \( X = 400 \). We use uniform space and time steps. Our reference prices computed using fine discretizations are given in Table 1.

![Kou’s jump density f(y)](image)

Table 1: Reference prices for American put options.

<table>
<thead>
<tr>
<th>Dividend</th>
<th>( x = 90 )</th>
<th>( x = 100 )</th>
<th>( x = 110 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>10.005071</td>
<td>2.807879</td>
<td>0.561876</td>
</tr>
<tr>
<td>Given</td>
<td>10.118920</td>
<td>3.022921</td>
<td>0.605666</td>
</tr>
<tr>
<td>Stochastic</td>
<td>10.189990</td>
<td>3.064947</td>
<td>0.614725</td>
</tr>
</tbody>
</table>

The integrals resulting from Kou’s model are computed using the recursion formulas in [27]. With the stochastic dividend the integration (8) needs to be performed once for all grid points when the dividend is paid. Due to the exponential decay of the probability density function \( s \), a good approximation is obtained by integrating only over a reasonably small number of grid intervals. Thus, the jump due to the stochastic dividend payment can be computed as a matrix multiplication by a banded matrix. The experiments are performed on a PC with 2 GHz Intel Core i7 processor and the implementation was done using C language.
Tables 2, 3, and 4 report the results for the three examples on five different space-time grids. The columns of the tables give the $l_2$ pricing errors for $x = 90, 100, 110$, their ratios for consecutive discretizations, the number of total iterations, and run times in milliseconds. For the iterations (42) with the implicit time discretization, we have used the stopping criterion $\|\text{residual vector}\|_2 \leq 10^{-8}\|\text{right hand side vector}\|_2$. Based on the ratios close to four on finer grids, we conclude the discretization are roughly second-order accurate. With a given dividend, the accuracies are fairly good when compared to the two other examples, but the reduction of error is more erratic. With the implicit time discretization on finer grids, two iterations per time step are required to satisfy the stopping criterion, while on coarser grids three iterations are required. The accuracies of the implicit and IMEX discretizations are very similar, but the computations with the IMEX discretizations are about 1.5 times faster.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$m$</th>
<th>$\text{Error}$</th>
<th>$\text{Ratio}$</th>
<th>$\text{Iter}$</th>
<th>$\text{Time}$</th>
<th>$\text{Error}$</th>
<th>$\text{Ratio}$</th>
<th>$\text{Time}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>36</td>
<td>0.1242</td>
<td>4.20</td>
<td>108</td>
<td>0.7</td>
<td>0.1232</td>
<td>4.25</td>
<td>0.4</td>
</tr>
<tr>
<td>201</td>
<td>72</td>
<td>0.0310</td>
<td>4.01</td>
<td>216</td>
<td>2.3</td>
<td>0.0306</td>
<td>4.03</td>
<td>1.0</td>
</tr>
<tr>
<td>401</td>
<td>144</td>
<td>0.0089</td>
<td>3.48</td>
<td>289</td>
<td>4.6</td>
<td>0.0087</td>
<td>3.52</td>
<td>3.0</td>
</tr>
<tr>
<td>801</td>
<td>288</td>
<td>0.0020</td>
<td>4.39</td>
<td>576</td>
<td>14.8</td>
<td>0.0019</td>
<td>4.52</td>
<td>10.6</td>
</tr>
<tr>
<td>1601</td>
<td>576</td>
<td>0.0005</td>
<td>3.87</td>
<td>1152</td>
<td>56.0</td>
<td>0.0005</td>
<td>3.96</td>
<td>39.6</td>
</tr>
</tbody>
</table>

Table 2: American put options without dividend.

Figure 3 shows the early exercise boundaries for the three examples. The early exercise is not optimal before the dividend payment. The time interval without early exercise coincides well with the one given by a formula in [22]. The proposed operator splitting methods are robust, accurate, and fast also with given or stochastic dividends. Thus, the more complicated structure of the early exercise region does not cause difficulties with the operator splitting method.
Table 3: American put options with given discrete dividend.

| $n$ | $m$ | Implicit | | Implicit Explicit |
|-----|-----|----------|-----------------|
|     |     | Error    | Ratio | Iter | Time | Error | Ratio | Time |
| 101 | 36  | 0.0951   | 6.56  | 108  | 0.7  | 0.0947| 6.60  | 0.4  |
| 201 | 72  | 0.0106   | 8.96  | 210  | 2.2  | 0.0105| 9.03  | 1.3  |
| 401 | 144 | 0.0021   | 5.05  | 346  | 5.1  | 0.0022| 4.77  | 3.6  |
| 801 | 288 | 0.0021   | 1.02  | 576  | 13.9 | 0.0020| 1.10  | 10.4 |
| 1601| 576 | 0.0005   | 3.92  | 1152 | 55.2 | 0.0005| 3.94  | 38.7 |

Table 4: American put options with stochastic discrete dividend.

| $n$ | $m$ | Implicit | | Implicit Explicit |
|-----|-----|----------|-----------------|
|     |     | Error    | Ratio | Iter | Time | Error | Ratio | Time |
| 101 | 36  | 0.0915   | 6.96  | 108  | 0.7  | 0.0912| 6.99  | 0.4  |
| 201 | 72  | 0.0115   | 9.97  | 209  | 2.2  | 0.0113| 8.10  | 1.4  |
| 401 | 144 | 0.0031   | 3.73  | 356  | 5.2  | 0.0029| 3.83  | 3.7  |
| 801 | 288 | 0.0009   | 3.50  | 576  | 14.3 | 0.0008| 3.53  | 10.8 |
| 1601| 576 | 0.0002   | 3.78  | 1152 | 54.7 | 0.0002| 3.75  | 39.0 |

Figure 3: Early exercise boundaries
8 CONCLUSIONS

An efficient finite difference method for pricing American options with and without dividends is described. The early exercise constraint is treated using an operator splitting method which gives an easy, robust, and accurate way to price American options.

The implicit treatment of the jump term leads to solution of problems with full matrices. These problems can be solved iteratively requiring only multiplications by a full matrix. Such iterations converge rapidly with a few iterations per time step. The finite activity jump term can be treated explicitly as long as the differential part is treated implicitly. The explicit treatment of jumps leads roughly to the same accuracy, but it is much faster than the implicit treatment.

A stochastic discrete dividend is formulated using a distribution for the expected dividend. Fixed and stochastic discrete dividends can be incorporated into the finite difference method easily via a jump condition at the dividend date. The increase of computation due the dividend is small and the accuracy of the discretizations is roughly the same with dividends. In the numerical experiments, penny accuracy for prices is obtained in a few milliseconds on a PC for American put options under Kou’s model.

REFERENCES


