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EVOLUTIONARY SHAPE OPTIMIZATION IN CFD WITH INDUSTRIAL APPLICATIONS

Jari P. Hämäläinen*, Raino A.E. Mäkinen†, Pasi Tarvainen‡, and Jari Toivanen†

*Valmet Corporation, P.O. Box 587, FIN-40101 Jyväskylä, Finland, E-mail: Jari.P.Hamalainen@valmet.com, Web page: http://www.valmet.com

†University of Jyväskylä, Department of Mathematical Information Technology, P.O. Box 35 (MaE), FIN-40351 Jyväskylä, Finland, E-mail: Raino.Makinen@mit.jyu.fi, Jari.Toivanen@mit.jyu.fi, Web page: http://www.mit.jyu.fi

‡Numerola Oy (Ltd.), Väinönkatu 11A, FIN-40100 Jyväskylä, Finland, E-mail: Pasi.Tarvainen@numerola.fi, Web page: http://www.numerola.fi

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Abstract. *The solution of two optimal shape design problems related to industrial CFD using genetic algorithms are considered. The first one is a single objective optimization problem, where the geometry of a flow divider of a paper machine headbox is designed subject to prescribed goals and restrictions. The second problem is a two-dimensional airfoil design problem, where the objectives are to minimize the drag and the electromagnetic backscatter while the lift is larger than a given value. The flow and backscatter are modeled by the thin-layer Navier-Stokes equations and the time-harmonic Maxwell equations, respectively. Hence, this is a multiobjective and multidisciplinary shape optimization problem. Numerical experiments demonstrate capability of the genetic algorithms to solve these two industrial optimal shape design problems.*

1 INTRODUCTION

During the last decades, optimal shape design problems have been actively studied; see [13], [19], for example. Also in industry, numerical optimal shape design is becoming more and more important. This is especially the case with industrial problems related to fluid dynamics. In some cases, it has been possible to replace experimental work with the use of computational fluid dynamics (CFD). For example, the design of airplanes is nowadays largely based on CFD and optimal shape design. Also, in paper machine industry, the CFD-based optimal shape design is an important part of the design process.

Traditionally, the design has been optimized with respect to only one criterion and only one discipline such as elasticity, electromagnetics or aerodynamics. It has been recognized that, in many cases, this is not enough. For example, it is advantageous to consider the aerodynamic and elastic properties together during the design of airplane. Also, there are naturally several criteria in this kind of problems. The drag of an airplane should be minimal, but, at the same time, lift should be sufficient. Therefore, it is beneficial to study the tradeoff between drag and lift as a multicriteria or multiobjective optimization problem.

Optimal shape design problems lead to nonlinear optimization problems. Descent algorithms based on gradient information are typically used to numerically solve the resulting problems. The sensitivity analysis gives a way to compute the gradient of the objective function, but it is usually a very laborious and error prone task. Sometimes, it is practically impossible to do, since the analysis solver might have been designed in such a way that it does not allow to perform the sensitivity analysis without major rewriting of the solver. In such cases, gradient-free optimization methods, like genetic algorithms (GAs), are preferable choices. GAs offer additional robustness, since they do not impose any regularity requirements on objective functions. Moreover, as GAs are global optimization methods they can find new innovative designs instead of traditional designs corresponding to local minima.

In this paper, we consider two industrial optimal shape design problems that both are CFD-based problems of quite different nature. Namely, the first problem is based on internal incompressible flow and there is one objective function. In the second one, the first objective function is based on external and compressible flow while the other one is related to electromagnetics. Thus, the second problem is a multiobjective and multidisciplinary optimal shape design problem. Despite of different origins of the problems, they both can be solved using rather similar GAs.

The first problem deals with the design of a divider that directs laminar flow into three outlets evenly. The most important flow divider in a paper machine headbox is the header, which has been modeled and optimized in several references [2], [8], [9], [10], [11], [17], [23], [24]. There are also other flow divider problems in a paper machine, for example, dividing of dilution water feed into several outlets. In different applications, there are different dimensions from centimeter to metre scales, there might be a recirculation flow or not, but from shape optimization point of view, they are quite similar problems. Our goal here is to test GAs with an INGENET testcase flow divider problem in order to develop a generic shape optimization tool for different industrial applications. Instead of using a turbulent multiphase flow model (mixture of water, wood fibres, etc.), we are using a simpler one-phase laminar flow model.

In the second shape optimization problem, we solve a multidisciplinary problem in which an airfoil should have good aerodynamical and electromagnetic properties. More precisely, in this multiobjective optimization problem the drag and the electromagnetic backscatter of the airfoil is minimized while keeping sufficient lift. Thus, the solution of this problem requires a specialized method suitable for multiobjective problems; see, for example, [3]. Similar problems have been studied in [1], [15], [16], [20], for instance. The flow is modeled using a thin-layer Reynolds-averaged Navier-Stokes equations, while the computation of backscatter is based on the time-harmonic Maxwell equations.

The paper is organized as follows: The following section describes the use of genetic algorithms for the solution of a single objective shape optimization and their modifications for the multiobjective case. In Section 3, we consider a shape optimization problem for a flow divider. In the following section, we solve a multidisciplinary and multiobjective INGENET testcase shape optimization problem for an airfoil profile. Some concluding remarks and suggestions are given in Section 5.

2 GENETIC ALGORITHMS APPLIED TO SHAPE OPTIMIZATION

2.1 Single objective shape optimization using GAs

We consider the following (abstract) shape optimization problem

$$\text{minimize } f(a,u) \tag{1}$$

$$\text{subject to } c(a,u) = 0 \tag{2}$$

$$a \in U_{ad}, \tag{3}$$

where a is the design variable, U_{ad} is the set of admissible designs, $u = u(a)$ is the state of the system to be optimized, (2) is the state equation, and $f(a,u)$ is the objective (or cost) function.

GAs give a robust way of representing solutions of a problem in terms of a population of digital chromosomes that are modified during the reproduction by the random operations of crossover and mutation. In shape optimization problems, the fitness function measuring the goodness of design is naturally defined by the objective function. GAs work with a population of individuals which are designs in our case. We use a traditional GA which replaces the entire parent population by its offsprings. An exception to this is the elitism mechanism which copies a few best parents to the new population. In this way, we do not lose the best designs in reproduction and convergence becomes monotonic.

In classical GAs, a binary coding is used for genes [6]. Instead, we use here floating point coding [18]. This is more natural, since the genes are here real-valued design variables. An offspring is obtained by performing crossover of two parents and, possibly, a mutation. We have decided to use one point crossover and a special mutation promoting small mutations; see [15]. The parents for the reproduction are chosen using a tournament selection.

2.2 Multiobjective shape optimization using GAs

We consider the following multiobjective shape optimization problem

$$\text{minimize } \{f_1(a, u_1, \ldots, u_n),\ \ldots,\ f_m(a, u_1, \ldots, u_n)\} \tag{4}$$

$$\text{subject to } c_1(a, u_1) = 0,\ \ldots,\ c_n(a, u_n) = 0 \tag{5}$$

$$a \in U_{ad}. \tag{6}$$

Again, a is the design variable and U_{ad} is the set of admissible designs. Now there are several state variables $u_1 = u_1(a), \ldots, u_n = u_n(a)$ to be optimized and they satisfy the set of state equations (5) which may correspond to several different disciplines such as elasticity, electromagnetics or aerodynamics. The aim is to minimize the values of the objective functions $f_1(a, u_1, \ldots, u_n), \ldots, f_m(a, u_1, \ldots, u_n)$.

In the multiobjective problems, a design is optimal when it is nondominated or, in other words, Pareto optimal. A design is nondominated, if no other feasible design exists which is better with respect to any objective, and it is at least equally good with respect to all the other objectives. Usually, in the practical solution of multiobjective problems, the task is to find several Pareto optimal solutions. This can be computationally a very laborious task. GAs can be adapted to this kind of problems and they are naturally parallel. Thus, GAs can be used efficiently in parallel computers which offer the required computing power. In this paper, we consider the solution of multiobjective optimization problem using a GA in a parallel computer.

For the multiobjective optimization problems, it is necessary to make some modifications to the basic GA. Our algorithm is based on the Nondominated Sorting GA (NSGA) [21]. We next describe very shortly the modifications to our algorithm when compared to the NSGA; for more details see [15]. For general discussion on GAs for multiobjective optimization, see [4] and references therein.

The fitness values are computed using the standard nondominated sorting employed by the NSGA. Our modified algorithm employs the tournament selection, unlike the NSGA which uses the roulette wheel selection. For each tournament, a fixed number of individuals are selected randomly. The individual having the highest fitness value wins the tournament, that is, it is selected to be a parent in the breeding. Unfortunately, if there were no modifications to the previous tournament selection, the population would usually converge towards one point on the set of Pareto optimal solutions whereas the aim was to obtain several points from the Pareto set. In our modified algorithm, the diversity of the population is preserved using the so-called tournament slot sharing which was introduced in [15]. The basic idea of this is to diminish the probability of the individual to enter to a tournament if the genotypic distance to the other individuals is small.

An elitist mechanism is added to our algorithm since it guarantees the objective function values to decrease monotonically from the previous generation to the next one. Also, it usually accelerates the convergence. This is implemented by copying from the old population to the new population all individuals which would be nondominated in the new population. As a coding, we have used the floating point coding [18]. The crossover is made using one crossover site, and a special mutation described in [15] is utilized.

3 SHAPE OPTIMIZATION OF A FLOW DIVIDER

3.1 Setting of the state problem

Let us consider the flow divider with three pipes shown in Figure 1. The divider consists of the inlet, pipes, and recirculation sections. The pipes section is divided into subsections by the pipes of width T_1, T_2, and T_3, respectively. Parabolic inflow enters the divider through Γ_{in}. There is also (small) parabolic outflow through Γ_r, the so-called recirculation. At the main outlets Γ_1, Γ_2, and Γ_3 of lengths P_1, P_2 and P_3, respectively, the normal and tangential stresses are given leading to Robin type boundary conditions. We assume that the flow is laminar and satisfies the steady-state Navier-Stokes equations

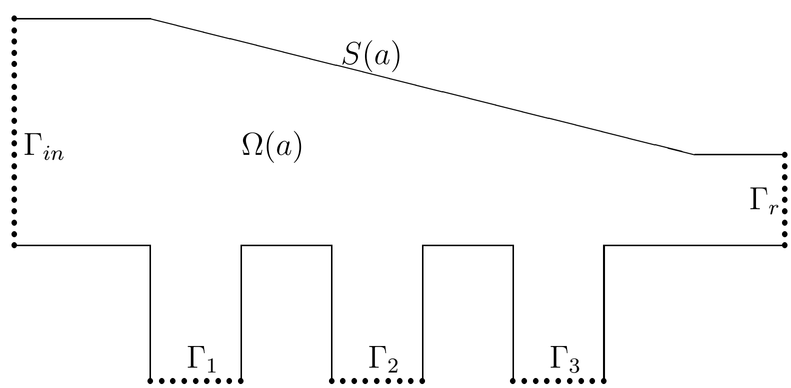

Figure 1: The geometry of the flow divider.

$$-\frac{1}{\mathrm{Re}}\Delta u + (u \cdot \nabla) u + \nabla p = 0 \quad \text{in } \Omega \tag{7}$$

$$\nabla \cdot u = 0 \quad \text{in } \Omega \tag{8}$$

with the boundary conditions

$$u = u_{in} \quad \text{on } \Gamma_{in} \tag{9}$$

$$u = u_r \quad \text{on } \Gamma_r \tag{10}$$

$$\nabla u_2 \cdot n - p\, n_2 = p_2 \quad \text{on } \Gamma_1 \cup \Gamma_2 \cup \Gamma_3, \tag{11}$$

where p_2 is an outlet pressure rate depending on the downsteam head losses and pressure rates [8], [11]. The Reynolds number defined at the inlet tube is Re = 100. In real situations, it is of order 10^5, but for simplicity, GAs are here tested only for a laminar case. For numerical simulation, the equations (7)–(11) are discretized by a stabilized FEM [5] and bilinear elements for the velocity components and pressure.

3.2 Setting of the shape optimization problem

The target of the design optimization is to find equal flow rates at the main outlets $\Gamma_i,\ i = 1, 2, 3$, by varying the back wall $S(a)$ of the divider. The shape of the back wall $S(a)$ is determined by the design variable vector $a = (a_1\ a_2\ \cdots\ a_m)^T$.

The object function measuring the quality of the outflow flow profile is determined by the standard deviation of the flow rates in the pipes, that is,

$$f(a) = s(Q_1(a), Q_2(a), Q_3(a)) = \sqrt{\frac{1}{2}\sum_{i=1}^{3}(Q_i(a) - \bar{Q}(a))^2}, \tag{12}$$

where Q_i, $i = 1, 2, 3$, denotes the flow rate associated with the pipe outlet Γ_i, defined by

$$Q_i = P_i \bar{u}_{i,2} = P_i \cdot \frac{1}{P_i}\int_{\Gamma_i} |u_{i,2}|\, ds = \int_{\Gamma_i} |u_{i,2}|\, ds,$$

and $\bar{Q}$ is the average outflow rate. The shape optimization problem can be formulated as a minimization problem

$$\min_{a\in U_{ad}} f(a) \text{ subject to (7)–(11)}, \tag{13}$$

where U_{ad} is the set of admissible design parameters.

Table 1: The parameters in the GA.

Parameter	Value
Population size	17
Generations	100
Tournament size	3
Crossover probability	0.8
Mutation probability	0.1
Mutation exponent	4

3.3 Numerical example

For the numerical example, we have chosen the following dimensions (in metres) for the divider: $H_1 = 1.0$, $L_1 = 1.0$, $L_2 = 7.0$, $L_3 = 0.5$ and $H_2 = 0.16$. The length and width of each pipe are $T_i = 0.5$ and $P_i = 1.0$ units ($i = 1, 2, 3$), respectively. The average

Table 2: Object function characteristics.

	Q_1	Q_2	Q_3	Mean value	Standard deviation
Initial flow rate	1.4842	1.2447	1.0514	1.2601	0.2168
Optimized flow rate	1.2546	1.2546	1.2546	1.2546	$1.7901 \cdot 10^{-5}$

inlet and recirculation velocities are 4.0 m/s and 0.5 m/s, respectively. The shape of the back wall $S(a)$ is defined by a Bézier curve with six control points, which are distributed uniformly in the x-direction and the y-coordinates of the control points are the six design variables. The problem is discretized on a mesh with about 2000 finite elements.

The input parameters for the GA are given in Table 1. The meaning of mutation exponent is explained in [15]. The first one of the initial shapes of the divider is the one corresponding to the linear back wall. The other 16 designs in the initial population are obtained by making some predetermined changes to the first shape.

According to Table 1, one optimization run forms 100 generations and each generation consists of 17 individuals. Since we use an elitism mechanism, the best individual is copied to a new generation and the other 16 are bred. Thus, the total number of flow problem solutions and object function evaluations is 1601. Due to the large amount of object function evaluations, one optimization run requires around 4 hours of wall clock time in an Intel Pentium PII 350 MHz with 128 MB memory. The flow rate profiles of the initial (linear back wall) and optimized dividers are shown in Figure 2 and the associated divider geometries with flow velocities in Figures 3 (initial) and 4 (optimized).

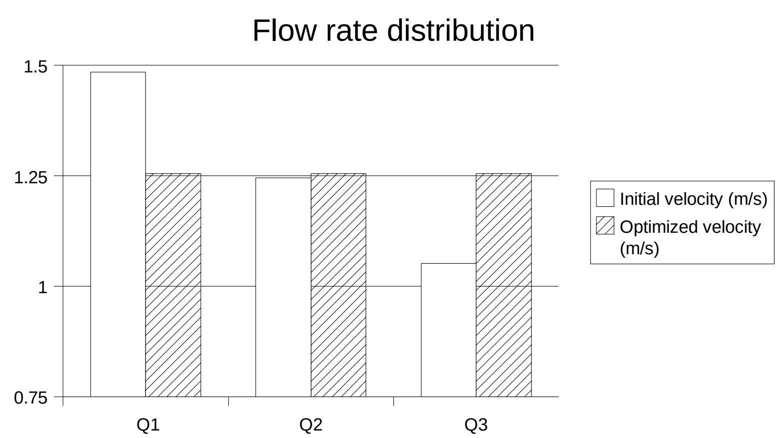

Figure 2: The flow rates in each pipe with initial and optimized geometries.

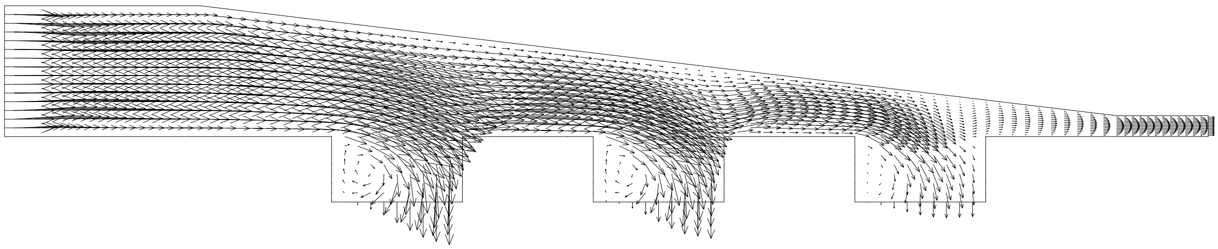

Figure 3: The velocity vectors of the initial flow divider.

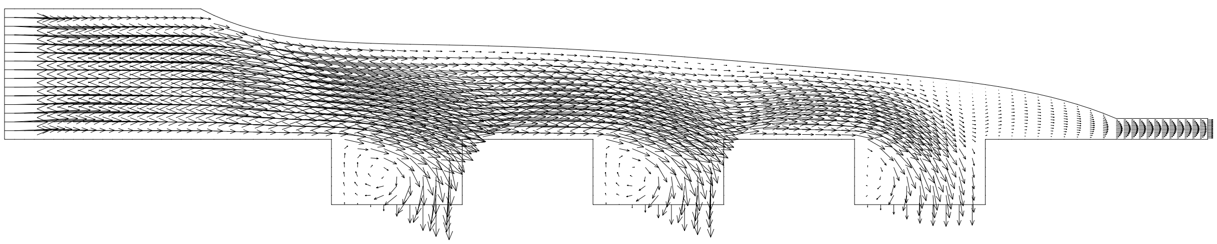

Figure 4: The velocity vectors of the optimized flow divider.

4 MULTIDISCIPLINARY SHAPE OPTIMIZATION OF AN AIRFOIL PROFILE

4.1 Setting of the state problems

Let us consider the airfoil shown in Figure 5. Its material is assumed to be perfectly conducting. An incoming electromagnetic wave is scattered from the airfoil. The aim of the numerical simulation is to determine the drag and lift coefficients and the radar cross section of the scattered wave.

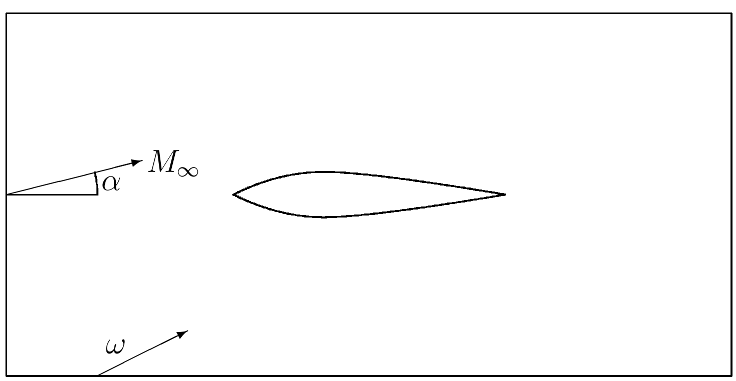

Figure 5: The problem geometry.

The flow is modeled by the two-dimensional Euler equations except a thin layer on the airfoil, where Reynolds-averaged Navier-Stokes equations based on the Cebeci-Smith model are used [22]. The Euler equations are

$$\frac{\partial W}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0, \tag{14}$$

where the vector of conservative variables W and the flux vectors F, G are

$$W = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix}, \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uH \end{pmatrix} \quad \text{and} \quad G = \begin{pmatrix} \rho v \\ \rho vu \\ \rho v^2 + p \\ \rho vH \end{pmatrix}, \tag{15}$$

where ρ is the density, u and v are the Cartesian velocity components, p is the pressure, E is the total energy and H is the total enthalpy. The problem (14) is solved numerically using FINFLO program [12] that uses the finite volume method, an implicit pseudo-time integration, and multigrid acceleration.

The wave scattering is modeled by the time-harmonic two-dimensional Maxwell equations, which can be reduced to the following Helmholtz equation with the Sommerfeld radiation condition on the far field boundary:

$$\Delta z + \omega^2 z = 0 \quad \text{in } \Omega \tag{16}$$

$$z = -z_0 \quad \text{on } \Gamma \tag{17}$$

$$\mathcal{S}(z) = 0 \quad \text{on } \Gamma_\infty. \tag{18}$$

The discretization is made using the linear finite elements. The domain is truncated and a second-order absorbing boundary condition is posed on the artificial far-field boundary. In (17), z_0 is the incident wave. A fictitious domain method is used to solve the resulting linear system [7], [16].

4.2 Setting of the optimization problem

In this multiobjective multidisciplinary design optimization problem, we minimize the drag coefficient C_d and the radar cross section at elevation angle of 10° of the backscattered wave while the lift coefficient C_l must be greater than a given constant C_l^0. This is one of the INGENET testcases defined by D. Spicer [20], except that we use an inequality constraint for the lift coefficient instead of an equality constraint. This modification makes the optimization problem easier to GAs.

The upper and lower surfaces of the airfoil are defined by

$$s_{upp}(x) = c(x) + t(x)$$
$$s_{low}(x) = c(x) - t(x),$$

respectively, where $0 \leq x \leq 1$ is the chordwise distance, $c(x)$ is the camber curve, and $t(x)$ is the half thickness of the airfoil. The camber curve is given by

$$c(x) = \begin{cases} \frac{1}{2}\beta - \frac{1}{8\beta} + \sqrt{\left(\frac{1}{2}\beta + \frac{1}{8\beta}\right)^2 - \frac{1}{4}(2x-1)^2}, & \beta > 0 \\ 0, & \beta = 0, \end{cases}$$

where β denotes the maximum camber. The thickness $t(x)$ is that of the RAE2822 airfoil.

In the corresponding optimization problem, the design variables are the angle α of attack and the maximum camber β. The set of admissible design variables is defined by

$$U_{ad} = \{a = (\alpha, \beta) \mid 0 \leq \alpha \leq \alpha_{max},\ 0 \leq \beta \leq \beta_{max}\},$$

where $\alpha_{\max} = 4$ and $\beta_{\max} = 0.04$. The set of the physically admissible designs is defined by

$$U^*_{ad} = \{a \in U_{ad} \mid C_l(a) \geq C_l^0\},$$

where $C_l^0 = 0.933$. Hence, we have a multiobjective minimization problem

$$\min_{\alpha \in U^*_{ad}} \{f_1(a),\ f_2(a)\}, \tag{19}$$

where $f_1(a) = C_d(a)$ and $f_2(a) = RCS(a, \theta_0 + \alpha)$, where $\theta_0 = 10$. The function $RCS(a, \theta_0 + \alpha)$ gives the radar cross section in the direction defined by the angle $\theta_0 + \alpha$. Thus, $f_2(a)$ measures the amplitude of backscattered electromagnetical wave.

The nonlinear lift constraint is taken into account by adding a quadratic penalty function to both object functions: Let ε be a small positive penalty parameter. Then, the penalized object functions are

$$f_i^\varepsilon(a) = f_i(a) + \frac{1}{\varepsilon}\max\{C_l(a) - C_l^0,\ 0\}^2, \quad i = 1, 2,$$

and the penalized multiobjective minimization problem reads

$$\min_{\alpha \in U_{ad}} \{f_1^\varepsilon(a),\ f_2^\varepsilon(a)\}. \tag{20}$$

4.3 Numerical results

In the flow model, the freestream Mach number is $M_\infty = 0.69$, and the Reynolds number is $\mathrm{Re} = 20 \times 10^6$. The transition from laminar flow to turbulent flow takes place at 5% on upper and lower surfaces. For discretization, we use 192×64 C-type grid with 128 grid points on the surface of airfoil. During the optimization process, the grid for the Navier-Stokes solver is depending continuously and smoothly on the design parameter β.

In the Helmholtz problem, the incident wave is given by

$$z_0 = \cos(\omega(\theta_0 + \alpha))\, x + i \sin(\omega(\theta_0 + \alpha))\, y \quad \text{with} \quad \omega = 40\pi.$$

This corresponds to a monostatic radar operating at 3 GHz. Thus, airfoils are 20 wave lengths long. The computational domain is truncated to be the rectangle $[-0.1, 1.1] \times [-0.3, 0.3]$ and the discretization is performed using 481×241 rectangular mesh with a local fitting on the surface of airfoil. Thus, there is 20 nodes per wave length. Due to the use of locally fitted mesh, the number of nodes and elements in the mesh usually varies during the optimization. Hence, the objective function f_2 computed using the finite element approximation is discontinuous.

We have scaled the second design variable β giving the maximum camber by 100 so that the admissible interval for both design variables is the same [0, 4]. The penalty parameter ε is 10^{-4}. The GA parameters are shown in Table 3. The sharing distance is related to the tournament slot sharing and the mutation exponent is related to the special mutation [15]. In one optimization run, 8192 fitness function values are computed. All designs in the initial population are randomly chosen.

Table 3: The parameters in the GA.

Parameter	Value
Population size	64
Generations	128
Tournament size	2
Sharing distance	0.25
Crossover probability	0.8
Mutation probability	0.2
Mutation exponent	4

The parallelization of the GA is done using a simple master-slave approach [14]. The computations are performed on a Beowulf cluster using 8 nodes with each having an Athlon 500 MHz processor with 128 MB memory and an Intel EtherExpress PRO/100 network card (100 Mbps) connected to a 3Com SuperStack II switch. The communication is performed using the MPI message passing library MPICH. The computation of one solution of the thin-layer Navier-Stokes equations and the Helmholtz equation required roughly 120 and 40 CPU seconds, respectively. The total wall clock time for one optimization run was about 50 hours.

After 128 generations, we obtained 254 nondominated designs which are shown in Figure 6. Due to the elitism, the number of nondominated designs is much larger than the size of population. This can be regarded as an indication that the optimization is approaching and may have already partly reached the Pareto set. As early as after 48 generations, the front defined by nondominated designs resembles closely the one obtained after 128 generations. The lift coefficients C_l of the nondominated designs in the last generation grow from 0.933 to 1.241 rather linearly with respect to the lift coefficients C_l. Thus, our designs do not satisfy the equality constraint imposed for the lift coefficient in [20].

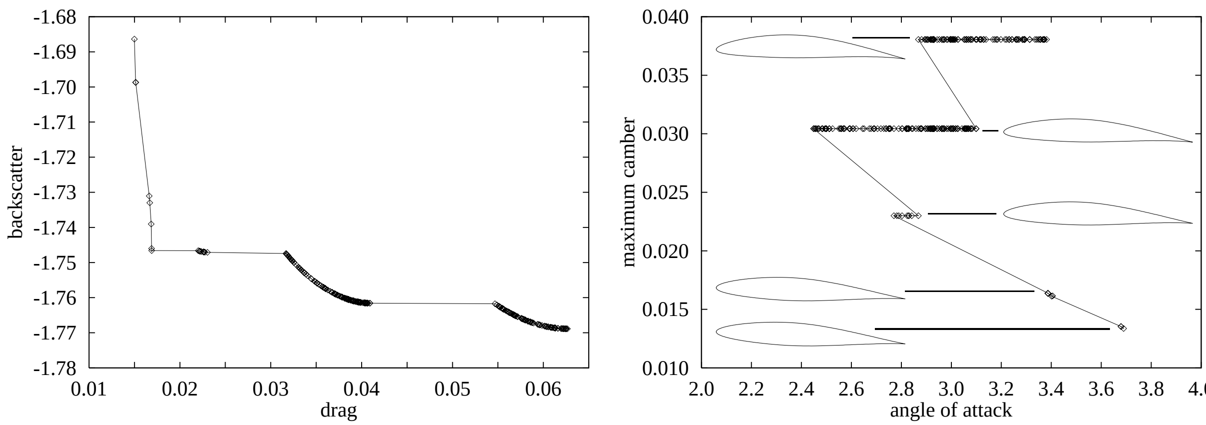

Figure 6: The nondominated designs from the last generation.

5 CONCLUSIONS

We have demonstrated capability of the GAs to treat industrial shape optimization problems in CFD with good results. Optimization together with CFD is still industrially and scientifically a very challenging problem. In the future, flow models will be refined and this makes the solution of CFD problems even more expensive. Thus, the effectiveness of the solution procedures for the nonlinear CFD and optimization problems must be increased in order to this approach to be costeffective also in the future for industrial design problems.

Since gradients are not required and the objective functions do not have to be continuous, we can use standard state solvers for shape optimization with GA. Also, it is easy to obtain a good speedup in parallel GA optimization with the standard sequential state solvers. GAs offer more robust and easy-to-use substitutes for gradient-based optimization methods. However, according to our experience, GAs cannot compete in efficiency with gradient based methods when the sensitivity analysis is performed for the state solver. Thus, more research is required in order to accelerate GAs convergence.

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