INVERTIBILITY VERSUS LAGRANGE EQUATION FOR TRACTION FREE ENERGY-MINIMAL DEFORMATIONS

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Let $\mathbb{X}, \mathbb{Y} \subset \mathbb{R}^2$ be bounded Jordan domains of the same topological type. We consider the class $\mathscr{M}_2(\mathbb{X}, \mathbb{Y})$ of monotone mappings $h : \mathbb{X} \xrightarrow{\text{onto}} \mathbb{Y}$ of finite Dirichlet energy. This class turns out to be the closure of homeomorphisms in the Sobolev $\mathscr{W}^{1,2}$-norm. We say that h is a traction free minimal deformation if

$$\int_{\mathbb{X}} |Dh(z)|^2 \, \mathrm{d}z \leqslant \int_{\mathbb{X}} |Dg(z)|^2 \, \mathrm{d}z \,, \quad \textit{for every } \; g \in \mathscr{M}_2(\mathbb{X}, \mathbb{Y}) \,.$$

The existence of such deformations is established for Lipschitz domains. Monotone Sobolev mappings need not be injective. However, for the energy-minimal deformations we show that $h : \mathcal{O} \to \mathbb{Y}$, restricted to any subdomain $\mathcal{O} \subset \mathbb{X}$, is injective if and only if it is harmonic in $\mathcal{O}$. Our intension is to endorse as yet unproved analogous principle of *interpenetration of matter* in hyperelastic materials of thin plates for more general variational integrals:

The loss of invertibility comes exactly with the loss of the Lagrange equation.