MAnET-workshop

Optimal mass transportation and related fields

Jyväskylä, August 12th 2017

Schedule

09:59 – 10:00	Opening
10:00 – 10:30	Anna Kausamo (Jyväskylä): *On multi-marginal optimal transport with repulsive costs*
10:30 – 11:00	Farhad Hatami (UAB): *Congested traffic problem; dynamics and regularity*
11:00 – 11:30	Coffee break
11:30 – 12:00	Luca Tamanini (SISSA and Paris-Ouest): *Schrödinger problem and W_2-geodesics*
12:00 – 12:30	Kinga Sipos (Bern): *Jacobian Determinant Inequality on Corank 1 Carnot Groups*
12:30 – 13:00	Gerardo Sosa (MPI Leipzig): *Group actions on metric measure spaces*
13:00 – 14:30	Lunch
14:30 – 15:00	Heikki Jylhä (UAB): *Uniqueness of solutions of continuity equations via optimal transport*
15:00 – 15:30	Walter Andrés Ortiz (UAB): *On the convex measure profile example : isoperimetry and symmetrization*
15:30 – 16:00	Coffee break
16:00 – 16:30	Giulio Trigila (NYU): *The barycenter problem for the explanation of variability in data sets*
16:30 – 17:00	Shirsho Mukherjee (Jyväskylä): *Regularity of Quasilinear equations on the Heisenberg group*

Abstracts

Anna Kausamo
(Jyväskylä)

On multi-marginal optimal transport with repulsive costs

Farhad Hatami (UAB)

Congested traffic problem; dynamics and regularity

During the last decade, transport phenomena and the Monge-Kantorovich mini- mization problem have been studied in a vast number of interdisciplinary areas. Our framework consists of the following very degenerate elliptic PDE

$$\int_{\Omega} \mathcal{A}(x, \nabla u(x)) = f,$$

where $\mathcal{A}$ is increasing with some ellipticity conditions and f is an L^s function. The next step will be to present our main theorem which gives an extended regularity for the gradient variable ∇u, and to see how this very degenerate elliptic PDE will appear in the traffic problem.

After introducing the scalar transport equation and its vectorial version, we aim at constructing the optimizer for each of these problems, using the optimizer of the scalar transport equation or the vectorial version respectively. Then we see the application of "Dacorogna-Moser" scheme to prove the equivalence between scalar and vector problem.

Extending the notion of transport problem to a non-autonomous setting, we con- sider our minimization problem dependence not only on "traffic intensity", but also on location x in the network.

Later on, in order to satisfy the regularity requirements for the "DiPerna-Lions" theory, we find a Sobolev regularity for our optimizer.

Luca Tamanini
(Université Paris Nanterre & SISSA)

Schrödinger problem and W_2-geodesics

Aim of this talk is to present the second order differentiation formula along geodesics in $\mathsf{RCD}^*(K, N)$ spaces with K allowed to be negative and $N < \infty$. This formula is new even in the context of Alexandrov spaces. In establishing such main theorem, we derive few auxiliary results which are interesting on their own, in particular:

- we obtain a wide range of new properties for the solutions of the dynamic Schrödinger problem, as for instance equiboundedness of the densities along the entropic interpolations and equi-Lipschitz continuity of the Schrödinger potentials;
- in accordance with the smooth case, we prove that the viscous solution of the Hamilton-Jacobi equation can be obtained, in the context of compact $\mathsf{RCD}^*(K, N)$ spaces, via a vanishing viscosity method.

Kinga Sipos
(Bern)

Jacobian Determinant Inequality on Corank 1 Carnot Groups

After establishing a weighted pointwise Jacobian determinant inequality on the Heisenberg group based on a mass transportation approach, we asked whether something similar holds also on the more general conrank 1 Carnot groups. In this new setting the abnormal geodesics need special treatment. Along them the Jacobian inequality exposes a purely Euclidean behavior, while out of abnormal geodesics - as in the Heisenberg group - a typical sub-Riemannian bahavior can be observed. Our proof shows that in addition to the Riemannian approximation argument - used earlier in case of Heisenberg groups -, a direct argument (similar to the one of Cordero-Erausquin, McCann and Schmuckenschläger) is also feasible. As applications, entropy, Brunn-Minkowski and Borell-Brascamp-Lieb inequalities can be concluded.

Gerardo Sosa
(MPI Leipzig)

Group actions on metric measure spaces

The goal is to give a short overview of recent research on metric measure spaces and symmetries of these spaces. More specifically, we will address the questions of when is the group of symmetries of a m.m. space *well-behaved*? And how can the analysis of these symmetries improve our comprehension of the spaces?

In the first part of the talk we study the existence of a differential structure on *symmetry groups* of metric measure spaces. For a class of m.m. spaces, we present a necessary and sufficient condition which guarantees the existence of such a smooth structure. As a consequence, we are able to recover classical results in Riemannian and metric geometry, and we provide new examples of spaces with smooth symmetry groups. Such is the case, for instance, of some spaces satisfying synthetic lower Ricci curvature bounds in the Lott-Sturm-Villani sense.

Motivated by these results, in the remainder of the talk we consider symmetric group actions on m.m. spaces with synthetic Ricci curvarture bounds. We show that synthetic weighted Ricci curvature bounds are preserved under the induced quotient maps. Finally, we glance into various novel applications of this previous result. The second part is based on a collaboration with Galaz-García, Kell, and Mondino.

Heikki Jylhä
(UAB)

Uniqueness of solutions of continuity equations via optimal transport

Walter Andrés Ortiz
(UAB)

On the convex measure profile example : isoperimetry and symmetrization

In the theory dedicated to metric spaces of probability, obtaninig Sobolev-Poincaré type inequalities, ussing isoperimetry and symmetrization. It is noteworthy that these have been studied for the isoperimetric concave profile; currently we focus in this type of isoperimetric inequalities with convex profile (Cauchy measure), adapting these characterizations in probability spaces with invariant rearrangement.

keywords: Rerragement invariant spaces, Convex measure, Isoperimetry, symmetrization

Giulio Trigila
(NYU)

The barycenter problem for the explanation of variability in data sets

A new data driven algorithm for the solution of the barycenter problem in optimal transport is presented. The algorithm is particularly well suited for the explanation of variability and removal of confounding factors from data sets defined in high dimensional spaces.

Shirsho Mukherjee
(Jyväskylä)

Regularity of Quasilinear equations on the Heisenberg group

A brief outline of local regularity of weak solutions for a wide class of Quasilinear equations, shall be discussed in this talk. This would be a generalization of the case for p-Laplace equations on the Heisenberg group.