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Comparison of DEC and DDA for electromagnetic scattering

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MATHEMATICAL MODEL

The time-dependent propagation of electromagnetic waves is presented by the hyperbolic system of the Maxwell equations

$$\varepsilon\frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{H} = -\sigma \mathbf{E} - \mathbf{J},$$
$$\mu\frac{\partial \mathbf{H}}{\partial t} + \nabla \times \mathbf{E} = \mathbf{0},$$

where ε is the electric permittivity, μ is the magnetic permeability, and σ is the electric conductivity.

DEC-based approach

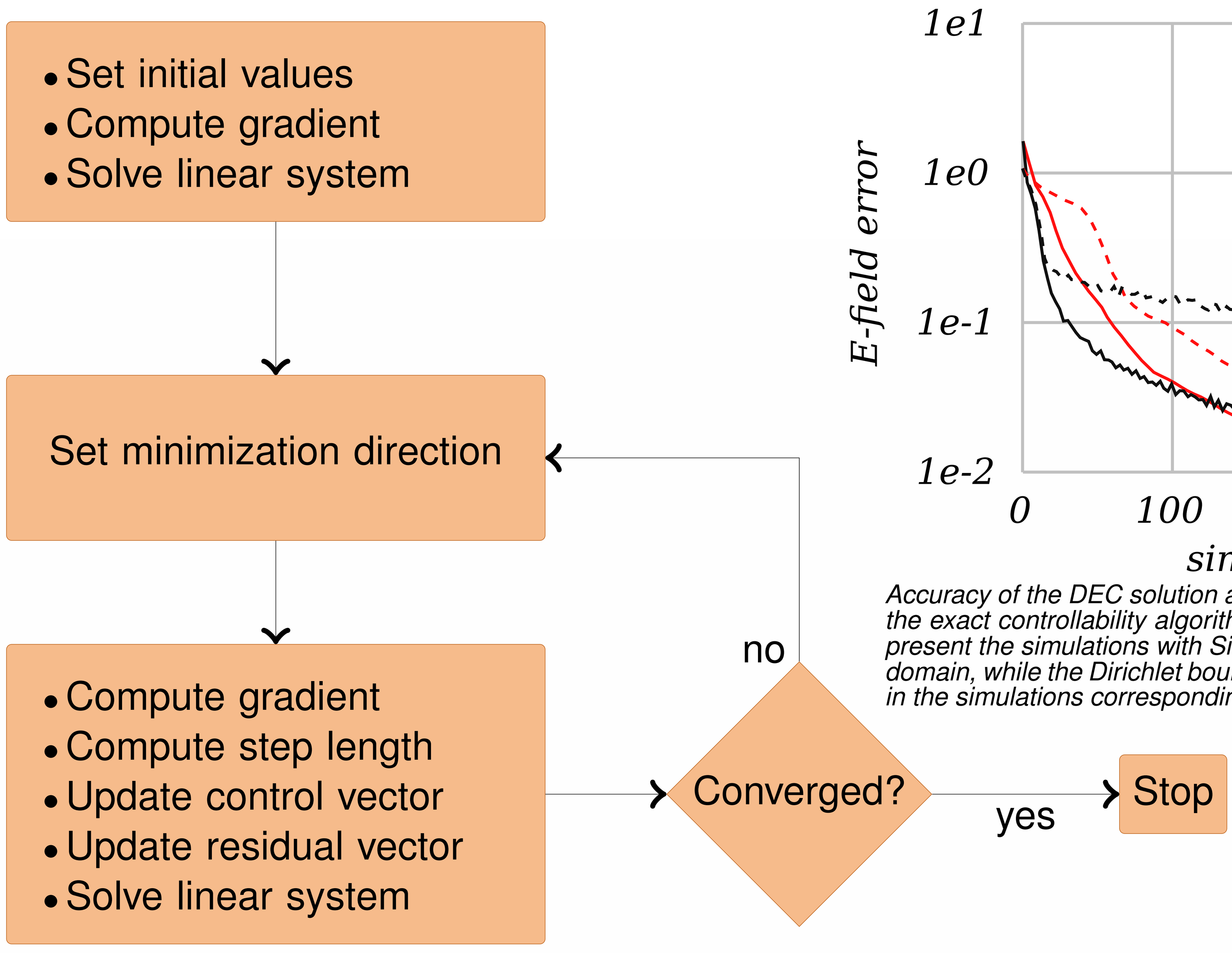
- The DEC (discrete exterior calculus) provides the properties and calculus of differential forms in a natural way at the discretization stage.
- By using the time-dependent equations, there are no challenges of solving large-scale indefinite systems.
  - Essentially, the time-harmonic solution can be reached by simple time integration (asymptotic approach).
  - We accelerate the convergence rate by using the exact controllability technique.
- The implementation used for the numerical experiments is developed at the Department of Mathematical Information Technology, University of Jyväskylä, Finland.

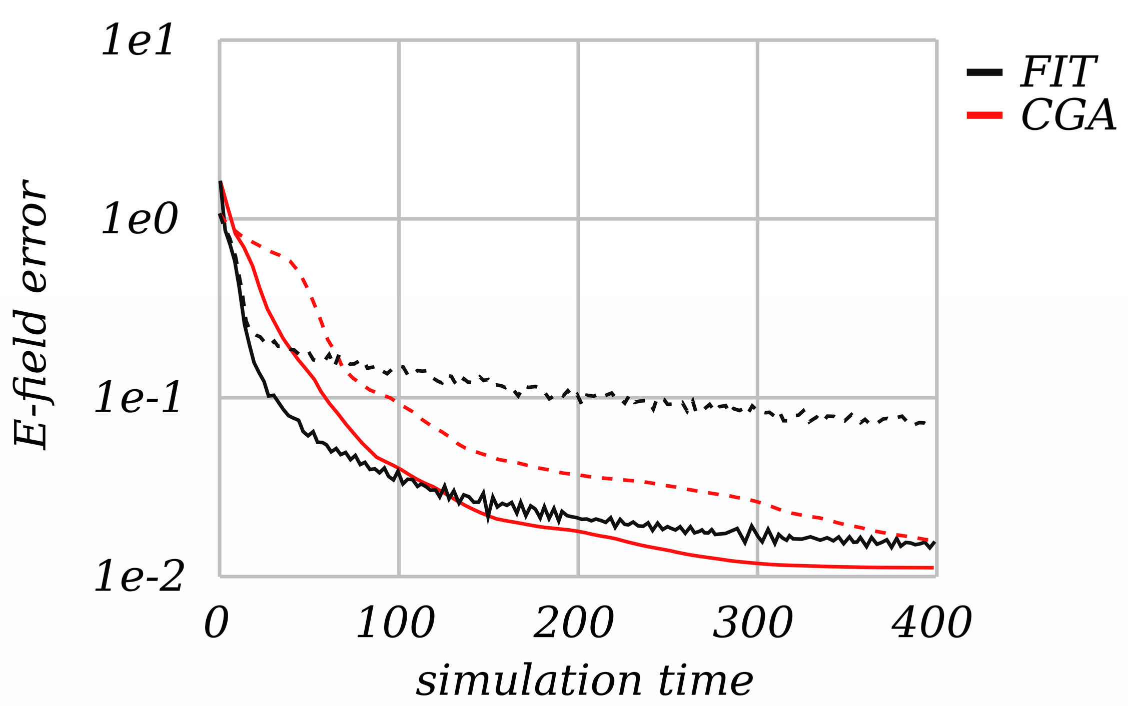
REFERENCES
D. Pauly and T. Rossi. Theoretical considerations on the computation of generalized time-periodic waves. Advances in Mathematical Sciences and Applications, 21(1), 105–131, 2011.

EXACT CONTROLLABILITY ALGORITHM

Searching for the periodic solution of the time-dependent wave equation is presented as a least-squares optimization problem.

- The initial conditions of the time-dependent problem are the control variables.
- A natural quadratic error functional is the squared energy norm of the system measuring how close the solution is to the corresponding time-harmonic solution.
- The energy norm is a weighted L^2-norm, implying that the discrete quadratic functional we minimize is spanned by a diagonal mass matrix.
- The minimization is performed by the conjugate gradient (CG) method operating in Hilbert spaces.
- The gradient is computed at each CG iteration by solving two time-dependent equations, the state equation advancing forward in time and the corresponding adjoint state equation advancing backward in time.
- The CG algorithm operating in a purely L^2-type Hilbert space does not need preconditioning.



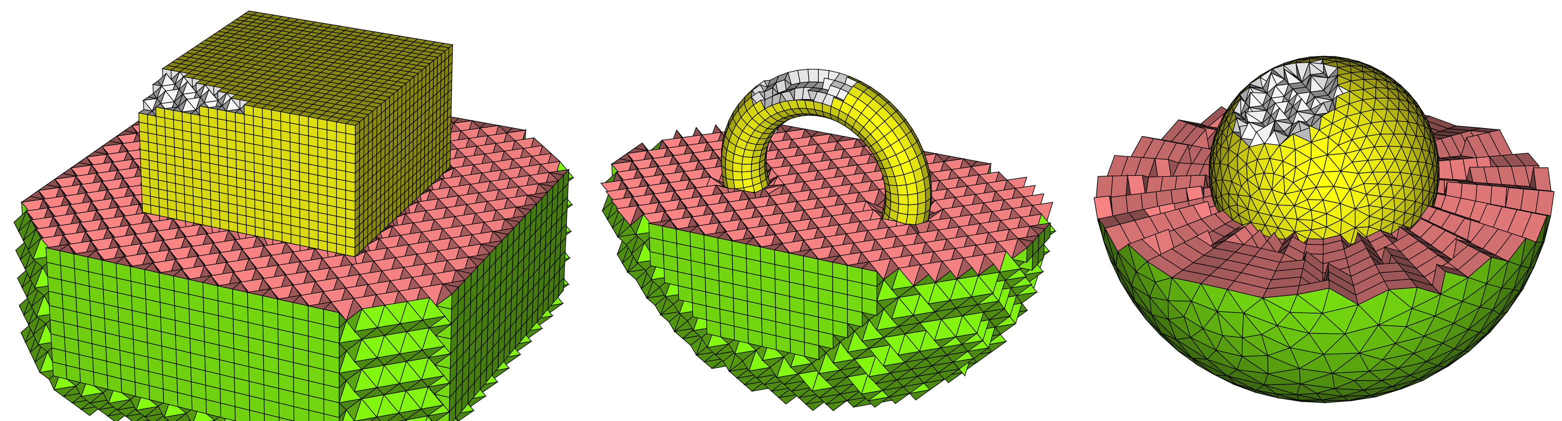


Accuracy of the DEC solution achieved by the asymptotic approach (FIT) and the exact controllability algorithm (CGA) in a cubical domain. The solid lines present the simulations with Silver–Müller boundary conditions truncating the domain, while the Dirichlet boundary conditions are applied on 5 of the 6 faces in the simulations corresponding to the dashed lines.

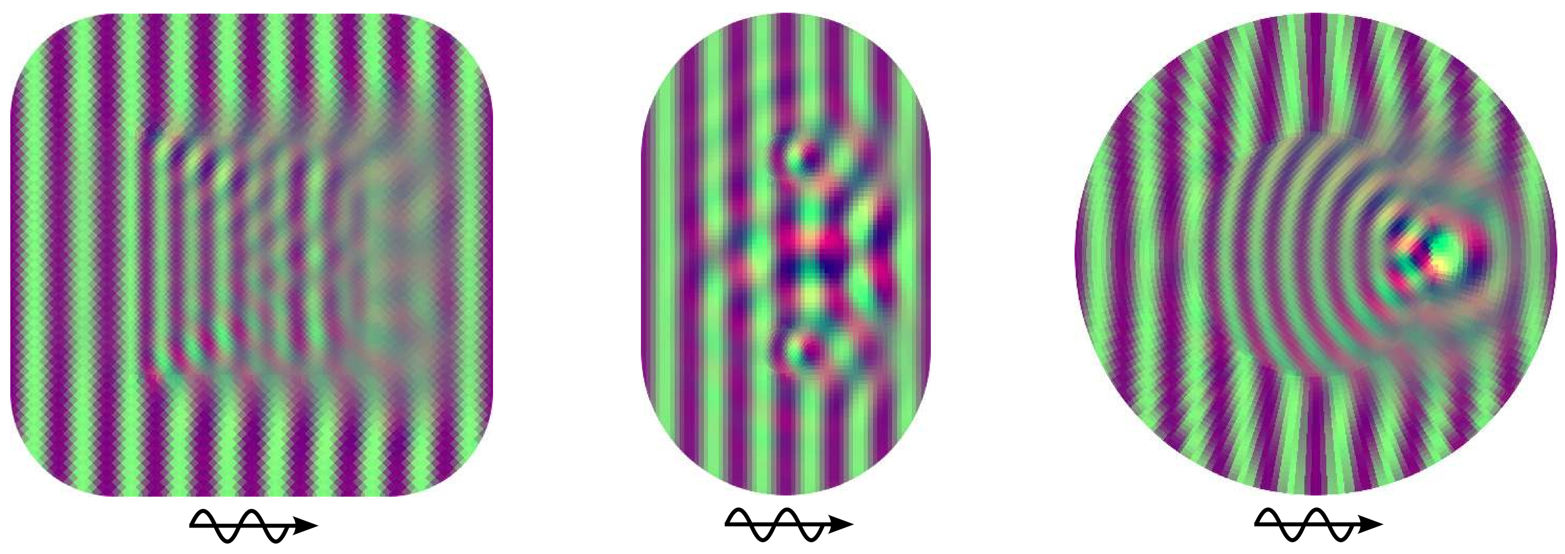
DDA-based approach

- The DDA (discrete-dipole approximation) is a well-known technique in the context of light scattering.
- The obstacle is represented as a set of dipole particles.
- The wave propagation is presented as a system of linear equations in the frequency domain.
- Numerical tests were run using the ADDA light scattering simulator.

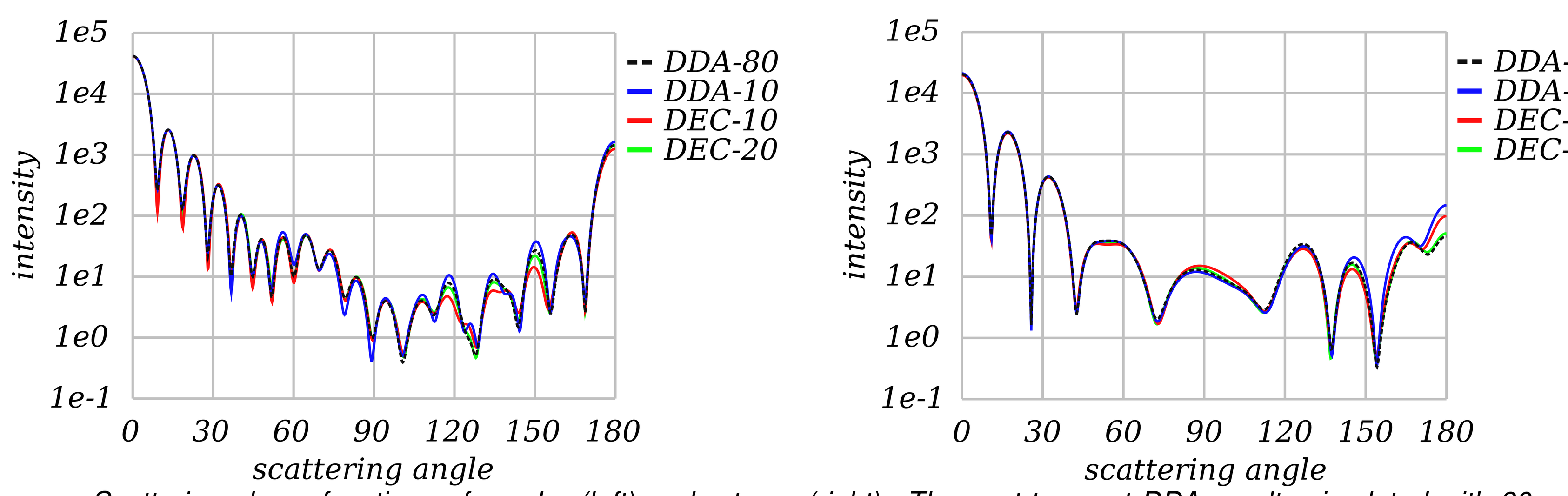
REFERENCES
M. A. Yurkin and A. G. Hoekstra, The discrete-dipole-approximation code ADDA: capabilities and known limitations, Journal of Quantitative Spectroscopy and Radiative Transfer, 112, 2234–2247, 2011.



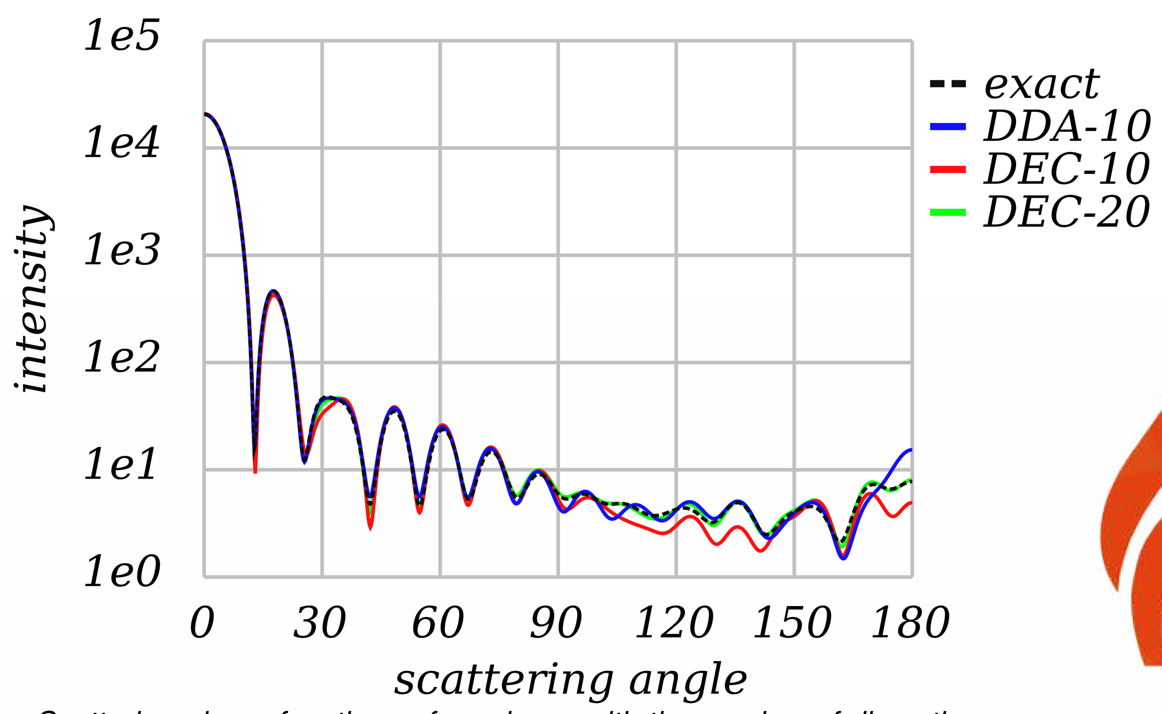
The DEC discretizations by polygons of different shapes (tetrahedron, prism, cube, octahedron). For clarity, these figures were made for 3 elements per wavelength $\lambda = 2\pi$, but 10 to 20 elements per wavelength are used in the simulations. A convex hull of thickness λ, surrounded by an absorbing layer of thickness 1.5λ preventing reflection from domain boundary, is used around the object.



Solutions of scattering problems solved by the DEC. The obstacles are a cube with edge length 5λ and a torus and a sphere with (outer) radius 5λ. The incident plane wave propagates in the x-direction (from left to right in this figure) and the total field is plotted on the x-y-plane. The refractive index is $1.6+0.05i$ and wavelength $\lambda = 2\pi$.



Scattering phase functions of a cube (left) and a torus (right). The next-to-exact DDA results simulated with 80 elements per wavelength (DDA-80) are compared to the numerical results with 10 (DDA-10 and DEC-10), and 20 (DEC-20) discretization elements per wavelength.



Scattering phase functions of a sphere, with the number of discretization elements per wavelength 10 (DDA-10 and DEC-10) and 20 (DEC-20), compared to the exact solution of the Mie theory.

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