Fourier analysis, period 1, Fall 2024

Presentation topics

The presentations are due on 24.10.2024. Please choose one of the topics below or agree with the instructor on another topic of interest, and prepare either an oral (around 30 min) or written (min 4 pages) presentation on it.

1. Periodic distributions [7, Section 2.4]
2. Gibbs phenomenon for Fourier series [3]
3. Fundamental solutions of classical PDE (Laplace, heat, wave equations)
4. Fundamental solutions of constant coefficient PDE (Malgrange-Ehrenpreis theorem) [6, chapter 8]
5. Distributions supported at a point [6, chapter 6]
6. Fourier transform of compactly supported distributions (Paley-Wiener and Paley-Wiener-Schwartz theorems) [6, chapter 7]
7. Fourier transform of positive measures (Bochner and Bochner-Schwartz theorems) [5, section IX.2]
8. Sobolev spaces $H^s(\mathbb{R}^n)$, Sobolev embeddings [8, chapter 8]
9. Restriction to submanifolds [5, section IX.9]
10. Elliptic regularity for constant coefficient PDE [8, chapter 8]
11. Central limit theorem in probability theory [4, section 7.6]
12. Spectral theorem for self-adjoint operators [9, section 8.1]
13. Prime number theorem [6, chapter 9]
14. Uncertainty principles [2]
15. Fourier methods in quantum mechanics
16. Littlewood-Paley theory (L^p spaces via Fourier transform)
17. Besov spaces B^s_{pq} and Triebel spaces F^s_{pq}
18. Method of stationary phase [1, section 4.5.3]
19. Microlocal analysis of singularities (wave front sets) [8, chapter 8]
20. Pseudodifferential operators [8, chapter 8]

References

[1] L.C. Evans, *Partial differential equations*, AMS, 1998.

[2] G. Folland, L. Sitaram, *The uncertainty principle: a mathematical survey*, J. Fourier. Anal. Appl. (1997), vol. 3, no. 3, 207–238.

[3] E. Hewitt, R.E. Hewitt, The Gibbs-Wilbraham phenomenon: an episode in Fourier analysis. *Arch. Hist. Exact Sci.*, 21(2):129–160, 1979.

[4] L. Hörmander, *The analysis on linear partial differential operators vol. I (Study edition)*, Springer-Verlag, 1990.

[5] M. Reed, B. Simon, *Methods of modern mathematical physics II*, Academic Press, 1975.

[6] W. Rudin, *Functional Analysis*, 2nd ed., McGraw-Hill, New York, 1991.

[7] M. Salo, *Fourier analysis and distribution theory*, lecture notes, 2013.

[8] R. Strichartz, *A guide to distribution theory and Fourier transforms*, CRC Press, 1994.

[9] M. Taylor, *Partial differential equations*, vols. 1-3.