A Genetic Algorithm for Multiobjective Design Optimization in Aerodynamics and Electromagnetics

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Abstract. The solution of a multiobjective multidisciplinary design optimization (MDO) using a genetic algorithm (GA) is considered. The objective functions in the optimization problem measure the aerodynamic feasibility based on the drag and lift coefficients and the electromagnetic feasibility based on the backscatter of the two-dimensional airfoil designs. The flow and backscatter are modeled by the thin-layer Navier–Stokes equations and the time-harmonic Maxwell equations, respectively. Numerical experiments illustrate the above evolutionary methodology on a parallel computer.

1 INTRODUCTION

Optimal shape design problems have been actively studied during the last decades; see [6], [9], [14], for example. Traditionally the design has been optimized with respect to only one discipline such as aerodynamics or electromagnetics. Although, it would be often highly desirable to consider multidisciplinary problems, that is, to consider several disciplines at once. Here, we solve a multidisciplinary problem in which one airfoil should have good aerodynamical and electromagnetic properties. More precisely, the drag and the electromagnetic backscatter of the airfoil is minimized while the lift is required to have at least a given minimum value [1], [11], [12].

The considered problem is also multiobjective optimization problem, since there is two objective functions, namely, the drag and the backscatter. Thus, the solution of this optimization problem requires specialized method suitable for multiobjective problems; see, for example, [3]. In this kind of problems, a design is optimal when it is *nondominated* or by using another term *Pareto optimal*. A design is nondominated if no other feasible design exists which is better with respect to any objective and at least equally good with respect to all the other objectives.

Usually, in the practical solution of multiobjective problems, the task is to find several Pareto optimal solutions. This can be computationally a very laborious task. Genetic algorithms (GA) can be adapted to this kind of problems and they are naturally parallel. Thus, GAs can be used efficiently in parallel computers which can offer the required computing power. In this paper, we consider the solution of multiobjective optimization problem using a GA in a parallel computer.

In [1], [12], the flow is modeled by the incompressible potential flow. The Euler equations are used in [11] which is one of the test cases in the Ingenet project. Here, we have chosen to use the Reynolds-averaged thin-layer Navier–Stokes equations. Also, we are continuing the development of genetic algorithms for optimal shape design problems; see [10], [11], [12], [15], for example.

The following section describes GAs with modifications for the problem under consideration. Then, the CFD and CEM solvers are shortly introduced and the actual multiobjective multidisciplinary optimization problem for the airfoil design is given. Lastly, we present the numerical results with some concluding remarks.

2 MULTIOBJECTIVE OPTIMIZATION WITH GENETIC ALGORITHM

Genetic algorithms (GAs) are stochastic processes mimicking the natural selection based on the Darwin's principle of survival of the fittest. A GA evolves from a gen-

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eration to another and each population is a set of solution candidates. The fitness function tells how good individual is. The selection of parents is performed so that more fit individuals are more likely to be chosen. The offsprings are formed from the parents by using the random operations of crossover and mutation. For more detailed description of the operations involved see [5], for example.

Without any gradient information, GAs can explore the search space in parallel with the population of individuals and exchange the beneficial information through crossover. GAs are shown to be robust adaptive optimization methods with inherent parallelism for problems where the traditional methods can fail, for example, when searching for a neighborhood of a global minimum [5].

For multiobjective optimization problems, it is necessary to make some modifications to the basic GA. Our algorithm is based on the Nondominated Sorting GA (NSGA) [16]. For general discussion on GAs for multiobjective optimization, see [4] and references therein. In the following, we describe the basic ideas of the NSGA and the modifications. The fitness values are computed using the following procedure in the NSGA and the modified algorithm:

Algorithm 1 Nondominated sorting
Choose a large dummy fitness value $\mathbf{F}$;
Repeat
  Find the nondominated individuals among
    the individuals whose fitness values are not set;
  Set the fitness value of individuals found in
    previous step to $\mathbf{F}$;
  Decrease the dummy fitness value $\mathbf{F}$;
Until (fitness values of all individuals are set).

Our modified algorithm employs the *tournament selection*, unlike the NSGA which uses the roulette wheel selection. For each tournament, a fixed number of individuals are selected randomly. The individual which has the highest fitness value wins the tournament, that is, it is selected to be a parent in the breeding.

Unfortunately, if there were no modifications to the previous tournament selection, the population would usually converge towards one point on the set of Pareto optimal solutions whereas the aim was to obtain several points from the Pareto set. In our modified algorithm, the diversity of the population is preserved using the so–called tournament slot sharing which was introduced in [11]. For this purpose, let us define the sharing function

$$\mathbf{Sh}(\mathbf{d_{ij}}) = \begin{cases} 1 - \left(\frac{\mathbf{d_{ij}}}{\sigma_{\text{share}}}\right)^2, & \text{if } \mathbf{d_{ij}} < \sigma_{\text{share}}, \\ 0, & \text{otherwise,} \end{cases}$$

where $\mathbf{d_{ij}}$ is the genotypic distance between the individuals $\mathbf{i}$ and $\mathbf{j}$, that is, in our case the euclidean distance between the vectors defining the designs $\mathbf{i}$ and $\mathbf{j}$. The parameter σ_{share} is the maximum *sharing distance* for a tournament slot. This is the very same sharing function as in the classical fitness value sharing. The probability for the individual $\mathbf{i}$ to enter a tournament is given by

$$\mathbf{p_i} = \frac{1/\sum_{\mathbf{j}=1}^{\mathbf{n}} \mathbf{Sh}(\mathbf{d_{ij}})}{\sum_{\mathbf{k}=1}^{\mathbf{n}}(1/\sum_{\mathbf{j}=1}^{\mathbf{n}} \mathbf{Sh}(\mathbf{d_{kj}}))},$$

where the parameter $\mathbf{n}$ is the size of population.

An elitist mechanism is added to our algorithm since it guarantees the cost function values to decrease monotonically from one generation to the next. Also, it usually accelerates the convergence. This is implemented by copying from the old population to the new population all the individuals which would be nondominated in the new population. As a coding, we have used the floating point coding [13]. The crossover is made using one crossover site and a special mutation considered in [11] is utilized. Thus, we are ready to present the modified algorithm:

Algorithm 2 The modified NSGA
Initialize population;
Compute object functions [in parallel];
Do generation:=2,number_of_generations
  Compute fitness values using nondominated sorting;
  Compute probabilities for each individual to
    enter tournament;
  Repeat
    Select two parents;
    Form two childrens using crossover;
  Until (new population is full);
  Perform mutations;
  Compute object functions [in parallel];
  Copy individuals from old population according
    to elitism;
End do.

3 CFD AND CEM SOLVERS

The flow is modeled by the two-dimensional Reynolds-averaged thin-layer Navier–Stokes equations. The discretization is made using the finite volume method. The steady state solution is obtained by an implicit pseudo-time integration. The convergence is accelerated using a multigrid algorithm. The flow solver is called FINFLOW [8].

The wave scattering is modeled by the time-harmonic two-dimensional Maxwell equations, which can be reduced to the Helmholtz equation with the Sommerfeld radiation condition. For the discretization made using the linear finite elements the domain is truncated and a second-order absorbing boundary condition is posed on the artificial far-field boundary. A fictitious domain method is used to solve the arising system of linear equations [7], [12].

4 DRAG AND BACKSCATTER REDUCTION

In this multiobjective multidisciplinary design optimization problem, we minimize the drag coefficient and the amplitude of the backscattered wave while the lift coefficient must not be less than a given value.

Let $\mathbf{U_{ad}}$ be the set of design variable vectors α in $\mathbf{R^n}$ which define the shapes of geometrically admissible airfoils. The set of the physically admissible designs is defined by

$$\mathbf{U^*_{ad}} = \left\{ \alpha \in \mathbf{U_{ad}} \mid \mathbf{C_l}(\alpha) \geq \mathbf{C_l^{min}} \right\},$$

where $\mathbf{C_l} = \mathbf{C_l}(\alpha)$ is the lift coefficient and $\mathbf{C_l^{min}}$ is the lower bound for the lift coefficient.

This problem can be formulated as a multiobjective minimization problem

$$\min_{\alpha \in \mathbf{U^*_{ad}}} \{\mathbf{C_d}(\alpha), \mathbf{J}(\alpha)\}, \tag{1}$$

where $\mathbf{C_d}(\alpha)$ is the drag coefficient and $\mathbf{J}(\alpha)$ measures the amplitude of backscattered electromagnetical wave. The function $\mathbf{J}$ is defined by the integral

$$\mathbf{J}(\alpha) = \int_\Theta |\mathbf{w}_\infty(\alpha)|^2 \, \mathbf{ds}, \tag{2}$$

where Θ is the sector where the backscatter is minimized and $\mathbf{w}_\infty$ is the far field pattern of the scattered electromagnetic wave.

The nonlinear lift constraint is taken into account by adding a quadratic penalty function to both object functions. Let ε be a small positive penalty parameter. Then, the penalized object functions are

$$\mathbf{C_d^\varepsilon}(\alpha) = \mathbf{C_d}(\alpha) + \frac{1}{\varepsilon}\left(\min\{\mathbf{C_l}(\alpha) - \mathbf{C_l^{min}}, 0\}\right)^2$$

and

$$\mathbf{J}^\varepsilon(\alpha) = \mathbf{J}(\alpha) + \frac{1}{\varepsilon}\left(\min\{\mathbf{C_l}(\alpha) - \mathbf{C_l^{min}}, 0\}\right)^2.$$

Now the penalized multiobjective minimization problem reads

$$\min_{\alpha \in \mathbf{U_{ad}}} \{\mathbf{C_d^\varepsilon}(\alpha), \mathbf{J}^\varepsilon(\alpha)\}. \tag{3}$$

5 NUMERICAL RESULTS

The airfoil shape is parametrized using two Bezier curves with each curve having nine control points [2]. There is one curve for the extrados and another one for the intrados. The control points on the leading and trailing edges are fixed and the other control points are moving in the $\mathbf{y}$–direction. The first two design variables are the $\mathbf{y}$–coordinates of the Bezier control points directly above and below the leading edge. The next 12 design variables are the sums and the differences of the $\mathbf{y}$–coordinates of the corresponding control points from upper and lower sides of airfoil. The angle of attack is the 15th design variable. By choosing the design variables this way, the geometrically feasible designs can be defined using box constraints. The two Bezier curves for the NACA64A410 airfoil are shown in Figure 1.

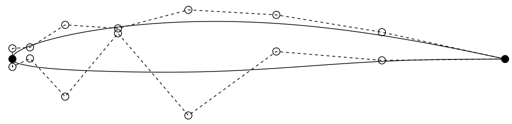

Figure 1. The Bezier curves for the NACA64A410 airfoil and their control points.

The airfoils are operating at Mach number $\mathbf{M}_\infty = 0.75$ and the Reynolds number is 10^6. The Navier–Stokes solver uses 192×48 C-type grid with 128 grid points on the surface of airfoil. During the optimization process, the grid for the Navier–Stokes solver is depending continuously and smoothly on the design parameters. For the Helmholtz solver the computational domain is truncated to be the rectangle $[-0.1, 1.1] \times [-0.3, 0.3]$. The airfoil is 20 wave lengths long and the mesh is 481×241 rectangular mesh with a local fitting on the surface of airfoil. Thus, there is 20 nodes per wave length. Due to the use of local fitted mesh, the number of nodes and elements in the mesh might vary during the optimization. Hence, the objective function $\mathbf{J}$ computed using the finite element approximation is discontinuous.

The lower limit for the lift coefficient $\mathbf{C_l^{min}}$ is set to be 0.5. The backscatter is measured in the sector $\Theta = [180, 200]^\circ$ in (2). The direction of the incident wave is 10°. The penalty parameter ε is 10^{-4}. The GA parameters are shown in Table 1. In the Section 2, the sharing distance was denoted by σ_{share}. The mutation exponent is related to the mutation and the exact meaning of it is explained in [11]. In one optimization run, 9600 fitness function values are computed. The initial population contains the NACA64A410 airfoil and 63 randomly chosen designs.

Table 1. The parameters in GA.

Population size	64
Generations	150
Tournament size	3
Sharing distance	0.25
Crossover probability	0.8
Mutation probability	0.2
Mutation exponent	4

The computations are performed on a Sun Ultra Enterprise 4000 server with eight 250 MHz processors using the MPI message passing library. The computation of one solution of the thin-layer Navier–Stokes equations and the Helmholtz equation required roughly 165 and 45 CPU seconds, respectively. The total wall clock time for one optimization run was approximately 75 hours.

The cost function values $(\mathbf{C}_\mathbf{d}^\varepsilon, \mathbf{J}^\varepsilon)$ for the initial design NACA64A410 are $(0.0171, 0.00167)$. After 150 generations we obtained 15 nondominated designs. These designs are sorted according to their $\mathbf{C}_\mathbf{d}^\varepsilon$ values and then they are referred using their ordinal number. The corresponding cost function values for the designs 1, 3, 7 and 11 are $(0.0046, 0.00098)$, $(0.0049, 0.00071)$, $(0.0058, 0.00048)$ and $(0.0071, 0.00046)$, respectively. The cost functions and the corresponding airfoils for these designs are shown in Figure 2.

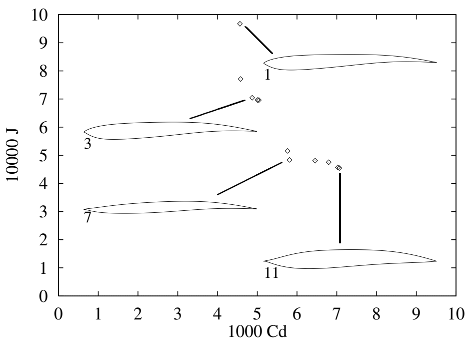

Figure 2. Some nondominated designs from the last generation.

In the remaining figures, we have examined three designs, namely the NACA64A410, and the design 1 and 11 from the nondominated designs in the last generation. The airfoils for the design 1 and 11 can be seen in Figure 2. The pressure coefficients $\mathbf{C_p}$ are shown in Figure 3 and in Figure 4, the radar cross sections (RCSs) are given in the sector where the backscatter is minimized.

The final generation is probably not fully convergent, that is, there is still a gap between the nondominated individuals in the last generation and the set of Pareto optimal solutions. Hence, more generations would probably improve the nondominated designs. Also due to the elitism mechanism in our GA the number of nondominated designs should start to grow when the individuals are reaching the Pareto set. Probably the tuning of the GA parameters are likely to accelerate the convergence. Unfortunately, the tuning is rather difficult, since each GA run requires extensive computational time.

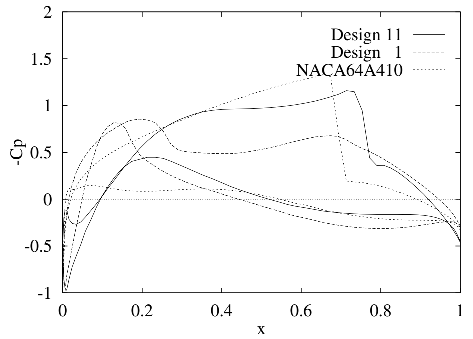

Figure 3. The pressure coefficients for some designs.

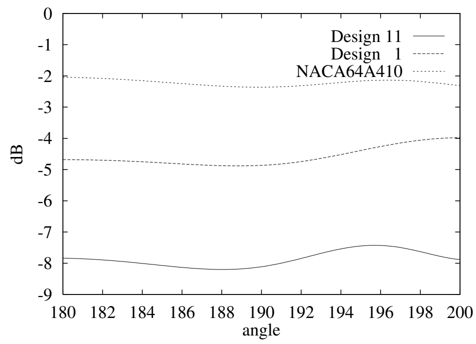

Figure 4. The RCSs for some designs.

6 CONCLUSIONS

In our numerical experiment, we were able to obtain several nondominated designs. Since gradients are not required and the cost functions do not have to be continuous, we can use any standard state solvers for shape optimization with GA. Also, it is easy to obtain a good speedup in parallel GA optimization with standard sequential state solvers. The number of performed cost function evaluations was rather high and thus, the optimization was computationally expensive. The GA should be further developed so that the convergence towards the set of Pareto optimal solutions would be improved.

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