Some examples related to eigenvalue computations. Although it is instructive to code the basic iterative eigenvalue algorithms in e.g. Saad's book, it is not good idea to include such a simple code in any production code. The reason is that the special cases (multiple or complex conjugate eigenvalues etc.) usually need some extra care (read: skill and lot of coding) in order to obtain correct and accurate results.

1. Compute the root nearest to $x = 3$ of the "sparse" polynomial

$$p(x) = x^{21} - 2x^{20} + 2x^{11} - 4x^{10} + x - 2$$

using Matlab's `eigs` function with sparse representation for the companion matrix.

2. Consider the matrix `A=gallery('neumann',n)` where n is a perfect square (i.e. $n = k \times k$ for some k). It is known that the eigenvalues are non-negative and 0 is also an eigenvalue. One wants to compute three smallest eigenvalues. However, using `eigs(A,3,'SM')` results in inaccurate results or even error. Why it is so and how to fix the problem?

3. The harmonic vibration of an elastic string can be modeled with the eigenvalue problem for an ordinary differential equation

$$-v''(x) = \lambda v(x), \quad 0 < x < 1, \quad v(0) = v(1) = 0.$$

Although this problem is easily solved analytically, let us discretize it using the Finite Element Method resulting in the *generalized eigenvalue problem*

$$Au = \lambda Bu.$$

Here A and B are $n \times n$ symmetric and positive definite matrices:

$$A = \frac{1}{h}\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & & \ddots & \\ & & & -1 & 2 \end{bmatrix}, \qquad B = h\begin{bmatrix} 2/3 & 1/6 & & & \\ 1/6 & 2/3 & 1/6 & & \\ & 1/6 & 2/3 & 1/6 & \\ & & & \ddots & \\ & & & 1/6 & 2/3 \end{bmatrix},$$

where $h = 1/(n+1)$ is the discretization parameter. The eigenvector u is the approximation of the (continuous) eigenmode v, i.e. $v_i \approx u(ih)$.

Compute using Matlab three smalles eigenvalues when $n = 9$ and $n = 99$. Use `eigs` function and pass to it the Cholesky factor of B instead of B. Read the documentation of `eigs` ;-)