Let $\mathscr{F} : \mathbb{R}^n \to \mathbb{R}^n$ be a vector valued function. Nonlinear algebraic system of equations $\mathscr{F}(x) = 0$ can be solved using Newton's method

$$x^{(k+1)} = x^{(k)} - \left[\mathscr{J}(x^{(k)})\right]^{-1} \mathscr{F}(x^{(k)}), \quad k = 0, 1, 2, \ldots$$

Here $\mathscr{J}(x^{(k)})$ is the Jacobian of $\mathscr{F}$ at $x^{(k)}$. Thus, at each Newton iteration one has to solve a linear system of equations. If the Jacobian is large and sparse one can use e.g. GMRES method to solve it. This linear system can be solved without forming the matrix $\mathscr{J}$ explicitly. Namely from Taylor expansion $\mathscr{F}(x + \epsilon\, v) \approx \mathscr{F}(x) + \epsilon\, \mathscr{J}(x)\, v$ it follows that the matrix-vector product needed in GMRES can be approximated by

$$\mathscr{J}(x)\, v \approx \frac{1}{\epsilon} \left(\mathscr{F}(x + \epsilon\, v) - \mathscr{F}(x)\right).$$