TIEA311
Tietokonegrafiikan perusteet
kevät 2019

("Principles of Computer Graphics" – Spring 2019)

TIEA311 Tietokonegrafiikan perusteet – kevät 2019 (“Principles of Computer Graphics” – Spring 2019)

Adapted from: *Wojciech Matusik*, and *Frédo Durand*: 6.837 Computer Graphics. Fall 2012. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu/.

Frontpage of the local course version, held during Spring 2019 at the Faculty of Information technology, University of Jyväskylä: `http://users.jyu.fi/~nieminen/tgp19/`

TIEA311 - Today in Jyväskylä

Plan for today:

- Usual warm-up and group discussion
- Try to address the most urgent issues
- Break – reset the brain.
- Then continue with the theory.

TIEA311 - Today in Jyväskylä

We start by discussion, reflection and **questions**!

Work in groups of 3 students if possible:

- Fast warm-up: 90 seconds evenly split between group members (30s each in groups of 3), no interruptions from others: Foremost feelings right now?
- Reflection: Silent work, solo, 1 minute, **list words on paper**: What have you learned during the last week? Or since the course started?
- Interaction: 1.5 minutes group discussion: Compare if you learned the same or different things? Do those things feel useful? Why or why not?
  $\rightarrow$ Sum it up classwide.
- Interaction: Group work, 1.5 minutes or less if talk ends: At the moment, what would be the most helpful thing to help you (or others!)?
  $\rightarrow$ Sum it up classwide, and try to address the findings.

TIEA311 - Today in Jyväskylä

What were the findings in group discussion?

What were found to be the most important issues to address right now?

$\rightarrow$ Classwide discussion is found on the lecture video.

NOTE: Even if you watch at home, please think about the same things and try to be in ”virtual dialogue” with those in classroom. Use pen and paper! I believe, more and more every day, that doing so will make your brain perform activities that help **your own learning**.

NOTE: Contemplate if you could watch the lecture videos with some friends who would also like to learn computer graphics? Get some pizza and coke if it helps you get to the mood(?).

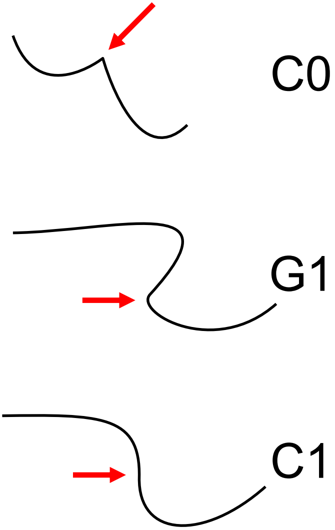
TIEA311 - Today in Jyväskylä

Plan for today:

- Usual warm-up and group discussion
- Try to address the most urgent issues
- Break – reset the brain.
- Then continue with the theory.

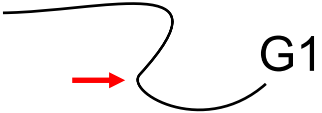
Orders of Continuity

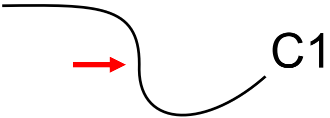
- C0 = continuous
  - The seam can be a sharp kink
- G1 = geometric continuity
  - Tangents **point to the same direction** at the seam
- C1 = parametric continuity
  - Tangents **are the same** at the seam, implies G1
- C2 = curvature continuity
  - Tangents and their derivatives are the same



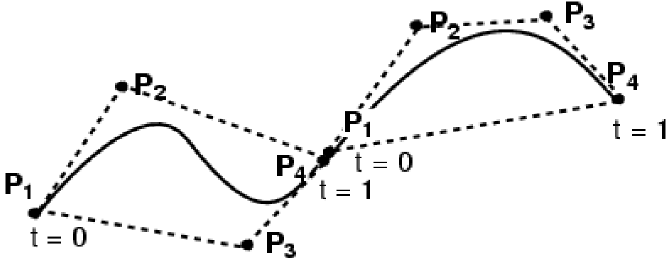
Orders of Continuity

- G1 = geometric continuity
  - Tangents **point to the same direction** at the seam
  - good enough for modeling
- C1 = parametric continuity
  - Tangents **are the same** at the seam, implies G1
  - often necessary for animation



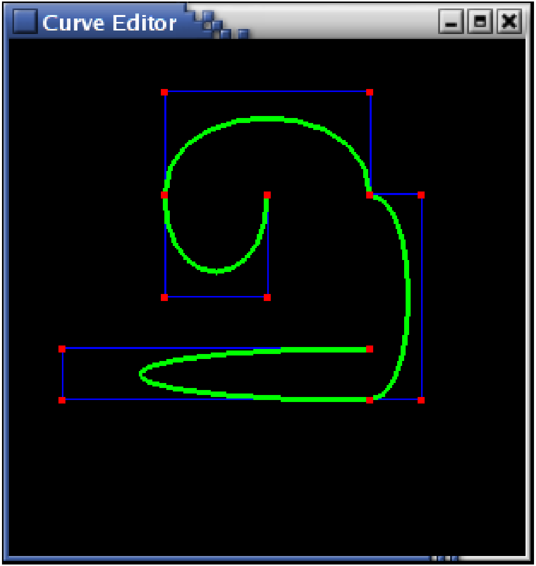


Connecting Cubic Bézier Curves



- How can we guarantee C0 continuity?
- How can we guarantee G1 continuity?
- How can we guarantee C1 continuity?
- C2 and above gets difficult

Connecting Cubic Bézier Curves



- Where is this curve
  - C0 continuous?
  - G1 continuous?
  - C1 continuous?
- What's the relationship between:
  - the # of control points, and the # of cubic Bézier subcurves?

Questions?

TIEA311 - Local plan for today

Oh, wait! Let us get one thing out of the way…

The teacher will now communicate to you at least three things simultaneously:

- How to pass our T2 with points 1/5, towards course grade 1/5.
- Some philosophy behind the definition of grade 1/5.
- Practically, a spoken-out definition of grade 1/5 on this course.
- One possible option of some steps that would need to be taken first in order to progress towards the higher grades, or in general "being a pro" in IT stuff.

→ Live coding and thinking aloud. See lecture video.

TIEA311

OK.

That went smoothly.

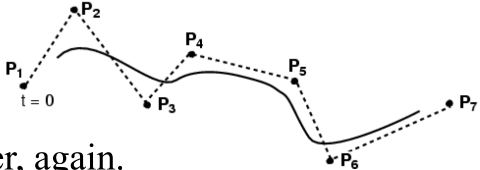
Like a smooth curve!

With score 1/5 secured, we can feel warm and happy, and stop worrying about “passing or not passing” T2 on TIEA311! That question about passing is foul in all contexts, anyway! Ugh!

This **enables us** to relax and spend some nice **focused learning time** in order to **understand more** about all this, and move towards not (just) copy-pasting, but at least copy-paste-modifying-with-some-idea-why. . .

Cubic B-Splines

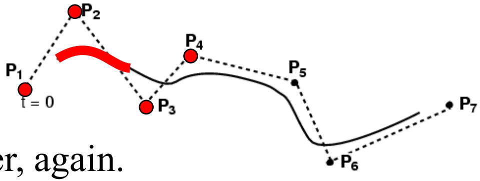
- $\geq$ 4 control points
- Locally cubic
  - Cubics chained together, again.



Courtesy of Seth Teller.

Cubic B-Splines

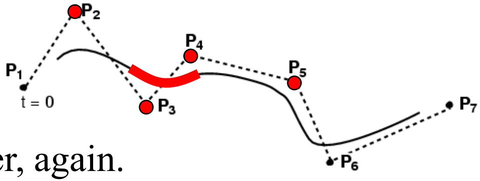
- $\geq$ 4 control points
- Locally cubic
  - Cubics chained together, again.



Courtesy of Seth Teller.

Cubic B-Splines

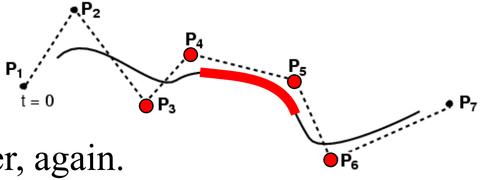
- $\geq$ 4 control points
- Locally cubic
  - Cubics chained together, again.



Courtesy of Seth Teller.

Cubic B-Splines

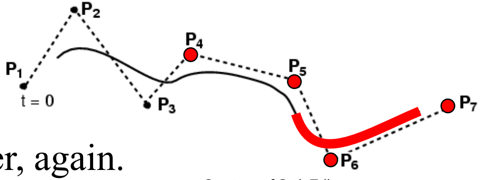
- $\geq$ 4 control points
- Locally cubic
  - Cubics chained together, again.



Courtesy of Seth Teller.

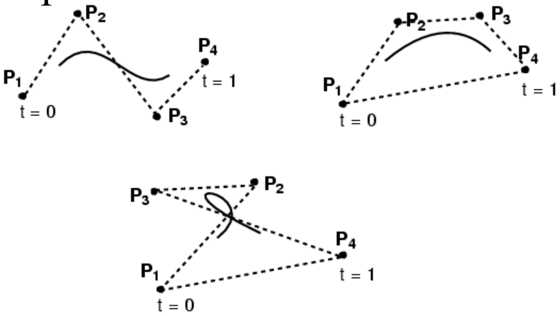
Cubic B-Splines

- $\geq$ 4 control points
- Locally cubic
  - Cubics chained together, again.

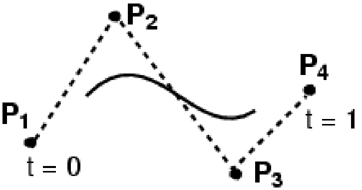


Courtesy of Seth Teller.

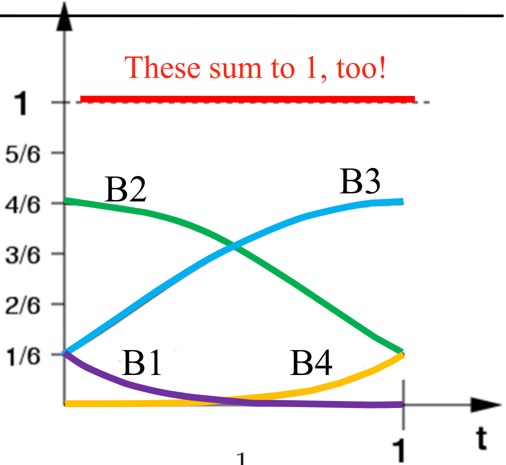
- Curve is not constrained to pass through any control points



Cubic B-Splines: Basis



A B-Spline curve is also bounded by the convex hull of its control points.



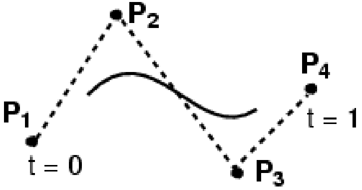
$$B_1(t) = \frac{1}{6}(1-t)^3$$

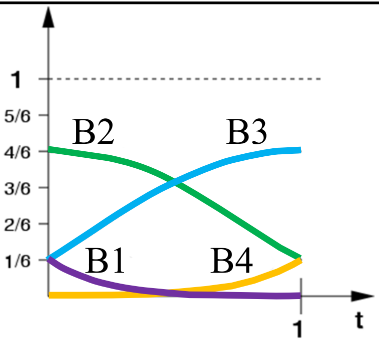
$$B_2(t) = \frac{1}{6}(3t^3 - 6t^2 + 4)$$

$$B_3(t) = \frac{1}{6}(-3t^3 + 3t^2 + 3t + 1)$$

$$B_4(t) = \frac{1}{6}t^3$$

Cubic B-Splines: Basis





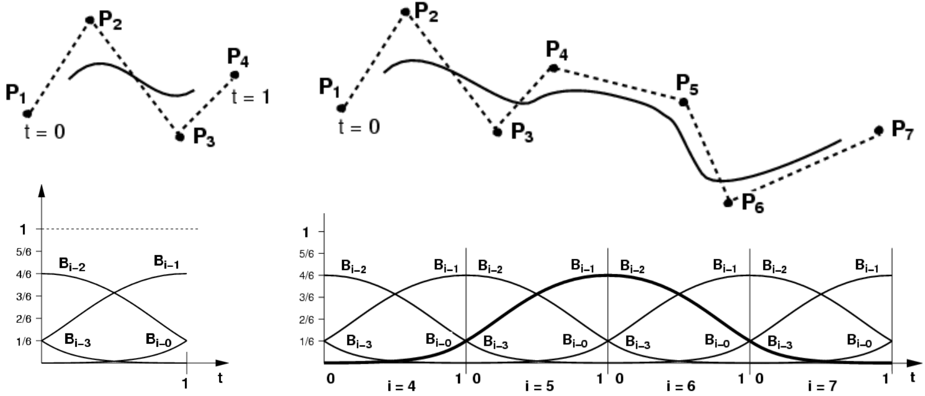
$$Q(t) = \frac{(1-t)^3}{6}P_1 + \frac{3t^3 - 6t^2 + 4}{6}P_2 + \frac{-3t^3 + 3t^2 + 3t + 1}{6}P_3 + \frac{t^3}{6}P_4$$

$$Q(t) = \mathbf{GBT(t)}$$

$$B_{B-Spline} = \frac{1}{6}\begin{pmatrix} 1 & -3 & 3 & -1 \\ 4 & 0 & -6 & 3 \\ 1 & 3 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

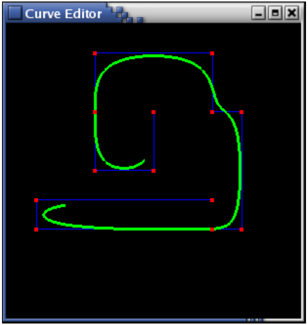
Cubic B-Splines

- Local control (windowing)
- Automatically C2, and no need to match tangents!

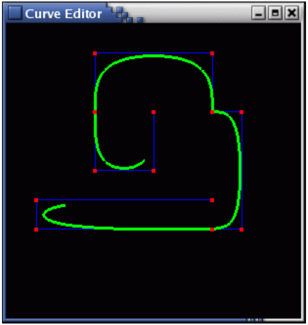


Courtesy of Seth Teller. Used with permission.

B-Spline Curve Control Points

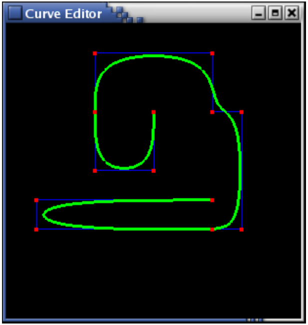


Default B-Spline



B-Spline with derivative discontinuity

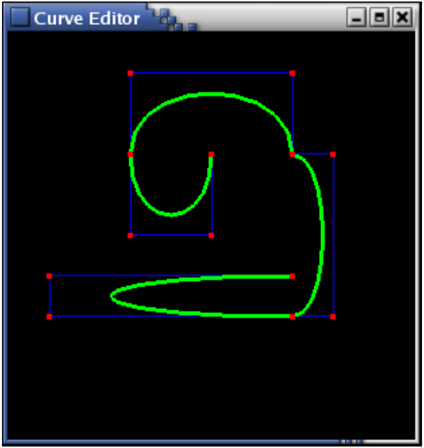
Repeat interior control point



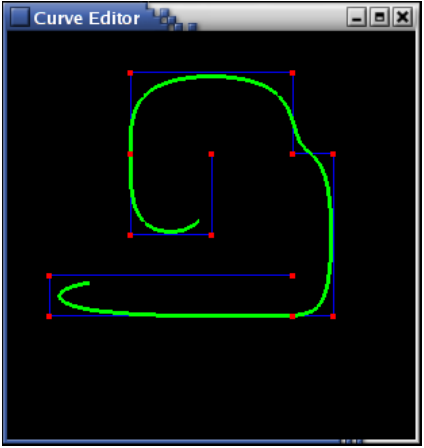
B-Spline which passes through end points

Repeat end points

Bézier ≠ B-Spline



Bézier



B-Spline

But both are cubics, so one can be converted into the other!

Converting between Bézier & BSpline

$Q(t) = \mathbf{GBT(t)}$ = Geometry $\mathbf{G}$ · Spline Basis $\mathbf{B}$ · Power Basis $\mathbf{T(t)}$

- Simple with the basis matrices!
  - Note that this only works for a single segment of 4 control points

$$B_{Bezier} = \begin{pmatrix} 1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- P(t) = **G B**1 **T**(t) = **G B**1 **(B**2-1**B2)** **T**(t)= (**G B**1 **B**2-1) **B**2 **T**(t)

$$B_{B-Spline} = \frac{1}{6}\begin{pmatrix} 1 & -3 & 3 & -1 \\ 4 & 0 & -6 & 3 \\ 1 & 3 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- **G B**1 **B**2-1 are the control points for the segment in new basis.

In the previous slide, the minor inconvenience of misprinted subscripts and superscripts is especially harmful. The equation should read as:

$$\begin{aligned} P(t) &= GB_1T(t) \\ &= GB_1(B_2^{-1}B_2)T(t) \\ &= (GB_1B_2^{-1})B_2T(t) \end{aligned}$$

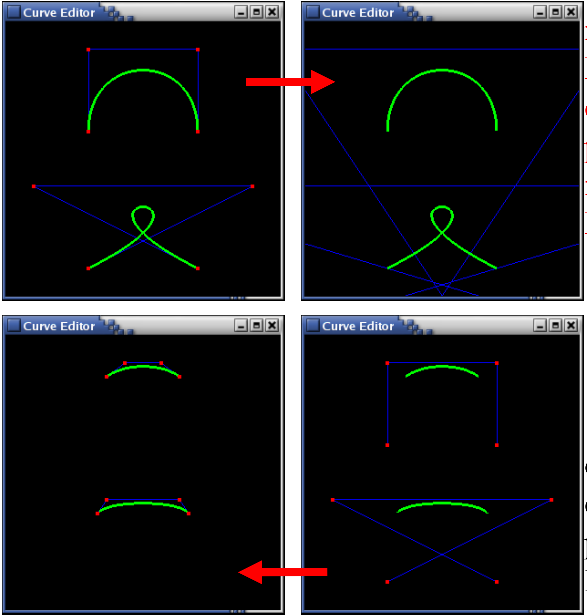
Then, we end up with $(GB_1B_2^{-1})$ as new control points.

"Unfortunately", you will need to do similar re-interpretation of many of the equations in the OpenCourseware slides to fully understand them.

"Fortunately", **doing this will actually make you understand each equation better** :). Pen and paper are your friends!

Converting between Bézier & B-Spline

original control points as Bézier



new BSpline control points to match Bézier

new Bézier control points to match B-Spline

original control points as B-Spline

NURBS (Generalized B-Splines)

- Rational cubics
  - Use homogeneous coordinates, just add *w* !
    - Provides an extra weight parameter to control points

- NURBS: Non-Uniform Rational B-Spline
  - **non-uniform** = different spacing between the blending functions, a.k.a. "knots"
  - **rational** = ratio of cubic polynomials (instead of just cubic)
    - implemented by adding the homogeneous coordinate *w* into the control points.

Demo

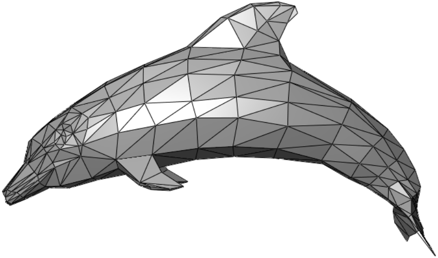
Questions?

Representing Surfaces

- Triangle meshes
  - Surface analogue of polylines, this is what GPUs draw
- **Tensor Product Splines**
  - Surface analogue of spline curves
- **Subdivision surfaces**
- **Implicit surfaces, e.g.** f(x,y,z)=0
- **Procedural**
  - e.g. surfaces of revolution, generalized cylinder
- From volume data (medical images, etc.)

Triangle Meshes

- What you've used so far in Assignment 0
- Triangle represented by 3 vertices
- **Pro**: simple, can be rendered directly
- **Cons**: not smooth, needs many triangles to approximate smooth surfaces (tessellation)



TIEA311 - Today in Jyväskylä

The time allotted for this lecture is now over.

Now: Break until tomorrow morning. Sleep if you have time.

But also **try to wake up and come to the lecture**!

We will pick up our thoughts soon enough!