# TIEA311 Tietokonegrafiikan perusteet

("Principles of Computer Graphics" – Spring 2019)

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# TIEA311 Tietokonegrafiikan perusteet – kevät 2019 ("Principles of Computer Graphics" – Spring 2019)

Adapted from: Wojciech Matusik, and Frédo Durand: 6.837 Computer Graphics. Fall 2012. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu/.

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Frontpage of the local course version, held during Spring 2019 at the Faculty of Information technology, University of Jyväskylä:

http://users.jyu.fi/~nieminen/tgp19/

### Greetings from YPE and APO (University Pedagogical Studies)



### TIEA311 - Today in Jyväskylä

#### Plan for today:

- ► Usual warm-up and group discussion
- ► Try to address the most urgent issues
- ► Break reset the brain.
- ► Then continue with the theory.

# TIEA311 - Today in Jyväskylä We start by discussion, reflection and questions!

Work in groups of 3 students if possible:

- ► Fast warm-up: 90 seconds evenly split between group members (30s each in groups of 3), no interruptions from others: Foremost feelings right now?
- ► Reflection: Silent work, solo, 1 minute, **list words on** paper: What have you learned during the last week? Or since the course started?
- ► Interaction: 1.5 minutes group discussion: Compare if you learned the same or different things? Do those things feel useful? Why or why not?
  - $\rightarrow$  Sum it up classwide.
- ► Interaction: Group work, 1.5 minutes or less if talk ends: At the moment, what would be the most helpful thing to help you (or others!)?
  - $\rightarrow$  Sum it up classwide, and try to address the findings.

### TIEA311 - Today in Jyväskylä

What were the findings in group discussion?

What were found to be the most important issues to address right now?

→ Classwide discussion is found on the lecture video.

NOTE: Even if you watch at home, please think about the same things and try to be in "virtual dialogue" with those in classroom. Use pen and paper! I believe, more and more every day, that doing so will make your brain perform activities that help **your own learning**.

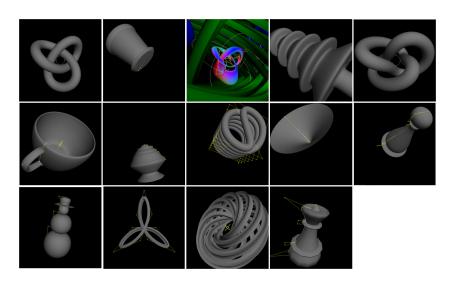
NOTE: Contemplate if you could watch the lecture videos with some friends who would also like to learn computer graphics? Get some pizza and coke if it helps you get to the mood(?).

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#### Plan for today:

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Towards Assignment 1, the first **real one**! Exercise answers from last year's students scoring 4/5 points or more:



#### Assignment 1 is about:

- ▶ What is a curve
- ► What is a surface
- ► How to model curved surfaces
- ▶ How to tessellate smooth shapes into a triangle mesh
- ▶ How to manage normals and orientations
- All this applicable to real-time animations, procedural content generation, and understanding the inner workings of modeling software we'd like to create for graphical artists and engineers.

Hold your horses! We must understand some stuff first!

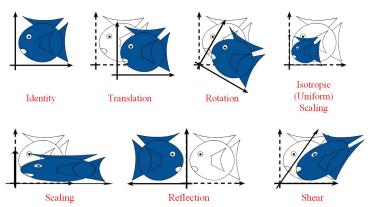
### TIEA311 - Today in Jyväskylä

Brief recap of last time - to turn our minds towards the new.

 $\rightarrow$  skim through previous slides  $\mbox{\it very}$   $\mbox{\it quickly}.$ 

### **Affine transformations**

- Include all linear transformations
  - · Applied to the vector basis
- Plus translation



### **Matrix notation**

We know how to transform the vector basis

$$\left[ \begin{array}{ccc} \mathcal{L}(\vec{b}_1) & \mathcal{L}(\vec{b}_2) & \mathcal{L}(\vec{b}_3) \end{array} \right] = \left[ \begin{array}{ccc} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{array} \right] \left[ \begin{array}{cccc} M_{1,1} & M_{1,2} & M_{1,3} \\ M_{2,1} & M_{2,2} & M_{2,3} \\ M_{3,1} & M_{3,2} & M_{3,3} \end{array} \right]$$

• We will soon add translation by a vector  $ec{t}$ 

$$\tilde{p} \Rightarrow \tilde{p} + \vec{t}$$

# Linear component

$$ilde{p} = ilde{o} + \sum_i c_i ec{b_i} = \left[ egin{array}{ccc} ec{b_1} & ec{b_2} & ec{b_3} & ec{o} \end{array} 
ight] \left[ egin{array}{ccc} c_1 \ c_2 \ c_3 \ 1 \end{array} 
ight]$$



$$ilde{o} + \sum_i c_i \mathcal{L}(ec{b_i}) = \left[ egin{array}{ccc} ec{b_1} & ec{b_2} & ec{b_3} & ilde{o} \end{array} 
ight] \left[ egin{array}{ccc} M_{11} & M_{12} & M_{13} & 0 \ M_{21} & M_{22} & M_{23} & 0 \ M_{31} & M_{32} & M_{33} & 0 \ 0 & 0 & 0 & 1 \end{array} 
ight] \left[ egin{array}{c} c_1 \ c_2 \ c_3 \ 1 \end{array} 
ight]$$

Note how we leave the fourth component alone

## **Translation component**

$$\tilde{p} \Rightarrow \tilde{p} + \vec{t}$$

Express translation vector t in the basis

$$ec{t} = \left[ egin{array}{ccc} ec{b_1} & ec{b_2} & ec{b_3} \end{array} 
ight] \left[ egin{array}{c} M_{14} \ M_{24} \ M_{34} \end{array} 
ight]$$

### **Translation**

$$ilde{p} = ilde{o} + \sum_i c_i ec{b_i} = \left[ egin{array}{ccc} ec{b_1} & ec{b_2} & ec{b_3} & ec{o} \end{array} 
ight] \left[ egin{array}{ccc} c_1 \ c_2 \ c_3 \ 1 \end{array} 
ight]$$



# Full affine expression

$$ilde{p} = ilde{o} + \sum_i c_i ec{b_i} = \left[ egin{array}{ccc} ec{b_1} & ec{b_2} & ec{b_3} & ilde{o} \end{array} 
ight] \left[ egin{array}{ccc} c_1 \ c_2 \ c_3 \ 1 \end{array} 
ight]$$



$$ilde{o} + ec{t} + \sum_i c_i \mathcal{L}(ec{b_i}) = \left[ egin{array}{cccc} ec{b_1} & ec{b_2} & ec{b_3} & ilde{o} \end{array} 
ight] \left[ egin{array}{cccc} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ 0 & 0 & 0 & 1 \end{array} 
ight] \left[ egin{array}{cccc} c_2 \\ c_3 \\ 1 \end{array} 
ight]$$
 Which tells us both how to get a new frame ftM

or how to get the coordinates Mc after transformation

# **Questions?**

## More notation properties

- If the fourth coordinate is zero, we get a vector
- Subtracting two points:

$$ilde{p}=ec{f^t} \left[egin{array}{c} c_1 \ c_2 \ c_3 \ 1 \end{array}
ight] \qquad \qquad ilde{p'}=ec{f^t} \left[egin{array}{c} c_1' \ c_2' \ c_3' \ 1 \end{array}
ight]$$

• Gives us 
$$\tilde{p}-\tilde{p'}=ar{f^t}egin{bmatrix} c_1-c_1'\\c_2-c_2'\\c_3-c_3'\\0 \end{bmatrix}$$
 a vector (last coordinate = 0)

## More notation properties

Adding a point

$$ilde{p}=ar{f}^t \left[egin{array}{c} c_1 \ c_2 \ c_3 \ 1 \end{array}
ight] \qquad ext{to a vector}$$

$$ec{v}=ec{f}^t \left[egin{array}{cc} c_1' \ c_2' \ c_3' \ 0 \end{array}
ight]$$

• Gives us 
$$\widetilde{p}+\vec{v}=\vec{f}^t \left[ \begin{array}{c} c_1+c_1'\\ c_2+c_2'\\ c_3+c_3'\\ 1 \end{array} \right]$$

a point (4th coordinate=1)

## More notation properties

vectors are not affected by the translation part

$$\left[\begin{array}{ccccc} \vec{b_1} & \vec{b_2} & \vec{b_3} & \tilde{o} \end{array}\right] \left[\begin{array}{ccccc} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} c_1 \\ c_2 \\ c_3 \\ 0 \end{array}\right]$$

- because their 4th coordinate is 0
- If I rotate my moving car in the world, I want its motion to rotate
- If I translate it, motion should be unaffected

# **Questions?**

### Further Examples (Affine transforms)

Look at the implementation of the Matrix4f class in our example codes, found in the files Matrix4f.cpp and Matrix4f.h

Really, do it. (And keep doing it all the time!)

Observe how the fourth coordinate is used to implement 3D frames and affine transforms of points. And linear transforms of directions. A straightforward way to do many things is to build a proper 4x4 matrix and then multiply. **Not much code**, actually!

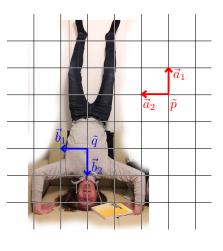
In Assignment 1 you will **avoid a lot of tears** by figuring out how (and when and why) to use the provided constructors and the operator  $\star$  to transform points and frames suitably.

Also, this stuff is a key thing in computer graphics!



Your teacher **did** promise to stand on his head, if it could help you to learn graphics?

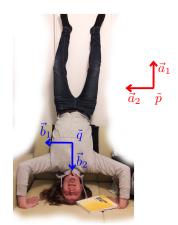
In order to avoid injury, the situation has been staged beforehand.



#### "Midterm!"

Using righthanded 2D coordinate system (y) opens "left" of x) and visual inspection of red "world" frame  $\vec{a}$  and blue "model" frame  $\vec{b}$  fill in the matrix:

$$\vec{\mathbf{b}}^T = \vec{\mathbf{a}}^T \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$



#### Observe:

- Affine frames transform fine without coordinates!
- Just scales and sums relative to parent.
- ► Only points and directions (origin and linear basis).

#### Distances?

- Need a concept of a metric. "Measure tape" in our usual, Euclidean 3D world.  $\rightarrow$  live example on
  - lecture video

Point of view ("independently discovered" by your teacher while thinking about this course and its primary target audience):

- ► Mathematics is structured much like careful object oriented design (and vice versa)! (Opinions may vary!)
- ► Math defines hierarchies of general and specific "classes" that model minimal, essential properties and methods found in worlds real and imagined.
- ➤ Vector space models "arrows", affine space also "points".

  Metric spaces model distances but not necessarily
  arrows, ...like "interfaces" of Java classes! (Can be
  combined or "multiply inherited" in a concrete class).
- ► Euclidean space R³ models distances and angles specifically in a Cartesian (orthonormal) coordinate system through dot product (which we'll learn soon-ish).
- ► Analogy from Assignment 0: C++ STL models a generic "stream" and specific "input stream" or "output stream" and finally an input stream instance called "cin" for you to "cin >> x >> y >> z;". This **behaviour** works for **any** istream!

Point of view cont'd ("independently discovered" by your teacher while thinking about this course and its primary target audience):

- ► Mathematics is nothing to be afraid of. Instead, we should embrace it. And, for example, in graphics, we must.
- Mathematics has been incrementally developed, peer reviewed, and **thoroughly tested** for the last 2500 years or so. (Compare with the 70 years we've had computer programming.)
- ► Resultingly, for many purposes, mathematics Just Works.
- ► We can apply math like we apply any class library: by learning how to read its documentation and constructing suitable instances for our current tasks!

That said, let us open the "API of mathematics", on page "vector spaces, linear maps (≈matrices), and their applications in cool CGI".

### TIEA311 - Today in Jyväskylä

The time allotted for this lecture is now over.

Now: Break until tomorrow morning. Sleep if you have time.

But also try to wake up and come to the lecture!

We will pick up our thoughts soon enough!