

# TIEA311

## Tietokonegrafiikan perusteet

kevät 2019

(“Principles of Computer Graphics” – Spring 2019)

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# TIEA311 Tietokonegrafiikan perusteet – kevät 2019 ("Principles of Computer Graphics" – Spring 2019)

Adapted from: *Wojciech Matusik*, and *Frédo Durand*: 6.837 Computer Graphics. Fall 2012. Massachusetts Institute of Technology: MIT OpenCourseWare, <https://ocw.mit.edu/>.

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Frontpage of the local course version, held during Spring 2019 at the Faculty of Information technology, University of Jyväskylä:

<http://users.jyu.fi/~nieminen/tgp19/>

# TIEA311 - Today in Jyväskylä

Super fast-forward!

Today we **rush** through the MIT OCW slides about **color**.

Notice that we'll end up with our "old friend": intensities of red, green, and blue (and "alpha" for transparency). But the following things are worth noticing:

- ▶ Color and the human visual processing system is a colorful research topic on its own
- ▶ Even as we use RGBA in real-time graphics, we need to know at least something of **why** we do that
- ▶ True "hardcore" photorealistic rendering needs more than just RGBA!
- ▶ Some of the things touched on the slides have quite interesting connections to our top research in Jyväskylä!!

# Color



*Wojciech Matusik MIT EECS*

Many slides courtesy of Victor Ostromoukhov, Leonard McMillan, Bill Freeman, Fredo Durand

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*Does color puzzle you?*

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# *Answer*

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- It's all linear algebra

# *Plan*

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- Spectra
- Cones and spectral response
- Color blindness and metamers
- Color matching
- Color spaces

# Color

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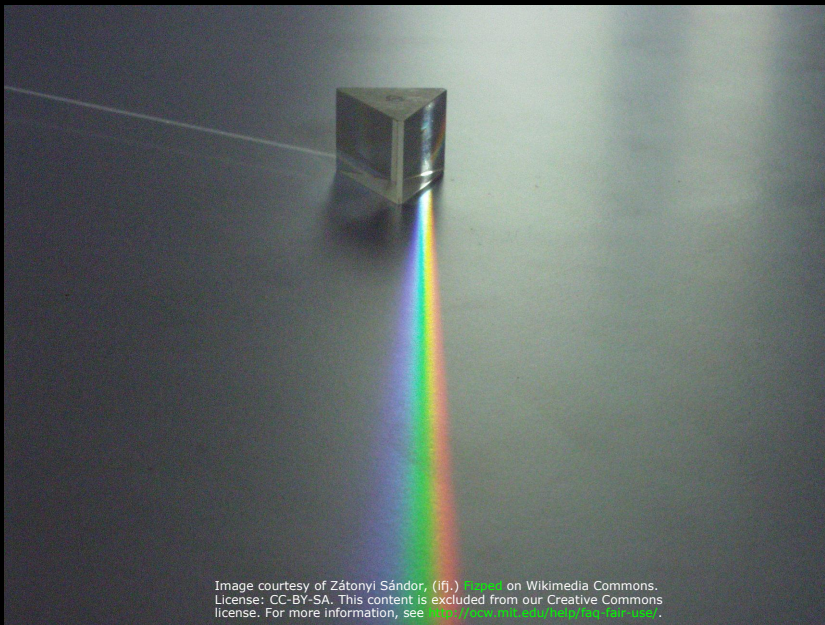
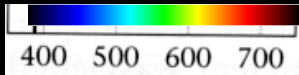


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# *Spectrum*

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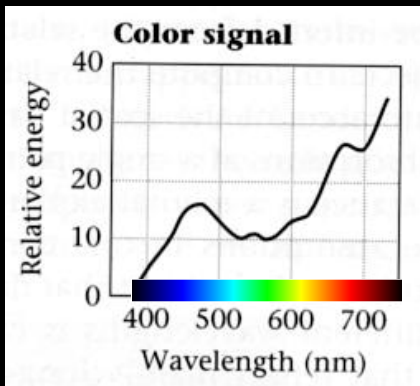


Light is a wave

Visible: between 450 and 700nm

# Spectrum

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Light is characterized by its spectrum:  
the amount of energy at each wavelength  
This is a full distribution:  
one value per wavelength  
(infinite number of values)

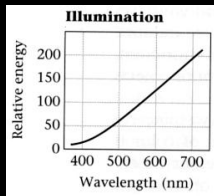
# Light-Matter Interaction

Where spectra come from:

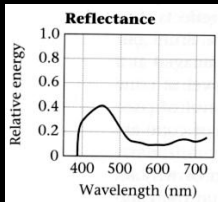
- light source spectrum
  - object reflectance (aka spectral albedo)
- get multiplied wavelength by wavelength

There are different physical processes that explain this multiplication

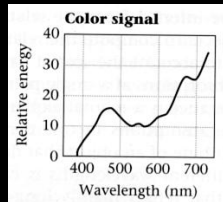
e.g. absorption, interferences



\*



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# Spectrum demo

- Diffraction grating:
  - shifts light as a function of wavelength
  - Allows you to see spectra
  - In particular, using a slit light source, we get a nice band showing the spectrum
- See the effect of filters
- See different light source spectra

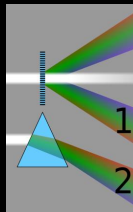
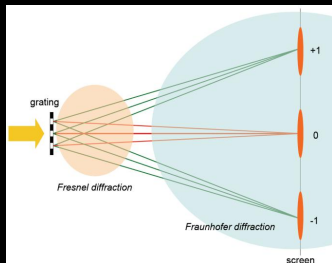


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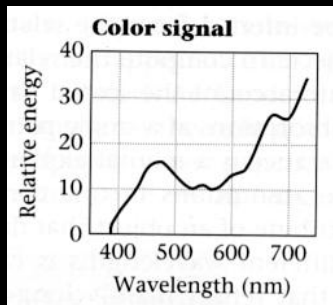
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# Questions?

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So far, physical side of colors: **spectra**

an infinite number of values  
(one per wavelength)



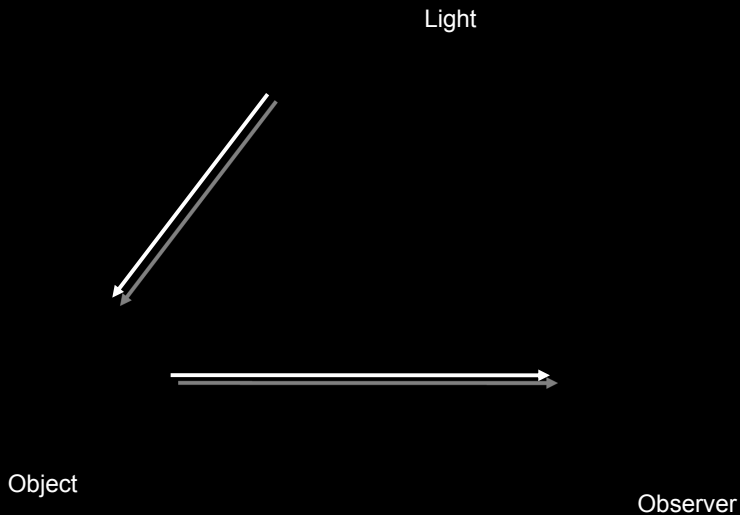
# *Plan*

---

- Spectra
- **Cones and spectral response**
- Color blindness and metamers
- Color matching
- Color spaces

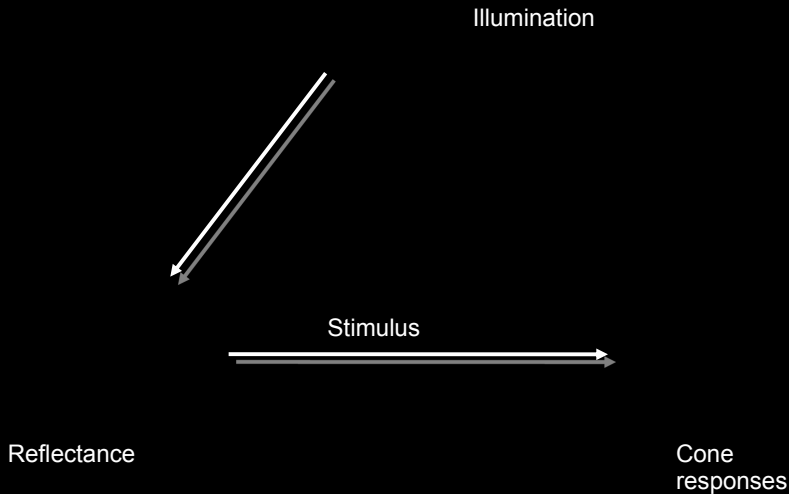
# *What is Color?*

---



# *What is Color?*

---



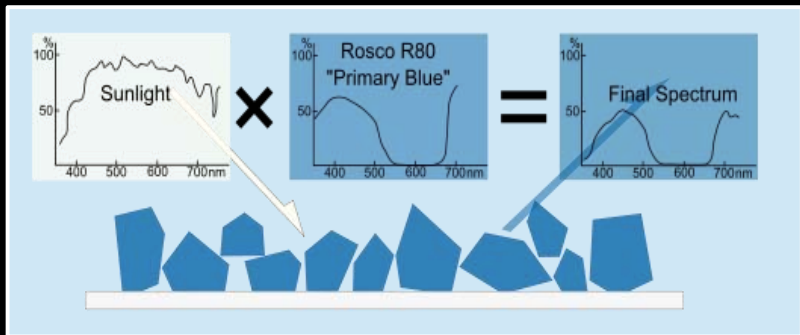


# What is Color?

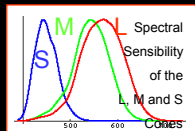
Light  
Illumination

Object  
Reflectance

Final stimulus

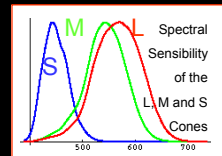
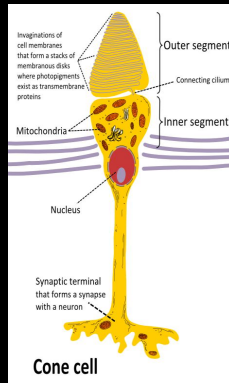
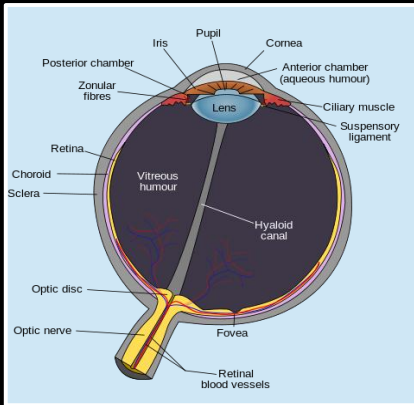


Then the cones in the eye interpret the stimulus



# Cones

- We focus on low-level aspects of color
  - Cones and early processing in the retina
- We won't talk about rods (night vision)



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## *Summary (and time for questions)*

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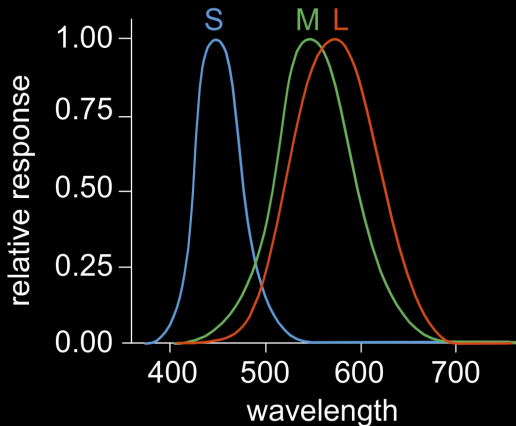
- Spectrum: infinite number of values
  - can be multiplied
  - can be added
- Light spectrum multiplied by reflectance spectrum
  - spectrum depends on illuminant
- Human visual system is complicated

# *Cone spectral sensitivity*

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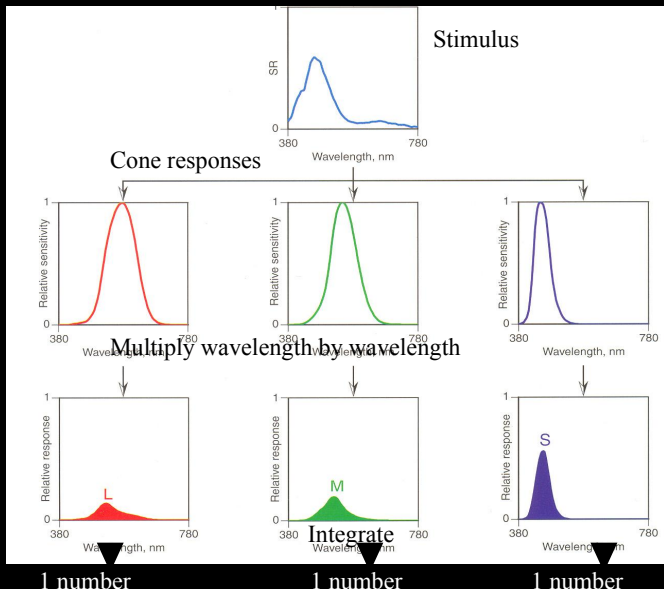
- Short, Medium and Long wavelength
- Response for a cone

$$= \int \lambda \text{ stimulus}(\lambda) * \text{response}(\lambda) d\lambda$$



# Cone response

Start from infinite number of values (one per wavelength)

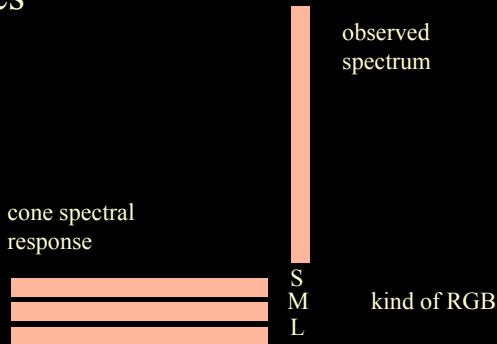


End up with 3 values (one per cone type)

# *For matrix lovers*

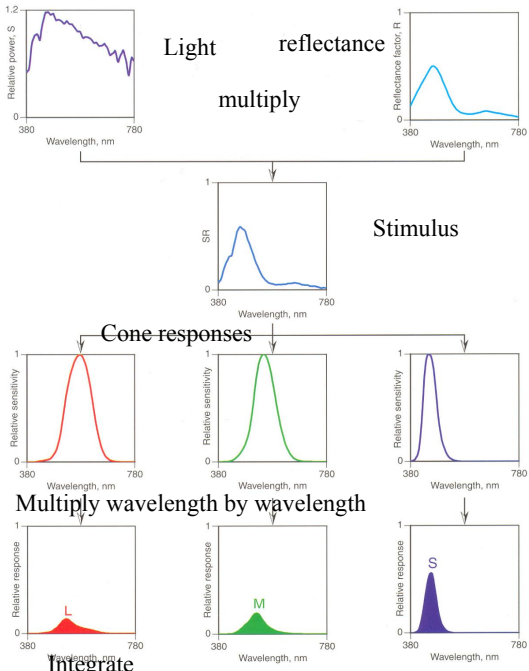
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- Spectrum: big long vector size  $N$  where  $N \rightarrow \infty$
- Cone response:  $3 \times N$  matrix of individual responses



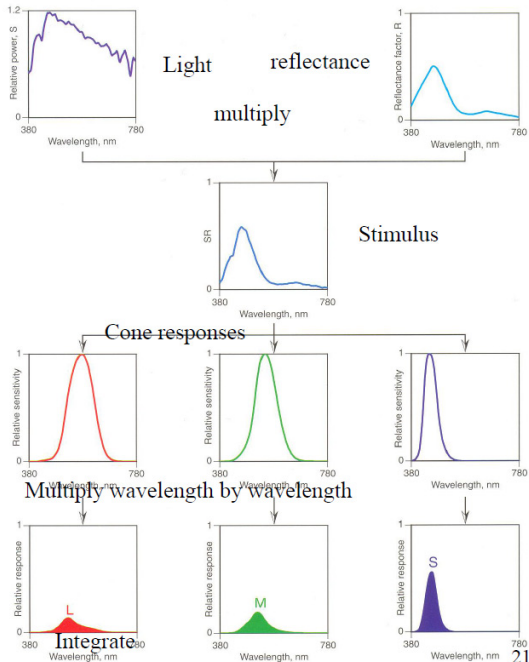
# Big picture

- It's all linear!



# Big picture

- It's all linear!
  - multiply
  - add
- But
  - non-orthogonal basis
  - infinite dimension
  - light must be positive
- Depends on light source





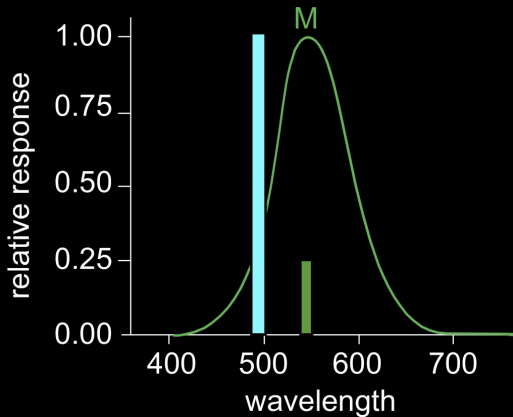
*Questions?*

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# *A cone does not “see” colors*

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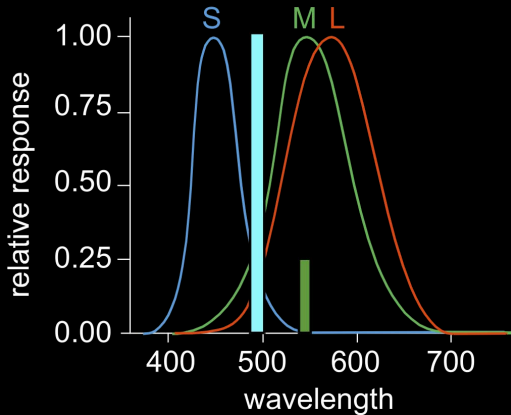
- Different wavelength, different intensity
- Same response



# Response comparison

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- Different wavelength, different intensity
- But different response for different cones

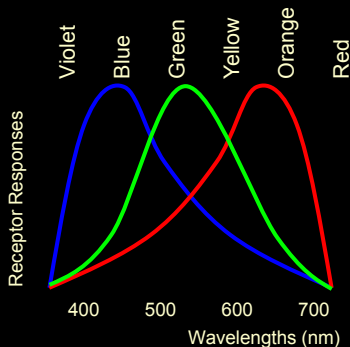


# *von Helmholtz 1859: Trichromatic theory*

- Colors as relative responses (ratios)



- Short wavelength receptors
- Medium wavelength receptors
- Long wavelength receptors



*Questions?*

---

# *Plan*

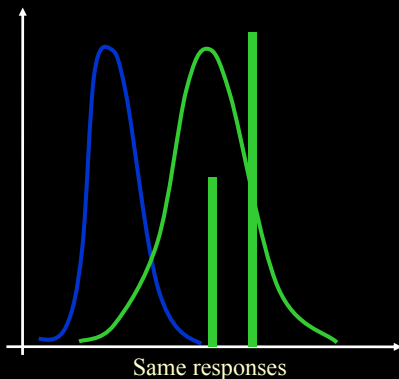
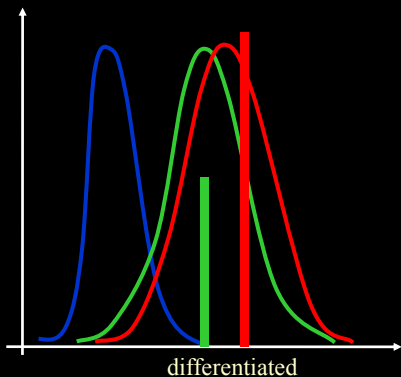
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- Spectra
- Cones and spectral response
- **Color blindness and metamers**
- Color matching
- Color spaces

# *Color blindness*

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- Classical case: 1 type of cone is missing (e.g. red)
- Makes it impossible to distinguish some spectra



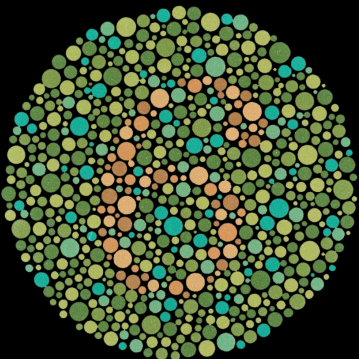
# *Color blindness – more general*

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- Dalton
- 8% male, 0.6% female
- Genetic
- Dichromate (2% male)
  - One type of cone missing
  - L (protanope), M (deuteranope), S (tritanope)
- Anomalous trichromat
  - Shifted sensitivity



# Color blindness test



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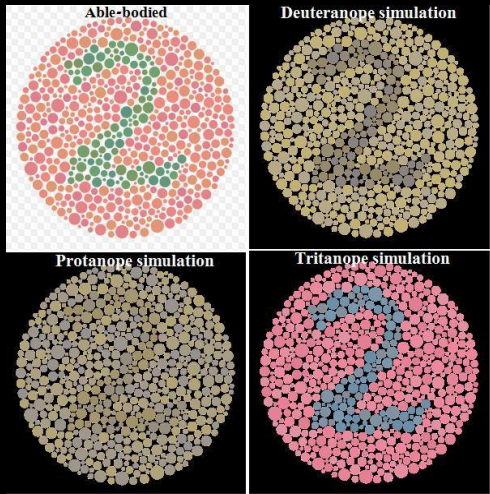
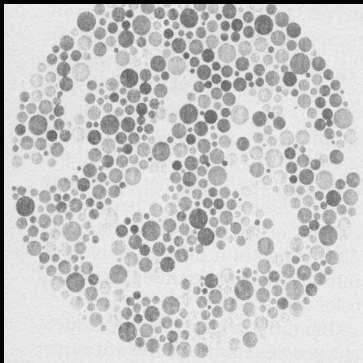
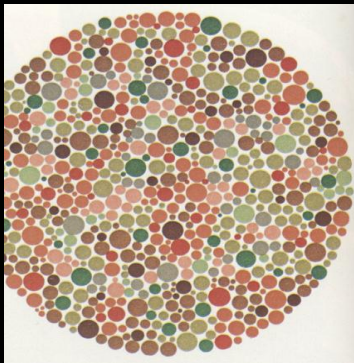


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# *Color blindness test*

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- Maze in subtle intensity contrast
- Visible only to color blinds
- Color contrast overrides intensity otherwise



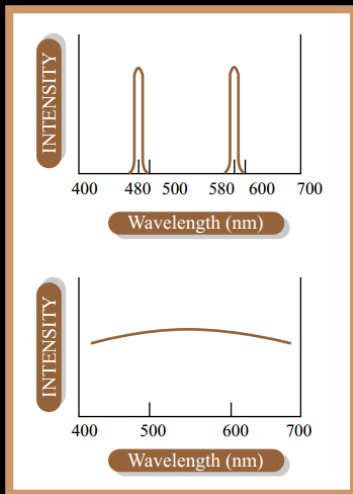
# Questions?

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- Links:
  - Vischeck shows you what an image looks like to someone who is colorblind.
  - <http://www.vischeck.com/vischeck/>
  - Daltonize, changes the red/green variation to brightness and blue/yellow variations.
  - <http://www.vischeck.com/dalton>
  - <http://www.vischeck.com/daltonize/runDaltonize.php>

# Metamers

- We are all color blind!
- These two different spectra elicit the same cone responses
- Called metamers



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# *Good news: color reproduction*

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- 3 primaries are (to a first order) enough to reproduce all colors

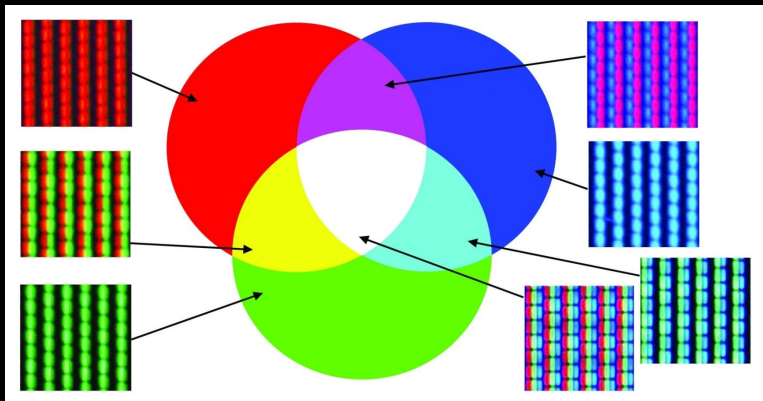


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# Recap

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- Spectrum: infinite number of values
- projected according to cone spectral response  
=> 3 values
- metamers: spectra that induce the same response  
(physically different but look the same)
  
- Questions?

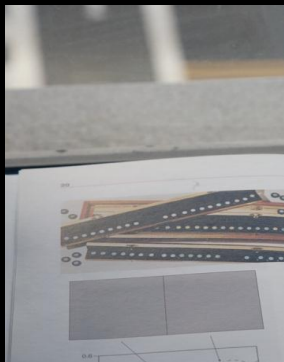
# *Metamerism & light source*

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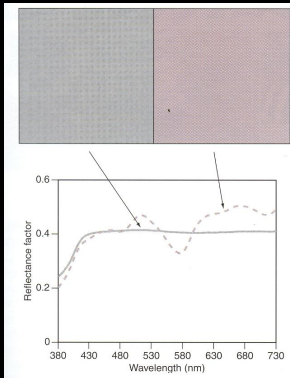
- Metamers under a given light source
- May not be metamers under a different lamp

# Illuminant metamerism example

- Two grey patches in Billmeyer & Saltzman's book look the same under daylight but different under neon or halogen (& my camera agrees ;-)



Daylight



Scan (neon)

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Hallogen



## *Bad consequence: cloth matching*

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- Clothes appear to match in store (e.g. under neon)
- Don't match outdoor

# Recap

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- Spectrum is an infinity of numbers
- Projected to 3D cone-response space
  - for each cone, multiply per wavelength and integrate
  - a.k.a. dot product
- Metamerism: infinite-D points projected to the same 3D point  
(different spectrum, same perceived color)
  - affected by illuminant
  - enables color reproduction with only 3 primaries

*Questions?*

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# *Analysis & Synthesis*

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- Now let's switch to technology
- We want to measure & reproduce color as seen by humans
- No need for full spectrum
- Only need to match up to metamerism

# Analysis & Synthesis

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- Focus on additive color synthesis
- We'll use 3 primaries (e.g. red green and blue) to match all colors

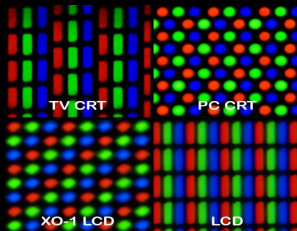


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- What should those primaries be?
- How do we tell the amount of each primary needed to reproduce a given target color?

# Warning

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Tricky thing with spectra & color:

- Spectrum for the stimulus / synthesis
  - Light, monitor, reflectance
- Response curve for receptor /analysis
  - Cones, camera, scanner



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They are usually not the same

There are good reasons for this

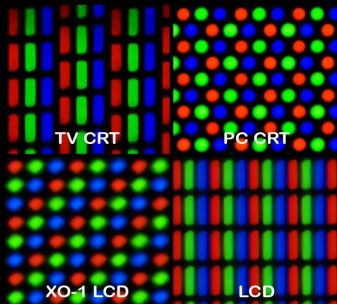
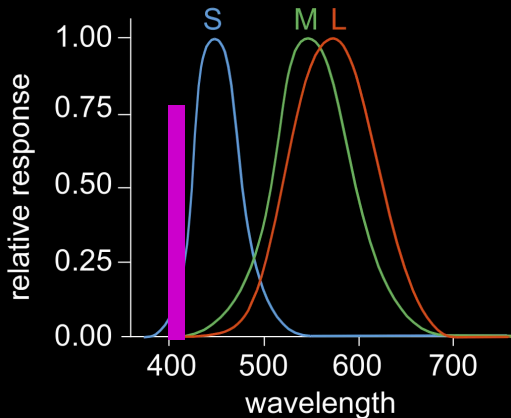


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# Additive Synthesis - wrong way

---

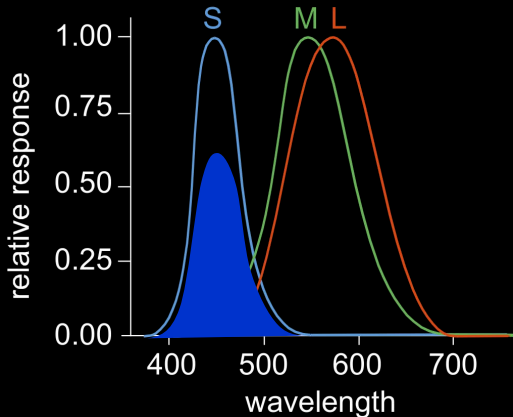
- Take a given stimulus and the corresponding responses  $s$ ,  $m$ ,  $l$  (here 0.5, 0, 0)



# Additive Synthesis - wrong way

---

- Use it to scale the cone spectra (here  $0.5 * S$ )
- You don't get the same cone response!  
(here 0.5, 0.1, 0.1)

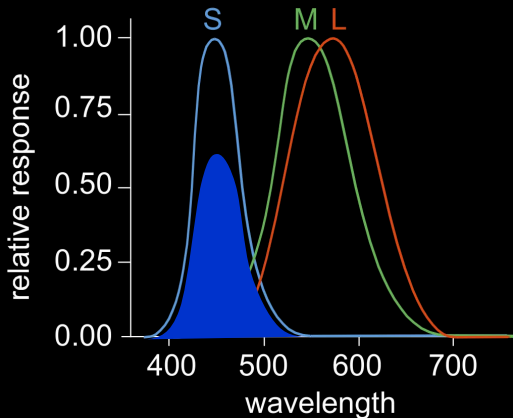




# What's going on?

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- The three cone responses are not orthogonal
- i.e. they overlap and “pollute” each other



# *Fundamental problems*

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- Spectra are infinite-dimensional
- Only positive values are allowed
- Cones are non-orthogonal/overlap

# Summary

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- Physical color
  - Spectrum
  - multiplication of light & reflectance spectrum
- Perceptual color
  - Cone spectral response: 3 numbers
  - Metamers: different spectrum, same responses
    - Color matching, enables color reproduction with 3 primaries
- Fundamental difficulty
  - Spectra are infinite-dimensional (full function)
  - Projected to only 3 types of cones
  - Cone responses overlap / they are non-orthogonal
    - Means different primaries for analysis and synthesis
  - Negative numbers are not physical

*Questions?*

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# *Standard color spaces*

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- We need a principled color space
- Many possible definition
  - Including cone response (LMS)
  - Unfortunately not really used, (because not known at the time)
- The good news is that color vision is linear and 3-dimensional, so any new color space based on color matching can be obtained using 3x3 matrix
  - But there are also non-linear color spaces (e.g. Hue Saturation Value, Lab)

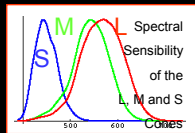
# Overview

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- Most standard color space: CIE XYZ
- LMS and the various flavor of RGB are just linear transformations of the XYZ basis
  - 3x3 matrices

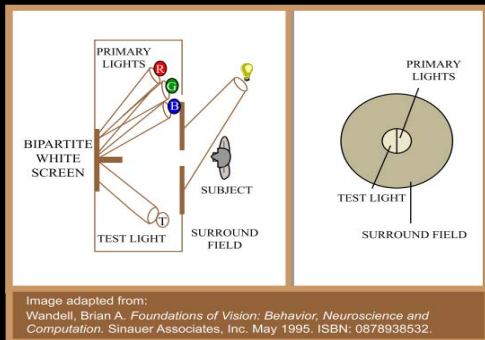
# Why not measure cone sensitivity?

- Less directly measurable
  - electrode in photoreceptor?
  - not available when color spaces were defined
- Most directly available measurement:
  - notion of metamers & color matching
  - directly in terms of color reproduction:  
**given an input color,**  
**how to reproduce it with 3 primary colors?**
  - Commission Internationale de l'Eclairage  
(International Lighting Commission)
  - Circa 1920



# CIE color matching

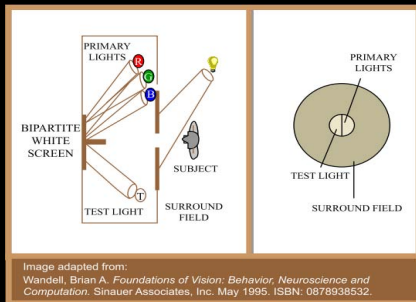
- Choose 3 synthesis primaries
- Seek to match any monochromatic light (400 to 700nm)
  - Record the 3 values for each wavelength
- By linearity, this tells us how to match any light





# CIE color matching

- Primaries (synthesis) at 435.8, 546.1 and 700nm
  - Chosen for robust reproduction, good separation in red-green
  - Don't worry, we'll be able to convert it to any other set of primaries (Linear algebra to the rescue!)
- Resulting 3 numbers for each input wavelength are called tristimulus values



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**Now, our interactive  
feature!**

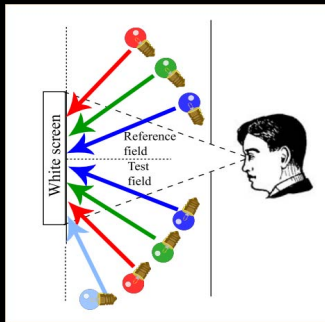
**You are...**

**THE LAB RAT**



# Color Matching Problem

- Some colors cannot be produced using only positively weighted primaries
- Solution: add light on the other side!

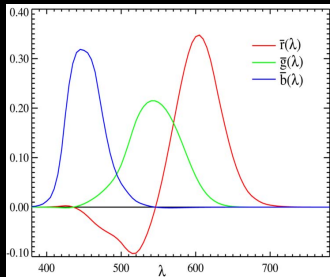


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# CIE color matching

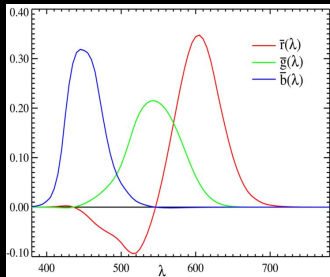
---

- Meaning of these curves: a monochromatic wavelength  $\lambda$  can be reproduced with  $b(\lambda)$  amount of the 435.8nm primary,  $+g(\lambda)$  amount of the 546.1 primary,  $+r(\lambda)$  amount of the 700 nm primary
- This fully specifies the color perceived by a human
- Careful: this is not your usual rgb



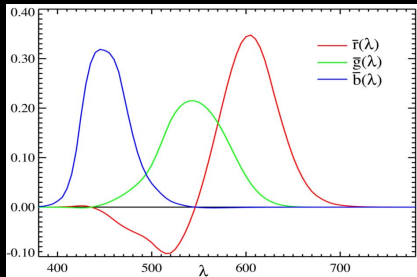
# CIE color matching

- Meaning of these curves: a monochromatic wavelength  $\lambda$  can be reproduced with  $b(\lambda)$  amount of the 435.8nm primary,  $+g(\lambda)$  amount of the 546.1 primary,  $+r(\lambda)$  amount of the 700 nm primary
- This fully specifies the color perceived by a human
- However, note that one of the responses can be negative
  - Those colors cannot be reproduced by those 3 primaries.



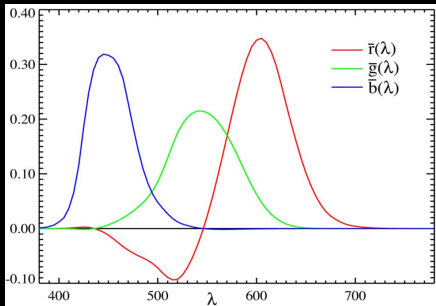
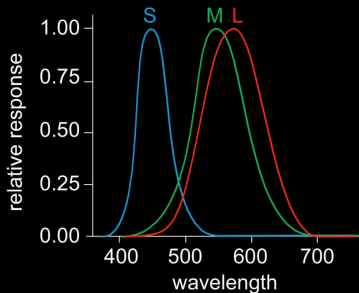
# CIE color matching: what does it mean?

- If I have a given spectrum  $X$
- I compute its response to the 3 matching curves (multiply and integrate)
- I use these 3 responses to scale my 3 primaries (435.8, 546.1 and 700nm)
- I get a metamer of  $X$  (perfect color reproduction)



# Relation to cone curves

- Project to the same subspace
  - b, g, and r are linear combinations of S, M and L
- Related by 3x3 matrix.
- Unfortunately unknown at that time. This would have made life a lot easier!





# Recap

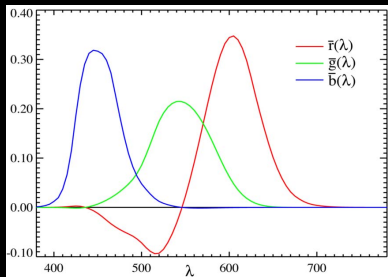
---

- Spectra : infinite dimensional
- Cones: 3 spectral responses
- Metamers: spectra that look the same (same projection onto cone responses)
- CIE measured color response:
  - chose 3 primaries
  - tristimulus curves to reproduce any wavelength
  
- Questions?

# How to build a measurement device?

---

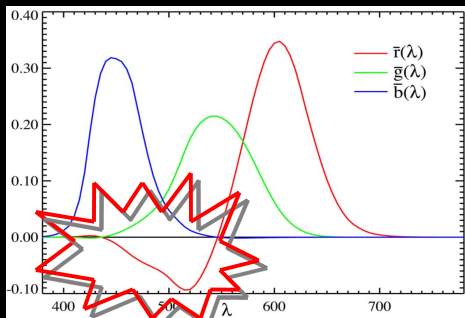
- Idea:
  - Start with light sensor sensitive to all wavelength
  - Use three filters with spectra  $b$ ,  $r$ ,  $g$
  - measure 3 numbers
- This is pretty much what the eyes do!



# CIE's problem

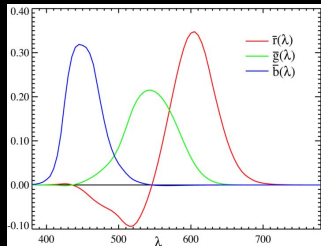
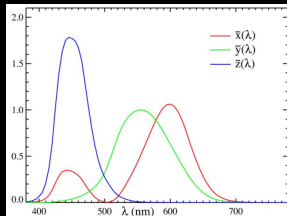
---

- Idea:
  - Start with light sensor sensitive to all wavelength
  - Use three filters with spectra  $b$ ,  $r$ ,  $g$
  - measure 3 numbers
- But for those primaries, we need negative spectra



# CIE's problem

- Obvious solution:  
use cone response!
  - but unknown at the time
- => new set of tristimulus curves
  - linear combinations of b, g, r
  - pretty much add enough b and g until r is positive



# Chromaticity diagrams

---

- 3D space are tough to visualize
- Usually project to 2D for clarity
- Chromaticity diagram:
  - normalize against  $X + Y + Z$ :



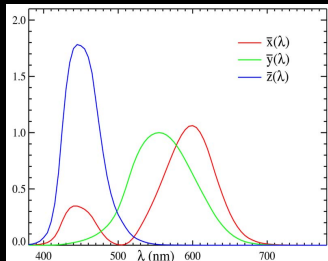
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$$x = \frac{X}{X + Y + Z}; \quad y = \frac{Y}{X + Y + Z}; \quad z = \frac{Z}{X + Y + Z}$$

# CIE XYZ -recap

---

- THE standard for color specification
- Lots of legacy decision - I wish it were LMS
- Based on color matching
  - 3 monochromatic primaries
  - Subjects matched every wavelength
  - Tricks to avoid negative numbers
  - These 3 values “measure” or describe a perceived color.



*Questions?*

---

## Other primaries

---

- We want to use a new set of primaries
  - e.g. the spectra of R, G & B in a projector or monitor
- By linearity of color matching, can be obtained from XYZ by a 3x3 matrix

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.24 & -1.54 & -0.50 \\ -0.97 & 1.88 & 0.04 \\ 0.06 & -0.20 & 1.06 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.41 & 0.36 & 0.18 \\ 0.21 & 0.72 & 0.07 \\ 0.02 & 0.12 & 0.95 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

one example RGB space



## *Other primaries*

---

- We want to use a new set of primaries
  - e.g. the spectra of R, G & B in a projector or monitor
- By linearity of color matching, can be obtained from XYZ by a 3x3 matrix
- This matrix tells us how to match the 3 primary spectra from XYZ using the new 3 primaries

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.24 & -1.54 & -0.50 \\ -0.97 & 1.88 & 0.04 \\ 0.06 & -0.20 & 1.06 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.41 & 0.36 & 0.18 \\ 0.21 & 0.72 & 0.07 \\ 0.02 & 0.12 & 0.95 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

one example RGB space

# *XYZ to RGB & back*

---

- e.g.

[http://www.brucelindbloom.com/index.html?Eqn\\_RGB\\_XYZ\\_Matrix.html](http://www.brucelindbloom.com/index.html?Eqn_RGB_XYZ_Matrix.html)

- sRGB to XYZ

0.412424	0.212656	0.0193324
0.357579	0.715158	0.119193
0.180464	0.0721856	0.950444

## XYZ to sRGB

3.24071	-0.969258	0.0556352
-1.53726	1.87599	-0.203996
0.498571	0.0415557	1.05707

- Adobe RGB to XYZ

0.576700	0.297361	0.0270328
0.185556	0.627355	0.0706879
0.188212	0.0752847	0.991248

## XYZ to Adobe RGB

2.04148	-0.969258	0.0134455
-0.564977	1.87599	-0.118373
-0.344713	0.0415557	1.01527

- NTSC RGB to XYZ

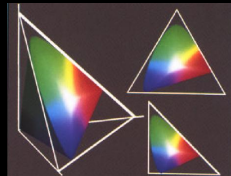
0.606734	0.298839	0.000000
0.173564	0.586811	0.0661196
0.200112	0.114350	1.11491

## XYZ to NTSC RGB

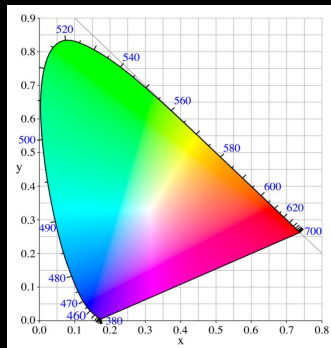
1.91049	-0.984310	0.0583744
-0.532592	1.99845	-0.118518
-0.288284	-0.0282980	0.898611

# Color gamut

- Given 3 primaries
- The realizable chromaticities lay in the triangle in  $xy$  chromaticity diagram
- Because we can only add light, no negative light



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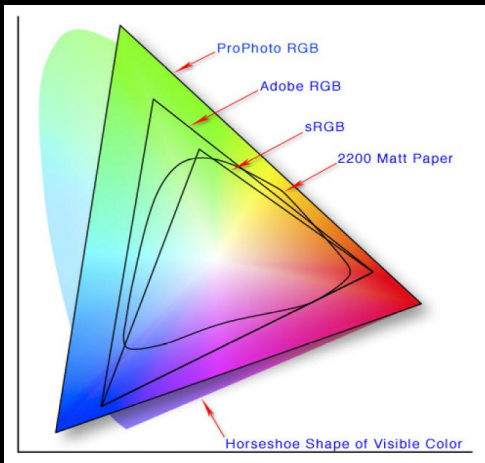


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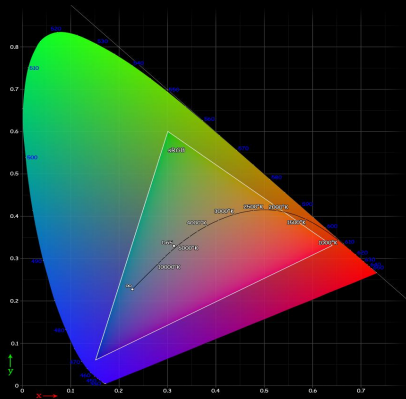


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## *In summary*

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- It's all about linear algebra
  - Projection from infinite-dimensional spectrum to a 3D response
  - Then any space based on color matching and metamerism can be converted by 3x3 matrix
- Complicated because
  - Projection from infinite-dimensional space
  - Non-orthogonal basis (cone responses overlap)
  - No negative light
- XYZ is the most standard color space
- RGB has many flavors

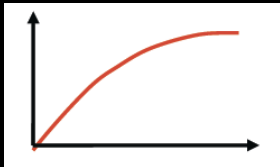
*Questions?*

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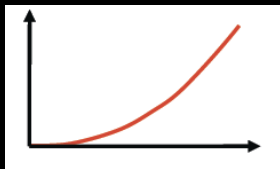
# *Gamma encoding overview*

---

- Digital images are usually not encoded linearly
- Instead, the value  $X^{1/\gamma}$  is stored



- Need to be decoded if we want linear values



# *Color quantization gamma*

---

- The human visual system is more sensitive to ratios
  - Is a grey twice as bright as another one?
- If we use linear encoding, we have tons of information between 128 and 255, but very little between 1 and 2!
- Ideal encoding?

Log

- Problems with log?
  - Gets crazy around zero

Solution: gamma



# *Color quantization gamma*

---

- The human visual system is more sensitive to ratios
  - Is a grey twice as bright as another one?
- If we use linear encoding, we have tons of information between 128 and 255, but very little between 1 and 2!
- This is why a non-linear gamma remapping of about 2.0 is applied before encoding
- True also of analog imaging to optimize signal-noise ratio

# *Color quantization gamma*

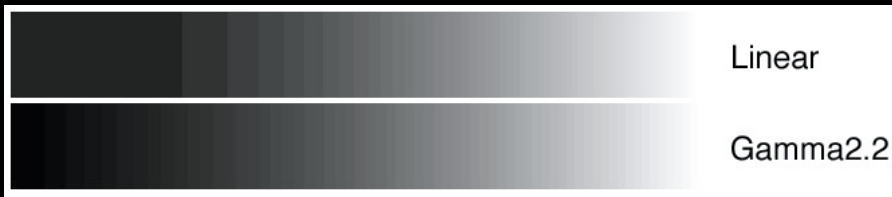
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- The human visual system is more sensitive to ratios
  - Is a grey twice as bright as another one?
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- This is why a non-linear gamma remapping of about 2.0 is applied before encoding
- True also of analog imaging to optimize signal-noise ratio

# *Gamma encoding*

---

- From Greg Ward
- Only 6 bits for emphasis



# *Important Message*

---

- Digital images are usually gamma encoded
  - Often  $\gamma = 2.2$  (but 1.8 for Profoto RGB)
- To get linear values, you must decode
  - apply  $x \Rightarrow x^\gamma$

*Questions?*

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# *Selected Bibliography*

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# Questions?

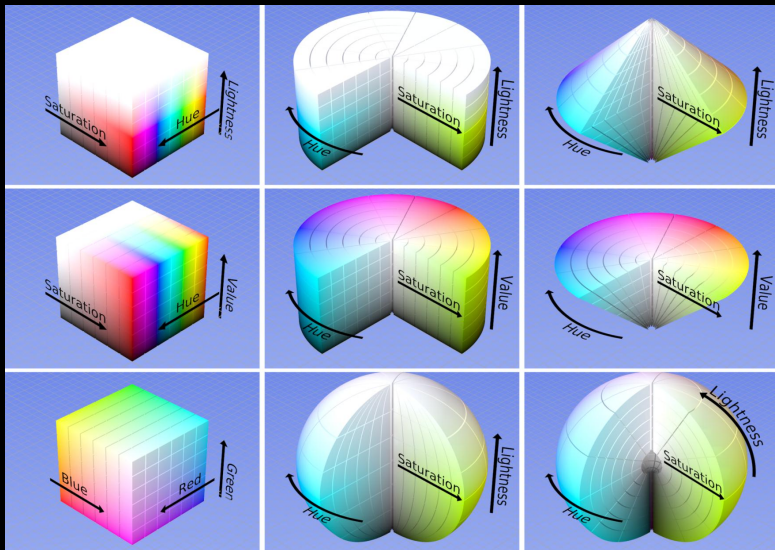


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