TIEA311
Tietokonegrafiikan perusteet
kevät 2018

("Principles of Computer Graphics" – Spring 2018)

TIEA311 Tietokonegrafiikan perusteet – kevät 2018
(“Principles of Computer Graphics” – Spring 2018)

Frontpage of the local course version, held during Spring 2018 at the Faculty of Information technology, University of Jyväskylä:
`http://users.jyu.fi/~nieminen/tgp18/`

TIEA311 - Agenda for Lecture 3 (Jan 17, 2018):

1. Discuss Assignment 0 (C++, Handout, terminology, searching the WWW)

2. Terminology and mathematics

Regarding math, we might start taking shortcuts and just "believe that the equations work", since this course is about an application, not theory itself.

NOTE: Don't worry about Assignment 1 yet! It's time will come very soon. Try to **understand as much as possible about the starter code of Assignment 0 and the included "vecmath" library**!

Let us try to “dissect” Assignment 0 somewhat together, on-screen . . .

(in 2018, we started this on lecture 2, and we’ll continue the dive on lecture 3)

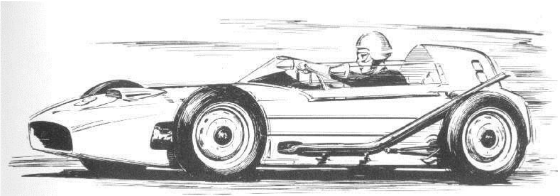
Coordinates and Transformations

MIT ECCS 6.837
Wojciech Matusik

many slides follow Steven Gortler's book

Hierarchical modeling

- **Many coordinate systems:**
  - Camera
  - Static scene
  - car
  - driver
  - arm
  - hand
  - ...



- **Makes it important to understand coordinate systems**

TIEA311 - Words of a wiser man

This course is not a math course – far from it!

But we are dealing with “math kinda stuff”, so let me cite Sheldon Axler from his “Preface to the Student” in his wonderful book “Linear Algebra Done Right”:

“You cannot expect to read mathematics the way you read a novel. If you zip through a page in less than an hour, you are probably going too fast. When you encounter the phrase “as you should verify”, you should indeed do the verification, which will usually require some writing on your part. When steps are left out, you need to supply the missing pieces. You should ponder and internalize each definition. For each theorem, you should seek examples to show why each hypothesis is necessary.”

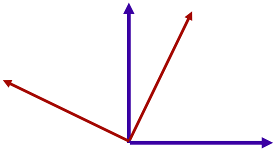
The next slide contains an obvious error. Let us find it together!

Coordinates

- We are used to represent points with tuples of coordinates such as $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$
- But the tuples are meaningless without a clear coordinate system

could be this point in the red coordinate system

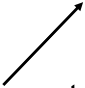
could be this point in the blue coordinate system



Different objects

- **Points**
  - represent locations

- **Vectors**
  - represent movement, force, displacement from A to B



- **Normals**
  - represent orientation, unit length

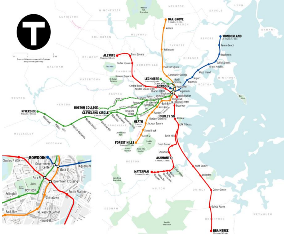
- **Coordinates**
  - numerical representation of the above objects
    in a given coordinate system

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Points & vectors are different

- The 0 vector has a fundamental meaning: no movement, no force
- Why would there be a special 0 point?

- It's meaningful to add vectors, not points
  - Boston location + NYC location =?

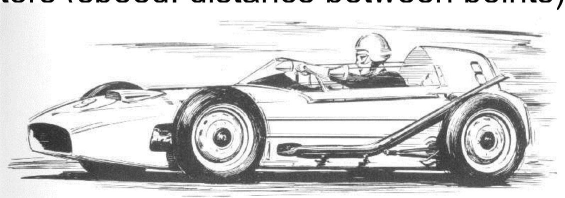


+

=?

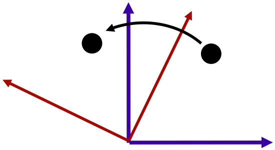
Points & vectors are different

- Moving car
  - points describe location of car elements
  - vectors describe velocity, distance between pairs of points
- If I **translate** the moving car to a different road
  - The points (location) change
  - The vectors (speed, distance between points) don't

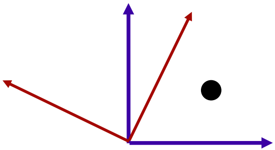


Matrices have two purposes

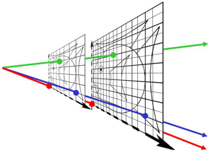
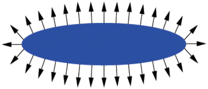
- (At least for geometry)
- Transform things
  - e.g. rotate the car from facing North to facing East



- Express coordinate system changes
  - e.g. given the driver's location in the coordinate system of the car, express it in the coordinate system of the world



Plan

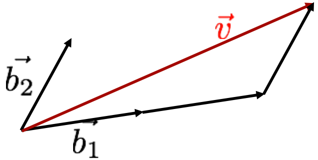
- Vectors 
- Points 
- Homogeneous coordinates 
- Normals (in the next lecture) 

Vectors (linear space)

- Formally, a set of elements equipped with addition and scalar multiplication
  - plus other nice properties
- There is a special element, the zero vector
  - no displacement, no force

Vectors (linear space)

- We can use a ***basis*** to produce all the vectors in the space:
  - Given n basis vectors $\vec{b_i}$ any vector $\vec{v}$ can be written as $$\vec{v} = \sum_i c_i \vec{b_i}$$



here:

$$\vec{v} = 2\vec{b_1} + \vec{b_2}$$

Linear algebra notation

$$\vec{v} = c_1\vec{b_1} + c_2\vec{b_2} + c_3\vec{b_3}$$

- can be written as

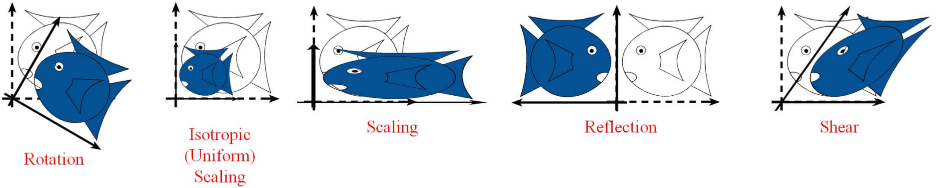
$$\begin{bmatrix} \vec{b_1} & \vec{b_2} & \vec{b_3} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

- Nice because it makes the basis (coordinate system) explicit
- Shorthand:

$$\vec{v} = \vec{\mathbf{b}}^t \mathbf{c}$$

- where bold means triplet, t is transpose

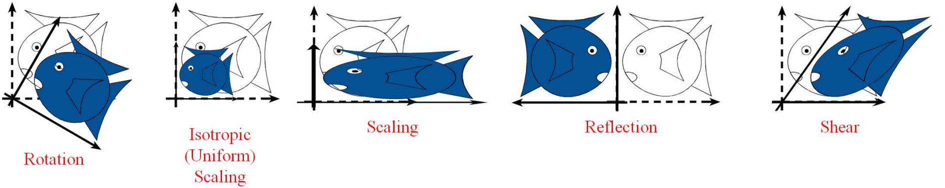
Linear transformation



Courtesy of Prof. Fredo Durand. Used with permission.

- Transformation $\mathcal{L}$ of the vector space

Linear transformation



- Transformation $\mathcal{L}$ of the vector space so that

$$\mathcal{L}(\vec{v}+\vec{u}) = \mathcal{L}(\vec{v}) + \mathcal{L}(\vec{u})$$

$$\mathcal{L}(\alpha\vec{v}) = \alpha\mathcal{L}(\vec{v})$$

- Note that it implies $\mathcal{L}(\vec{0}) = \vec{0}$
- Notation $\vec{v} \Rightarrow \mathcal{L}(\vec{v})$ for transformations

Matrix notation

- Linearity implies

$$\mathcal{L}(\vec{v}) = \mathcal{L}\left(\sum_i c_i \vec{b_i}\right) = \qquad ?$$

Matrix notation

- Linearity implies

$$\mathcal{L}(\vec{v}) = \mathcal{L}\left(\sum_i c_i \vec{b_i}\right) = \sum_i c_i \mathcal{L}(\vec{b_i})$$

- i.e. we only need to know the basis transformation
- or in algebra notation

$$\begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \Rightarrow \begin{bmatrix} \mathcal{L}(\vec{b}_1) & \mathcal{L}(\vec{b}_2) & \mathcal{L}(\vec{b}_3) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

Algebra notation

- The $\mathcal{L}(\vec{b_i})$ are also vectors of the space
- They can be expressed in the basis

...

Algebra notation

- The $\mathcal{L}(\vec{b_i})$ are also vectors of the space
- They can be expressed in the basis
  for example:

$$\mathcal{L}(\vec{b}_1) = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix} \begin{bmatrix} M_{1,1} \\ M_{2,1} \\ M_{3,1} \end{bmatrix}$$

- which gives us

$$\begin{bmatrix} \mathcal{L}(\vec{b}_1) & \mathcal{L}(\vec{b}_2) & \mathcal{L}(\vec{b}_3) \end{bmatrix} = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix} \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} \\ M_{2,1} & M_{2,2} & M_{2,3} \\ M_{3,1} & M_{3,2} & M_{3,3} \end{bmatrix}$$

Recap, matrix notation

$$\begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix} \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} \\ M_{2,1} & M_{2,2} & M_{2,3} \\ M_{3,1} & M_{3,2} & M_{3,3} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

- Given the coordinates ***c*** in basis $\vec{\mathrm{b}}$
  the transformed vector has coordinates Mc in $\vec{\mathrm{b}}$