TIEA311
Tietokonegrafiikan perusteet
kevät 2017

("Principles of Computer Graphics" – Spring 2017)

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Adapted from: *Wojciech Matusik*, and *Frédo Durand*: 6.837 Computer Graphics. Fall 2012. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu/.

Frontpage of the local course version, held during Spring 2017 at the Faculty of Information technology, University of Jyväskylä:
`http://users.jyu.fi/~nieminen/tgp17/`

TIEA311 - Event horizon of this course

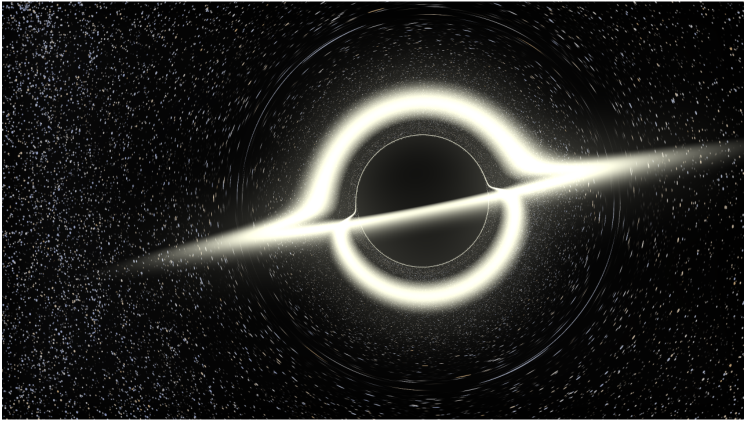


Image and rendering code by Sakari Kapanen. Used with permission. Source:

`https://github.com/flannelhead/blackstar/blob/master/example.png` Code repo:

`https://github.com/flannelhead/blackstar`

TIEA311 - Plan for the last lecture

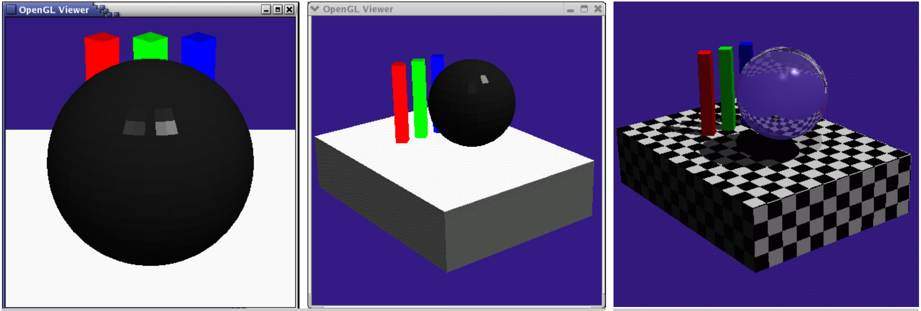
- First, lecture about rasterization.
- Break.
- Final discussion and reflection of what we may have learned about graphics, programming, and problem solving in our two months.

Graphics Pipeline & Rasterization

Image removed due to copyright restrictions.

How Do We Render Interactively?

- Use graphics hardware, via OpenGL or DirectX
  - OpenGL is multi-platform, DirectX is MS only



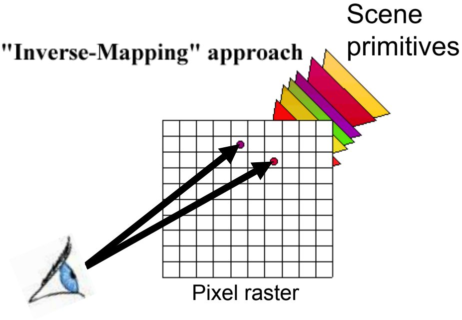
OpenGL rendering *Our ray tracer*

- Most global effects available in ray tracing will be sacrificed for speed, but some can be approximated

Ray Casting vs. GPUs for Triangles

Ray Casting

```
For each pixel (ray)
  For each triangle
    Does ray hit triangle?
```
Keep closest hit



Ray Casting vs. GPUs for Triangles

Ray Casting

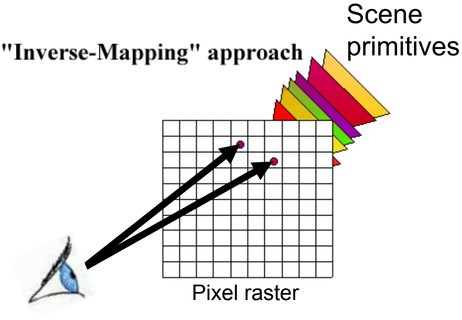
```
For each pixel (ray)
   For each triangle
      Does ray hit triangle?
```

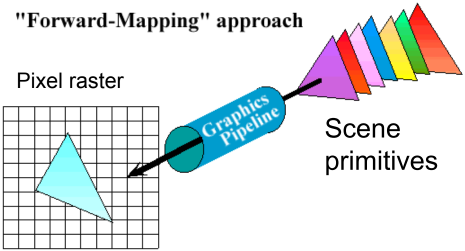
Keep closest hit

GPU

```
For each triangle
   For each pixel
      Does triangle cover pixel?
```

Keep closest hit





Ray Casting vs. GPUs for Triangles

Ray Casting

```
For each pixel (ray)
   For each triangle
      Does ray hit triangle?
```

Keep closest hit

GPU

```
For each triangle
   For each pixel
      Does triangle cover pixel?
```

Keep closest hit

It's just a different order of the loops!

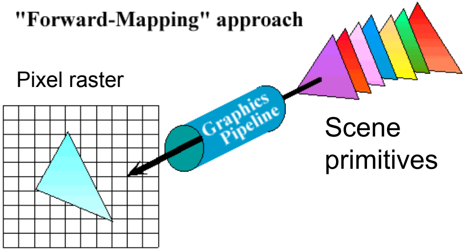
GPUs do Rasterization

- The process of taking a triangle and figuring out which pixels it covers is called **rasterization**

GPU

```
For each triangle
  For each pixel
    Does triangle cover pixel?
```

Keep closest hit

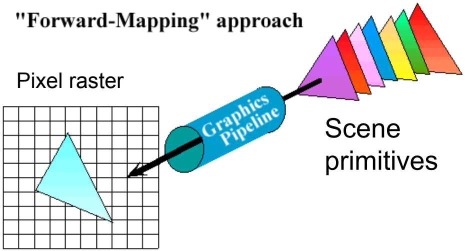


GPUs do Rasterization

- The process of taking a triangle and figuring out which pixels it covers is called **rasterization**
- We've seen acceleration structures for ray tracing; rasterization is not stupid either
  - We're not actually going to test *all* pixels for each triangle

GPU

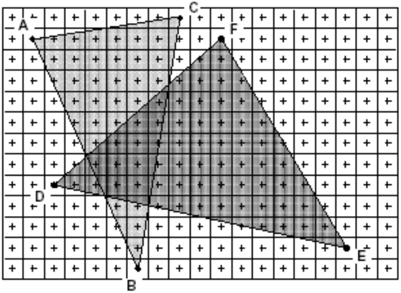
```
For each triangle
  For each pixel
    Does triangle cover pixel?
    Keep closest hit
```



Rasterization ("Scan Conversion")

- Given a triangle's vertices & extra info for shading, figure out which pixels to "turn on" to render the primitive
- Compute illumination values to "fill in" the pixels within the primitive
- At each pixel, keep track of the closest primitive (z-buffer)
  - Only overwrite if triangle being drawn is closer than the previous triangle in that pixel

```
glBegin(GL_TRIANGLES)
glNormal3f(...)
glVertex3f(...)
glVertex3f(...)
glVertex3f(...)
glEnd();
```



What are the Main Differences?

Ray Casting

```
For each pixel (ray)
  For each triangle
    Does ray hit triangle?
```

Keep closest hit

Ray-centric

GPU

```
For each triangle
  For each pixel
    Does triangle cover pixel
```

Keep closest hit

Triangle-centric

- What needs to be stored in memory in each case?

What are the Main Differences?

Ray Casting

```
For each pixel (ray)
   For each triangle
      Does ray hit triangle?
```
Keep closest hit

Ray-centric

GPU

```
For each triangle
   For each pixel
      Does triangle cover pixel
```
Keep closest hit

Triangle-centric

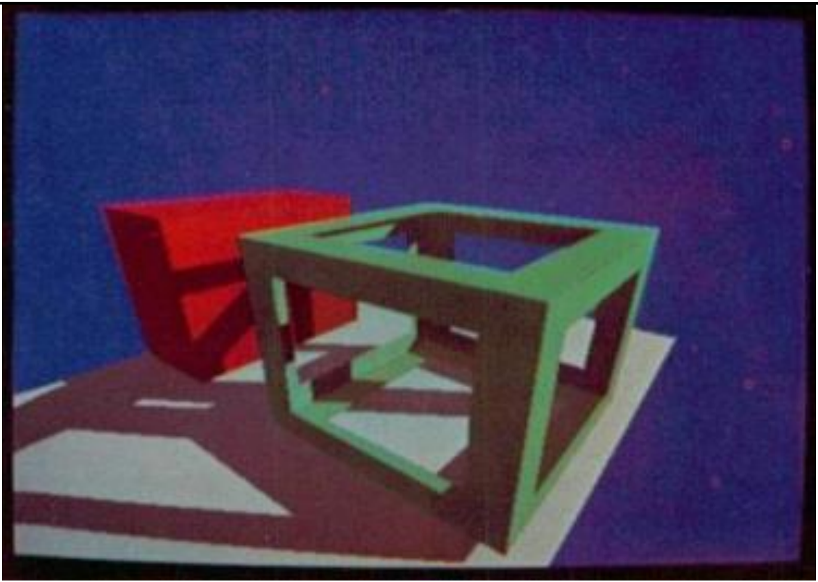
- In this basic form, ray tracing needs the entire scene description in memory at once
  - Then, can sample the image completely freely
- The rasterizer only needs one triangle at a time, *plus* the entire image and associated depth information for all pixels

Rasterization Advantages

- Modern scenes are more complicated than images
  - A 1920x1080 frame at 64-bit color and 32-bit depth per pixel is 24MB (not that much)
    - Of course, if we have more than one sample per pixel this gets larger, but e.g. 4x supersampling is still a relatively comfortable ~100MB
  - Our scenes are routinely larger than this
    - This wasn't always true

Rasterization Advantages

Weiler, Atherton 1977

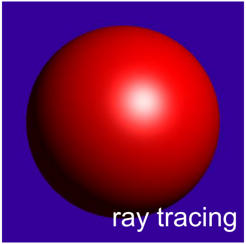


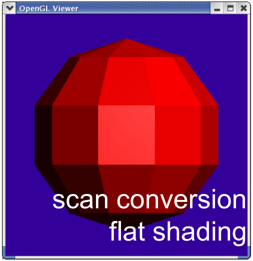
Rasterization Advantages

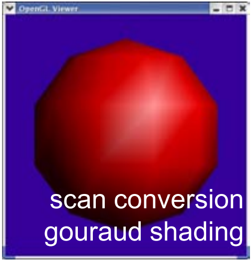
- Modern scenes are more complicated than images
  - A 1920x1080 frame (1080p) at 64-bit color and 32-bit depth per pixel is 24MB (not that much)
    - Of course, if we have more than one sample per pixel (later) this gets larger, but e.g. 4x supersampling is still a relatively comfortable ~100MB
  - Our scenes are routinely larger than this
    - This wasn’t always true
- A rasterization-based renderer can *stream* over the triangles, no need to keep entire dataset around
  - Allows parallelism and optimization of memory systems

Rasterization Limitations

- Restricted to scan-convertible primitives
  - Pretty much: triangles
- Faceting, shading artifacts
  - This is largely going away with programmable per-pixel shading, though
- No unified handling of shadows, reflection, transparency
- Potential problem of overdraw (high depth complexity)
  - Each pixel touched many times







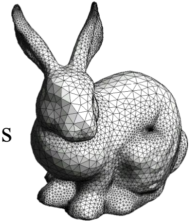
Ray Casting / Tracing

- Advantages
  - Generality: can render anything that can be intersected with a ray
  - Easily allows recursion (shadows, reflections, etc.)

- Disadvantages
  - Hard to implement in hardware (lacks computation coherence, must fit entire scene in memory, bad memory behavior)
    - Not such a big point any more given general purpose GPUs
  - Has traditionally been too slow for interactive applications
  - Both of the above are changing rather rapidly right now!

Modern Graphics Pipeline

- Input
  - Geometric model
    - Triangle vertices, vertex normals, texture coordinates
  - Lighting/material model (shader)
    - Light source positions, colors, intensities, etc.
    - Texture maps, specular/diffuse coefficients, etc.
  - Viewpoint + projection plane

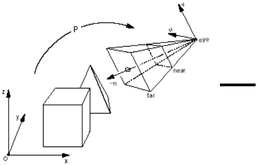


- Output
  - Color (+depth) per pixel

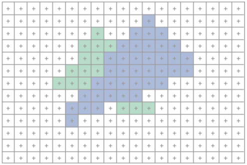
Colbert & Krivanek

Image of Real-Time Rendering of the Stanford Bunny with 40 Samples per Pixel removed due to copyright restrictions -- please see Fig. 20-1 from http://http.developer.nvidia.com/GPUGems3/gpugems3_ch20.html for further details.

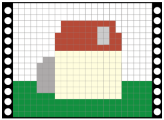
Modern Graphics Pipeline

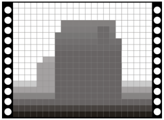


- Project vertices to 2D (image)

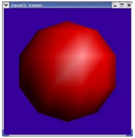


- Rasterize triangle: find which pixels should be lit



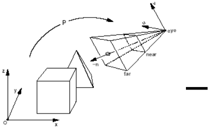


- Test visibility (Z-buffer), update frame buffer color

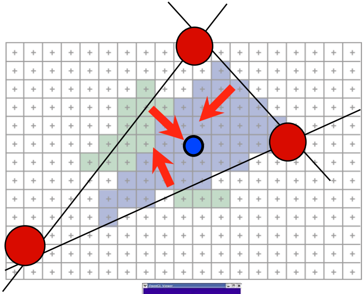


- Compute per-pixel color

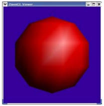
Modern Graphics Pipeline



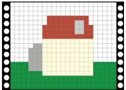
- Project vertices to 2D (image)

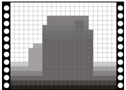


- Rasterize triangle: find which pixels should be lit
  - For each pixel, test 3 edge equations
    - if all pass, draw pixel

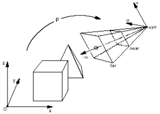


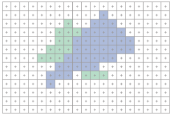
- Compute per-pixel color
- Test visibility (Z-buffer), update frame buffer color

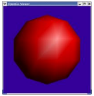




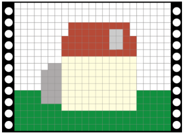
Modern Graphics Pipeline



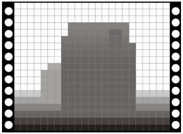




- Perform projection of vertices
- Rasterize triangle: find which pixels should be lit
- Compute per-pixel color
- Test visibility, update frame buffer color
  - Store minimum distance to camera for each pixel in "Z-buffer"
    - ~same as t_{min} in ray casting!
  - **if** *newz* < *zbuffer*[*x*,*y*]
    zbuffer[x,y]=new_z
    framebuffer[x,y]=new_color

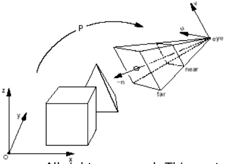


frame buffer

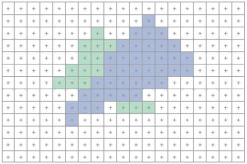


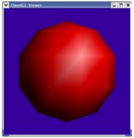
Z buffer

Modern Graphics Pipeline

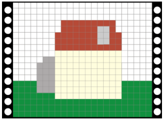


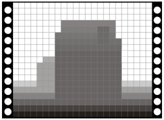
For each triangle
  transform into eye space
  (perform projection)
  setup 3 edge equations
  for each pixel x,y
    if passes all edge equations
      compute z
      if z<zbuffer[x,y]
        zbuffer[x,y]=z
        framebuffer[x,y]=shade()



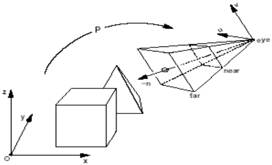


Questions?

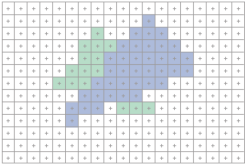


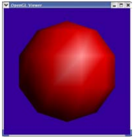


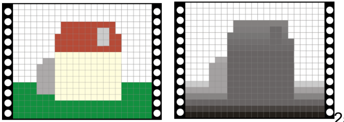
Projection



- Project vertices to 2D (image)
- Rasterize triangle: find which pixels should be lit
- Compute per-pixel color
- Test visibility (Z-buffer), update frame buffer

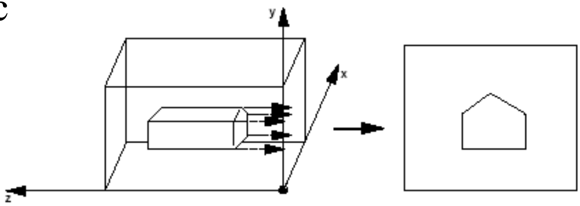




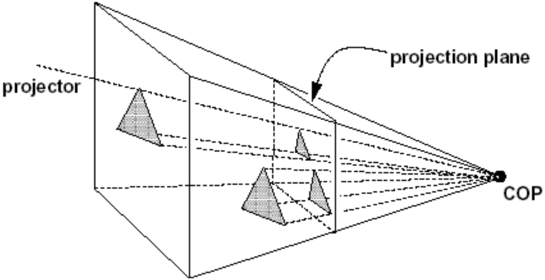


Orthographic vs. Perspective

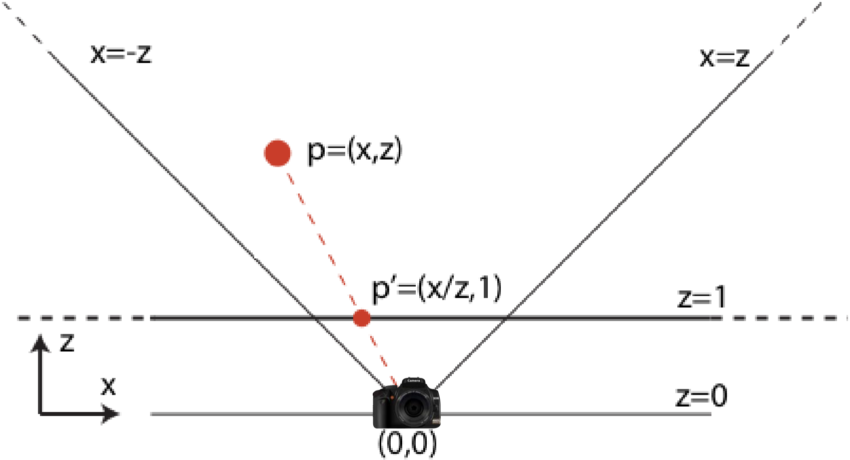
- Orthographic



- Perspective



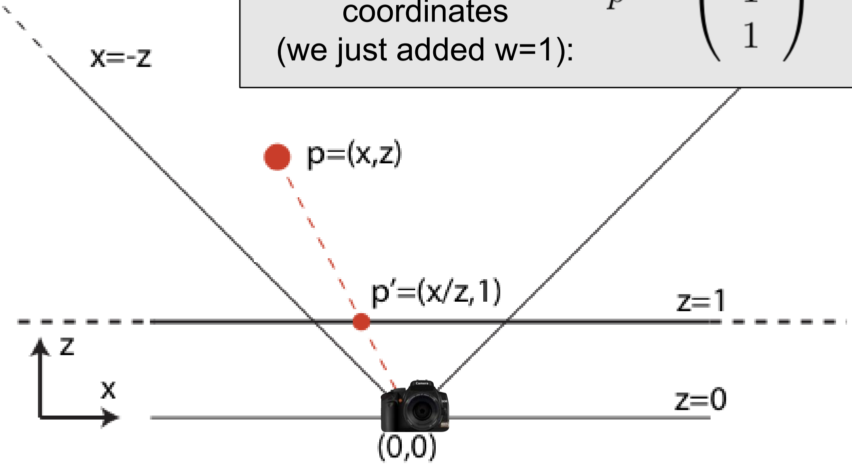
Perspective in 2D



Perspective in 2D

The projected point in homogeneous coordinates (we just added w=1):

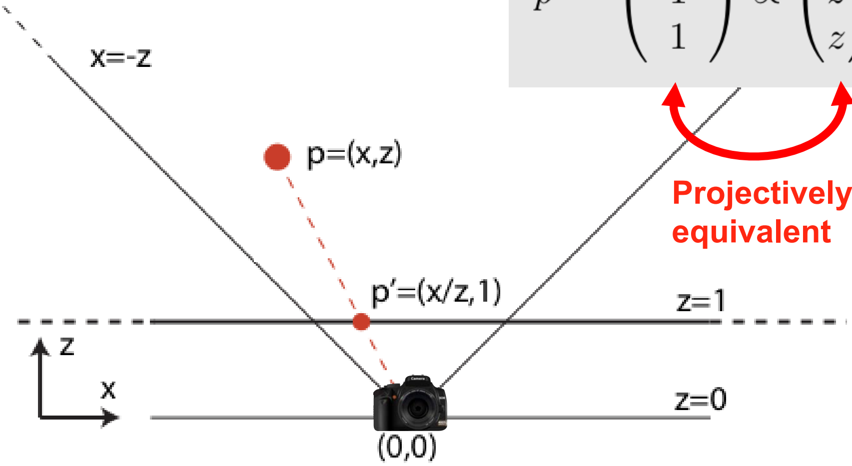
$$p' = \begin{pmatrix} x/z \\ 1 \\ 1 \end{pmatrix}$$



Perspective in 2D

$$p' = \begin{pmatrix} x/z \\ 1 \\ 1 \end{pmatrix} \propto \begin{pmatrix} x \\ z \\ z \end{pmatrix}$$

Projectively equivalent



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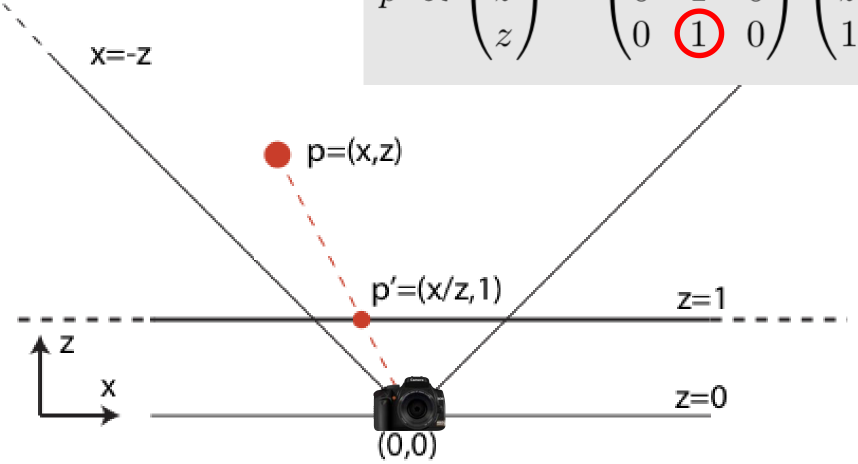
Perspective in 2D

We'll just copy z to w, and get the projected point after homogenization!

$$p' \propto \begin{pmatrix} x \\ z \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ z \\ 1 \end{pmatrix}$$



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Extension to 3D

- Trivial: Just ass another dimension *y* and treat it like *x*
- Different fields of view and non-square image aspect ratios can be accomplished by simple scaling of the *x* and *y* axes.

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Caveat

- These projections matrices work perfectly in the sense that you get the proper 2D projections of 3D points.
- However, since we are flattening the scene onto the z=1 plane, we've lost all information about the distance to camera.
  - We need the distance for Z buffering, i.e., figuring out what is in front of what!

Basic Idea: store 1/*z*

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Basic Idea: store 1/*z*

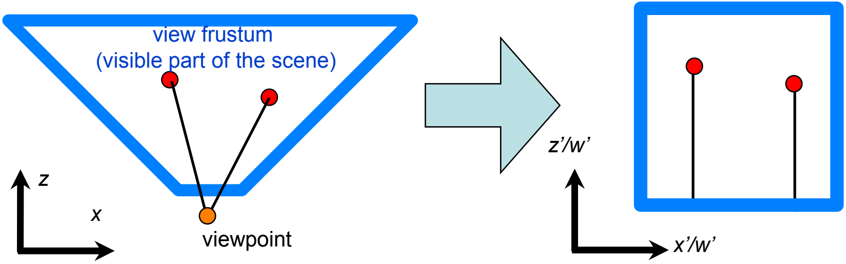
$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \\ z \end{pmatrix}$$

- $z' = 1$ before homogenization
- $z' = 1/z$ after homogenization

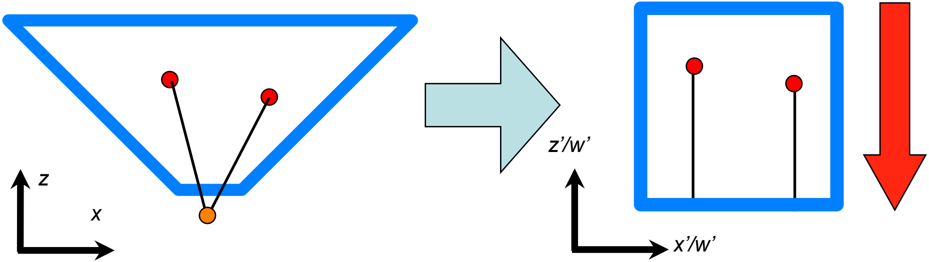
Full Idea: Remap the View Frustum

- We can transform the frustum by a modified projection in a way that makes it a square (cube in 3D) after division by *w*'.



The View Frustum in 2D

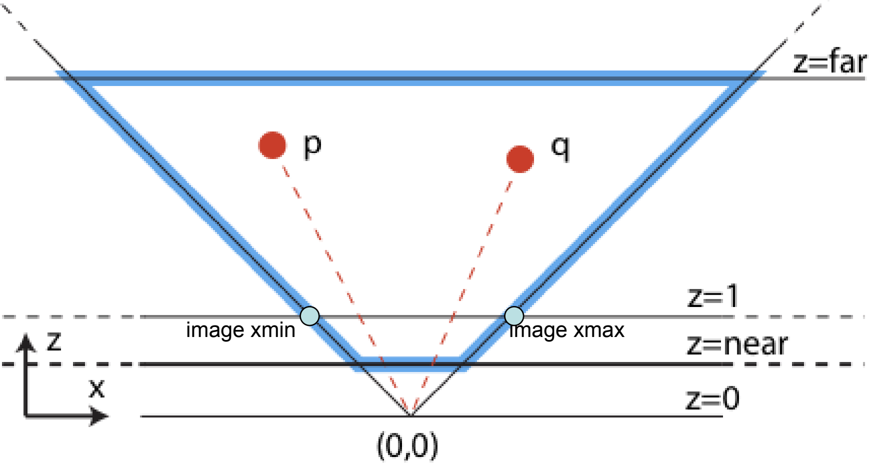
- We can transform the frustum by a modified projection in a way that makes it a square (cube in 3D) after division by *w*'.



The final image is obtained by merely dropping the z coordinate after projection (orthogonal projection)

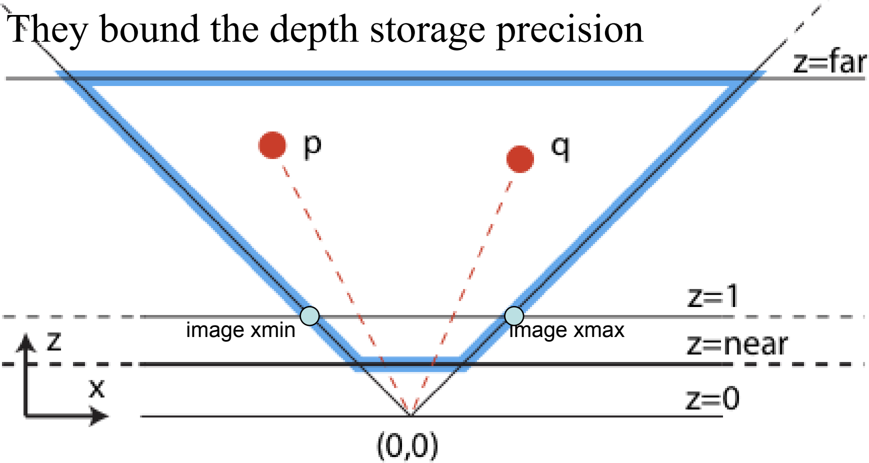
The View Frustum in 2D

- (In 3D this is a truncated pyramid.)

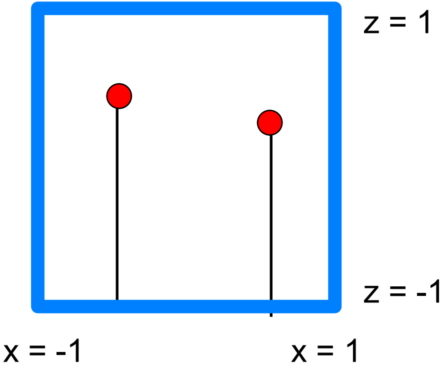


The View Frustum in 2D

- Far and near are kind of arbitrary
- They bound the depth storage precision



The Canonical View Volume



- Point of the exercise: This gives screen coordinates and depth values for Z-buffering with unified math
  - Caveat: OpenGL and DirectX define Z differently [0,1] vs.[-1,1]

OpenGL Form of the Projection

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\text{far}+\text{near}}{\text{far}-\text{near}} & -\frac{2*\text{far}*\text{near}}{\text{far}-\text{near}} \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Homogeneous coordinates within canonical view volume

Input point in view coordinates

OpenGL Form of the Projection

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\text{far}+\text{near}}{\text{far}-\text{near}} & -\frac{2*\text{far}*\text{near}}{\text{far}-\text{near}} \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- z'=(az+b)/z =a+b/z
  - where a & b depend on near & far
- Similar enough to our basic idea:
  - z'=1/z

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

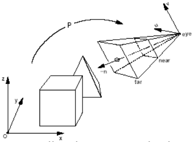
Recap: Projection Questions?

- Perform rotation/translation/other transforms to put viewpoint at origin and view direction along z axis
  - This is the OpenGL “modelview” matrix

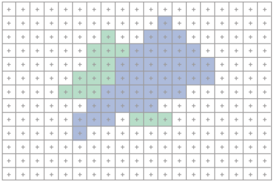
- Combine with projection matrix (perspective or orthographic)
  - Homogenization achieves foreshortening
  - This is the OpenGL “projection” matrix

- **Corollary**: The entire transform from object space to canonical view volume $[-1,1]^3$ is a single matrix

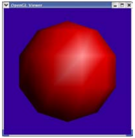
Modern Graphics Pipeline



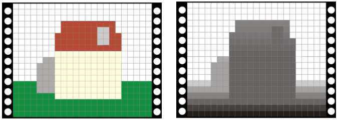
- Project vertices to 2D (image)
  - We now have screen coordinates
- Rasterize triangle: find which pixels should be lit



- Compute per-pixel color

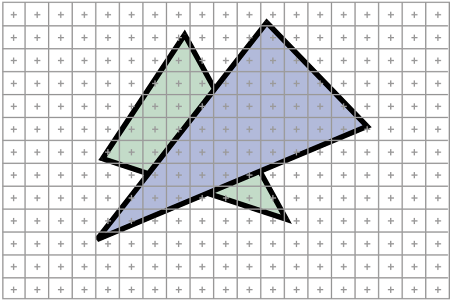


- Test visibility (Z-buffer), update frame buffer



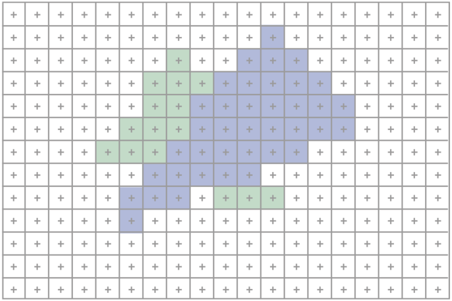
2D Scan Conversion

- Primitives are "continuous" geometric objects; screen is discrete (pixels)



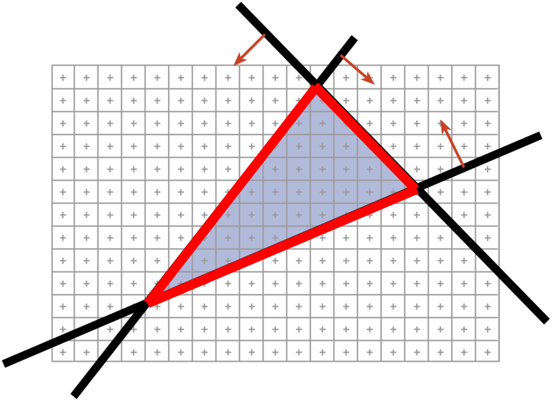
2D Scan Conversion

- Primitives are "continuous" geometric objects; screen is discrete (pixels)
- Rasterization computes a discrete approximation in terms of pixels **(how?)**



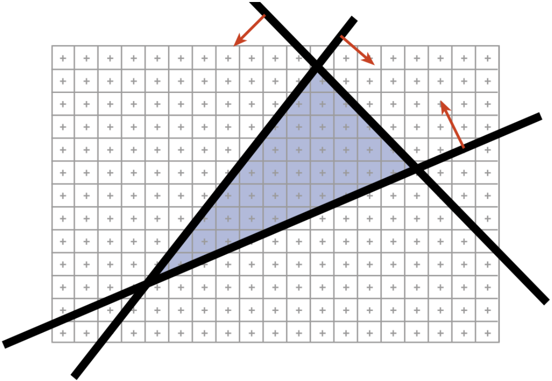
Edge Functions

- The triangle's 3D edges project to line segments in the image (thanks to planar perspective)
  - Lines map to lines, not curves



Edge Functions

- The triangle's 3D edges project to line segments in the image (thanks to planar perspective)
- The interior of the triangle is the set of points that is inside all three halfspaces defined by these lines



Edge Functions

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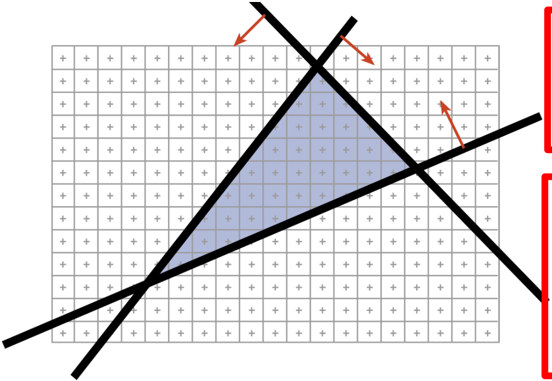
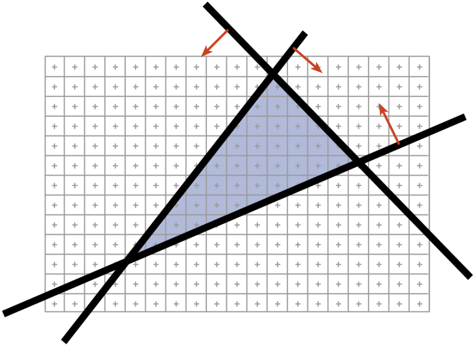


$$E_i(x, y) = a_i x + b_i y + c_i$$

$$(x, y) \text{ within triangle} \Leftrightarrow E_i(x, y) \geq 0, \quad \forall i = 1, 2, 3$$

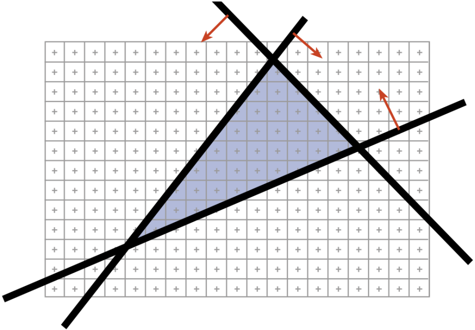
Brute Force Rasterizer

- Compute E_1, E_2 , E_3 coefficients from projected vertices
  - Called "triangle setup", yields a_i, b_i, c_i for i=1,2,3



Brute Force Rasterizer

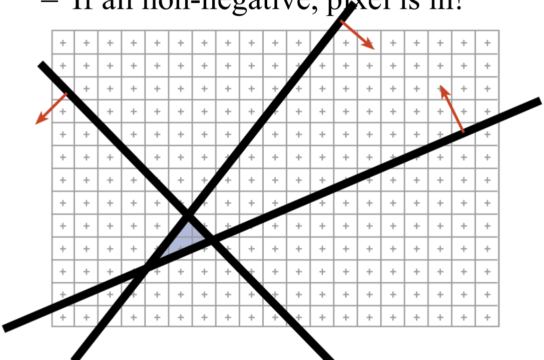
- Compute E_1, E_2 , E_3 coefficients from projected vertices
- For each pixel (x, y)
  - Evaluate edge functions at pixel center
  - If all non-negative, pixel is in!



Problem?

Brute Force Rasterizer

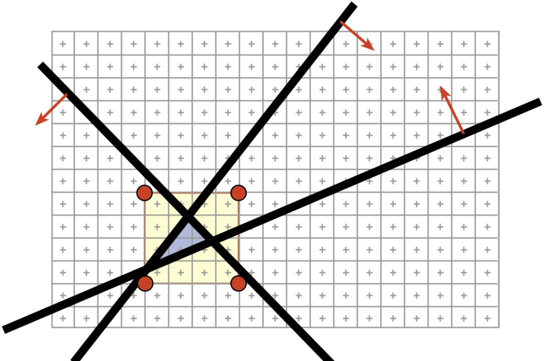
- Compute E_1, E_2 , E_3 coefficients from projected vertices
- For each pixel (x, y)
  - Evaluate edge functions at pixel center
  - If all non-negative, pixel is in!



If the triangle is small, lots of useless computation if we really test all pixels

Easy Optimization

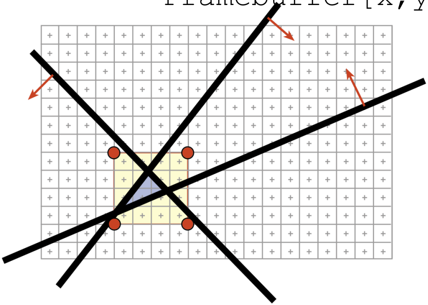
- Improvement: Scan over only the pixels that overlap the *screen bounding box* of the triangle
- How do we get such a bounding box?
  - X_{min}, X_{max}, Y_{min}, Y_{max} of the projected triangle vertices



Rasterization Pseudocode

Note: No visibility

```
For every triangle
   Compute projection for vertices, compute the Ei
   Compute bbox, clip bbox to screen limits
   For all pixels in bbox
      Evaluate edge functions Ei
      If all > 0
          Framebuffer[x,y ] = triangleColor
```

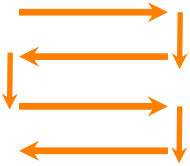


Bounding box clipping is easy, just clamp the coordinates to the screen rectangle

Questions?

Incremental Edge Functions

```
For every triangle
   ComputeProjection
   Compute bbox, clip bbox to screen limits
   For all scanlines y in bbox
      Evaluate all E_i's at (x0,y): E_i = a_i x0 + b_i y + c_i
      For all pixels x in bbox
          If all E_i>0
              Framebuffer[x,y ] = triangleColor
          Increment line equations: E_i += a_i
```

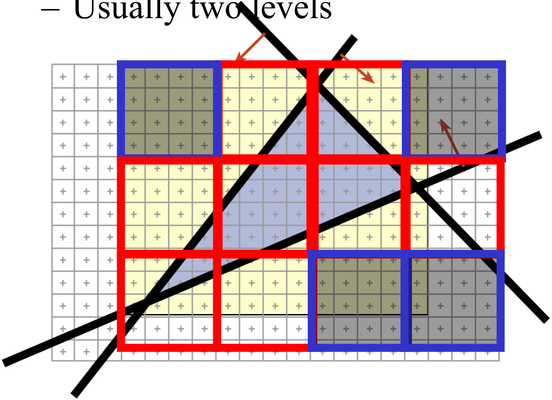


- We save ~two multiplications and two additions per pixel when the triangle is large

Can also zig-zag to avoid reinitialization per scanline, just initialize once at x0, y0

Indeed, We Can Be Smarter

- Hierarchical rasterization!
  - Conservatively test **blocks of pixels** before going to per-pixel level (can skip large blocks at once)
  - Usually two levels



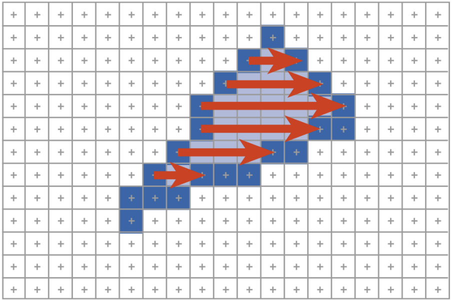
Can also test if an entire block is **inside** the triangle; then, can skip edge functions tests for all pixels for even further speedups.(Must still test Z, because they might still be occluded.)

Further References

- Henry Fuchs, Jack Goldfeather, Jeff Hultquist, Susan Spach, John Austin, Frederick Brooks, Jr., John Eyles and John Poulton, “Fast Spheres, Shadows, Textures, Transparencies, and Image Enhancements in Pixel-Planes”, Proceedings of SIGGRAPH ‘85 (San Francisco, CA, July 22–26, 1985). In *Computer Graphics*, v19n3 (July 1985), ACM SIGGRAPH, New York, NY, 1985.
- Juan Pineda, “A Parallel Algorithm for Polygon Rasterization”, Proceedings of SIGGRAPH ‘88 (Atlanta, GA, August 1–5, 1988). In *Computer Graphics*, v22n4 (August 1988), ACM SIGGRAPH, New York, NY, 1988. Figure 7: Image from the spinning teapot performance test.
- Marc Olano Trey Greer, “Triangle Scan Conversion using 2D Homogeneous Coordinates”, Graphics Hardware 97 http://www.cs.unc.edu/~olano/papers/2dh-tri/2dh-tri.pdf

Oldschool Rasterization

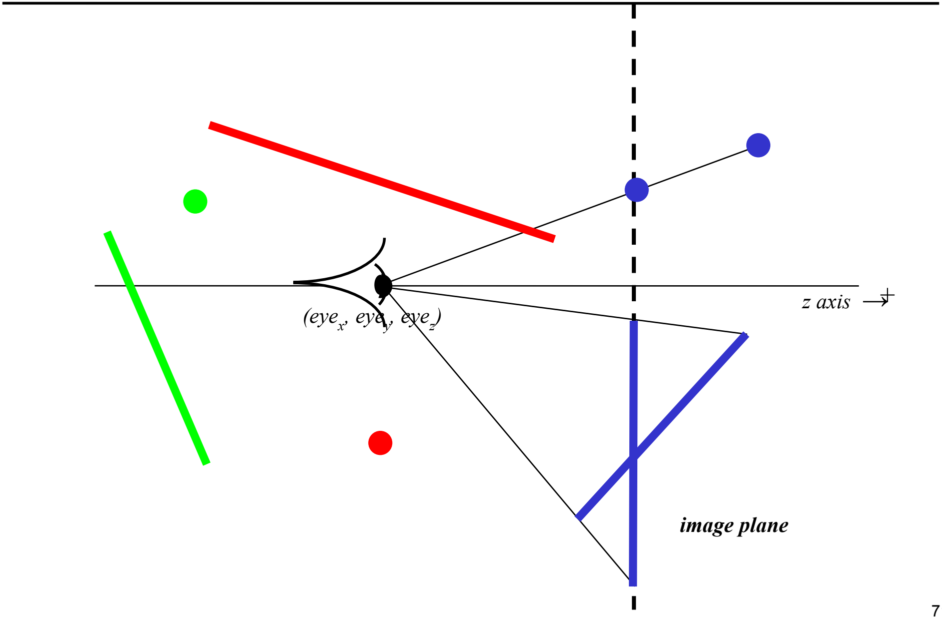
- Compute the boundary pixels using line rasterization
- Fill the spans



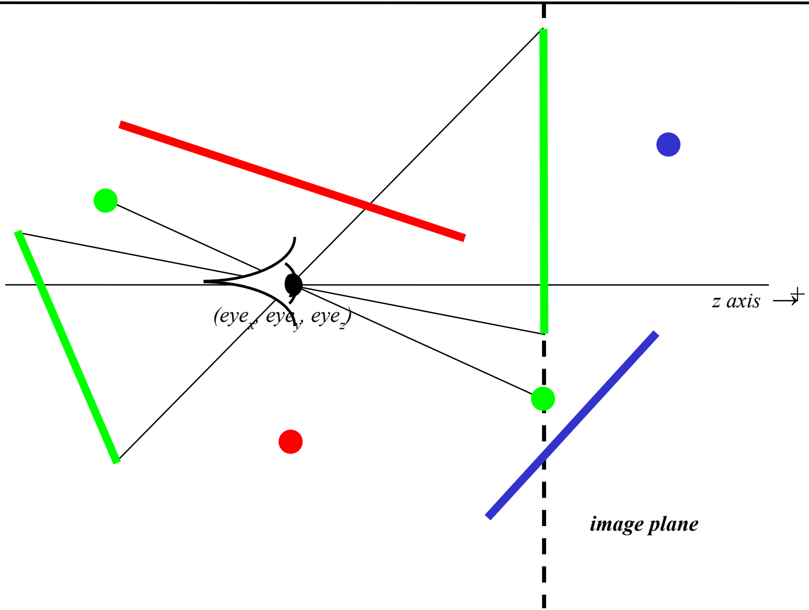
More annoying to implement than edge functions

Not faster unless triangles are huge

What if the p_z is > eye_z?

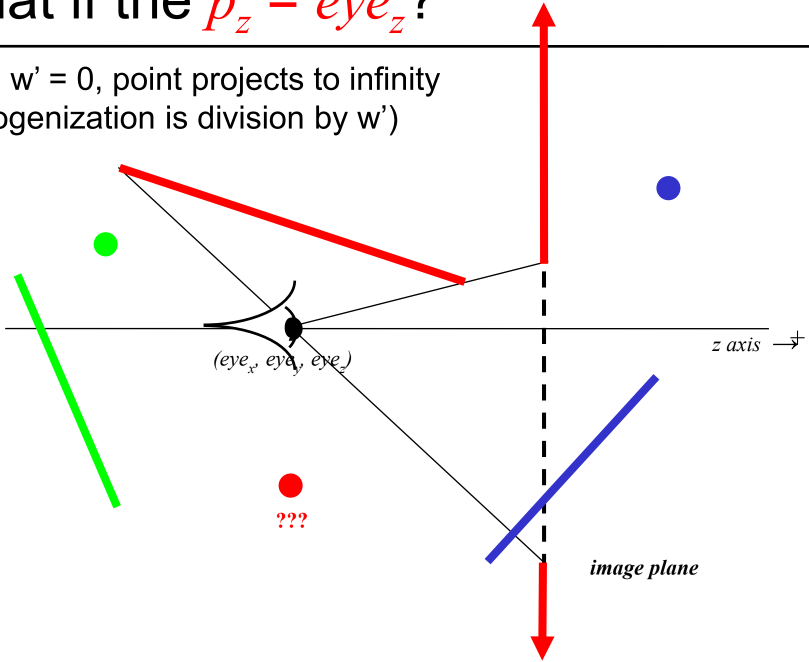


What if the p_z is < eye_z?

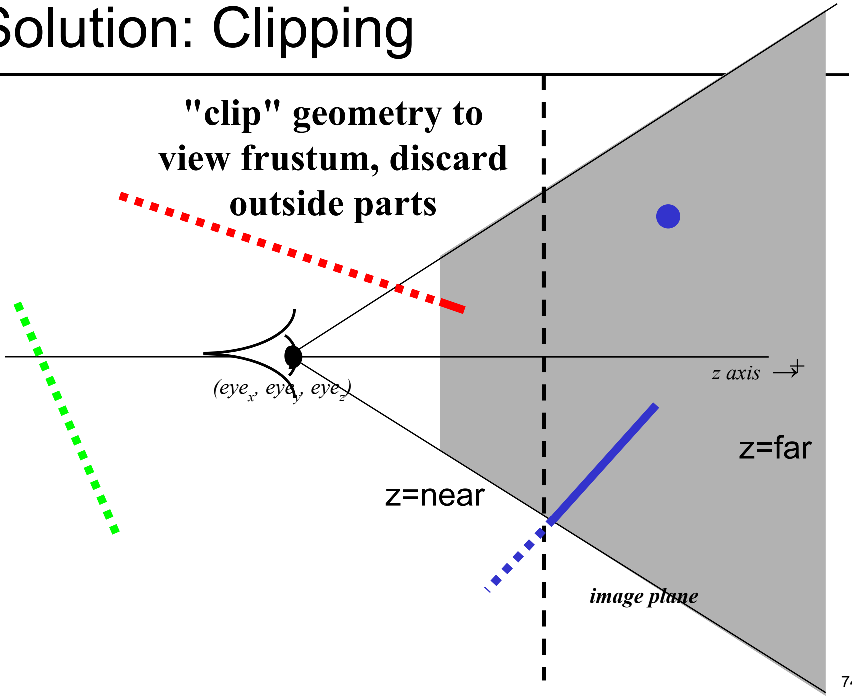


What if the $p_z = eye_z$?

When w' = 0, point projects to infinity
(homogenization is division by w')

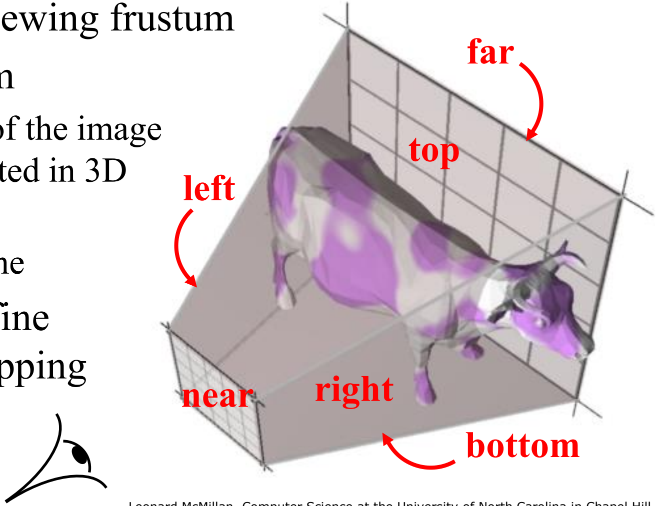


A Solution: Clipping



Clipping

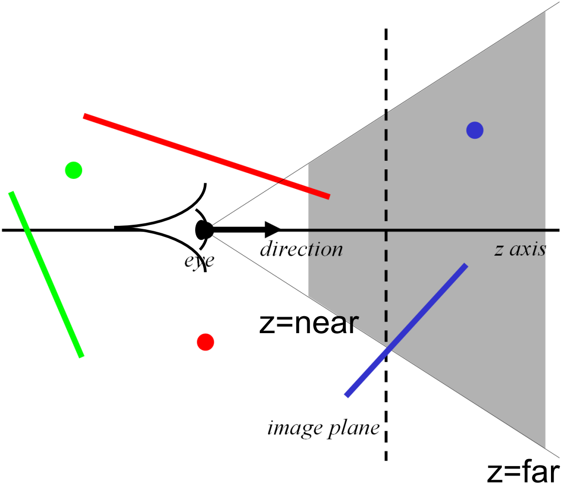
- Eliminate portions of objects outside the viewing frustum
- View Frustum
  - boundaries of the image plane projected in 3D
  - a near & far clipping plane
- User may define additional clipping planes



Leonard McMillan, Computer Science at the University of North Carolina in Chapel Hill.

Why Clip?

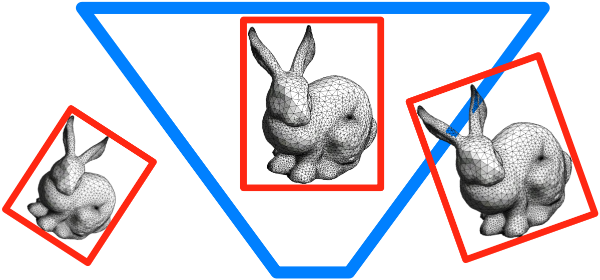
- Avoid degeneracies
  - Don't draw stuff behind the eye
  - Avoid division by 0 and overflow



Related Idea Questions?

- “View Frustum Culling”
  - Use bounding volumes/hierarchies to test whether any part of an object is within the view frustum
    - Need “frustum vs. bounding volume” intersection test
    - Crucial to do hierarchically when scene has *lots* of objects!
    - Early rejection (different from clipping)

See e.g. Optimized view frustum culling algorithms for bounding boxes, Ulf Assarsson and Tomas Möller, journal of graphics tools, 2000.



Homogeneous Rasterization

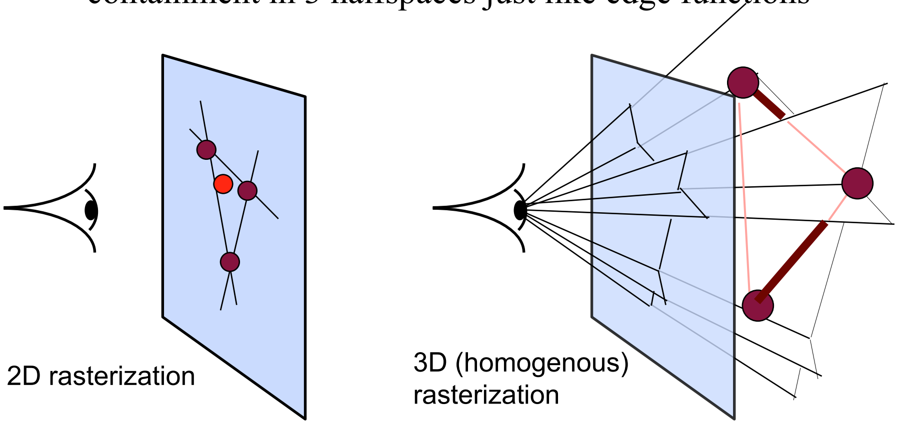
- Idea: avoid projection (and division by zero) by performing rasterization in 3D
  - Or equivalently, use 2D homogenous coordinates ($w'=z$ after the projection matrix, remember)

- **Motivation: clipping is annoying**

- Marc Olano, Trey Greer: Triangle scan conversion using 2D homogeneous coordinates, Proc. ACM SIGGRAPH/Eurographics Workshop on Graphics Hardware 1997

Homogeneous Rasterization

- Replace 2D edge equation by 3D plane equation
  - Treat pixels as 3D points $(x, y, 1)$ on image plane, test for containment in 3 halfspaces just like edge functions



Homogeneous Rasterization

Given 3D triangle

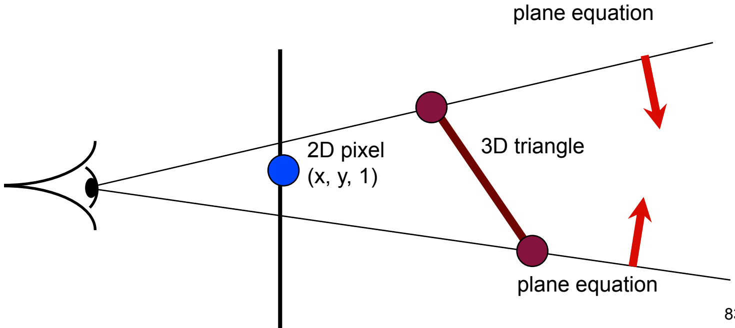
setup plane equations

(plane through viewpoint & triangle edge)

For each pixel x,y

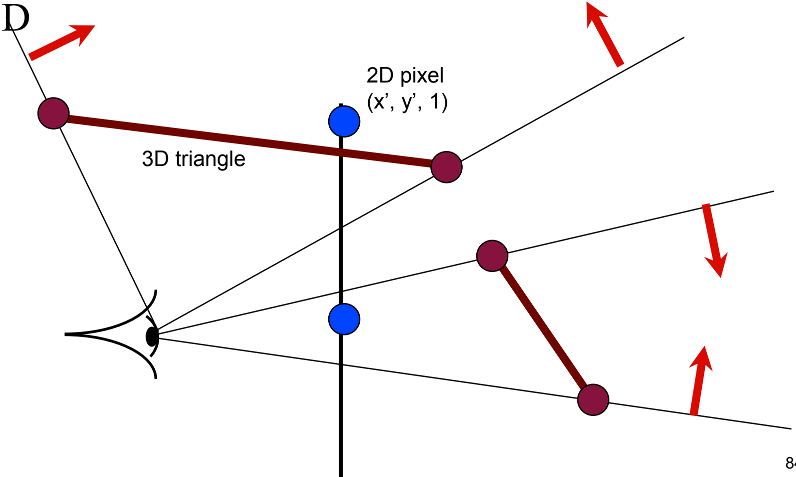
compute plane equations for (x,y,1)

if all pass, draw pixel

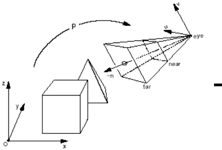


Homogeneous Rasterization

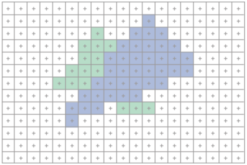
- Works for triangles behind eye
- Still linear, can evaluate incrementally/hierarchically like 2D



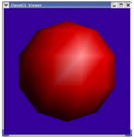
Modern Graphics Pipeline



- Perform projection of vertices

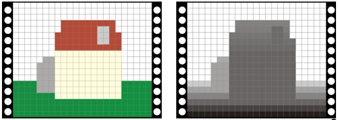


- Rasterize triangle: find which pixels should be lit



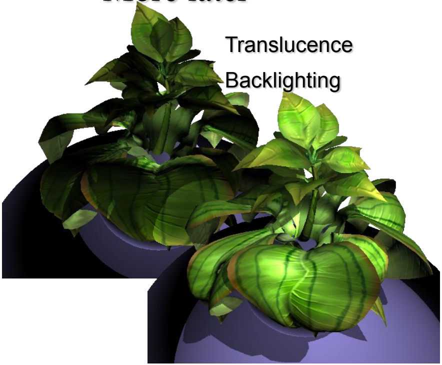
- Compute per-pixel color

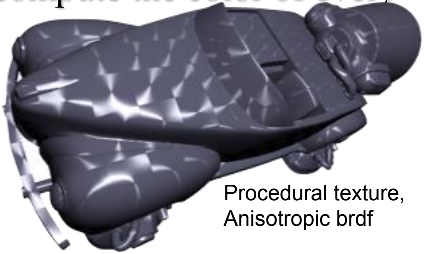
- Test visibility, update frame buffer

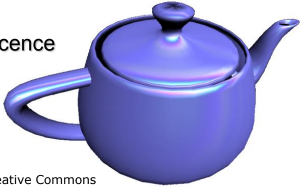


Pixel Shaders

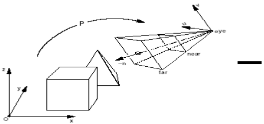
- Modern graphics hardware enables the execution of rather complex programs to compute the color of every single pixel
  - More later







Modern Graphics Pipeline



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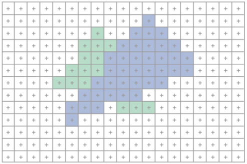
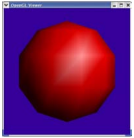


- Rasterize triangle: find which pixels should be lit



- Compute per-pixel color

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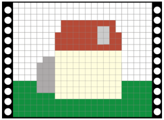
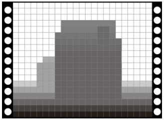
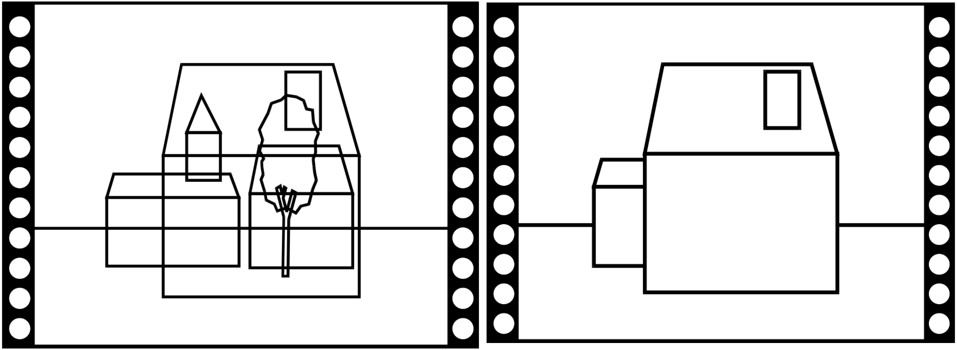




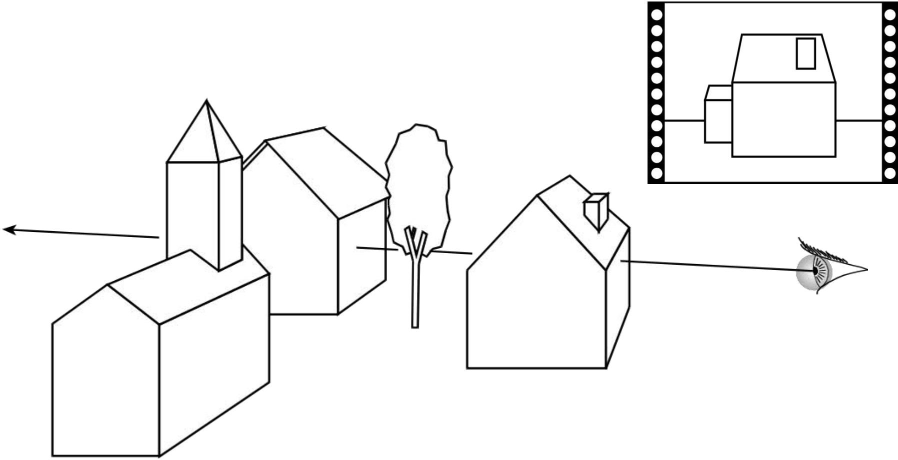
Visibility

- How do we know which parts are visible/in front?



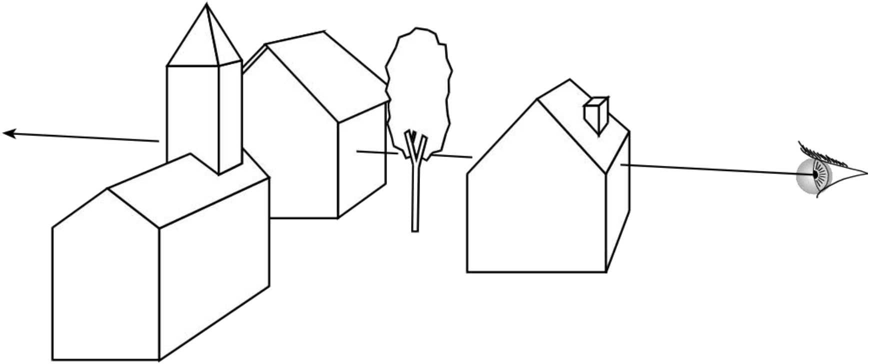
Ray Casting

- Maintain intersection with closest object



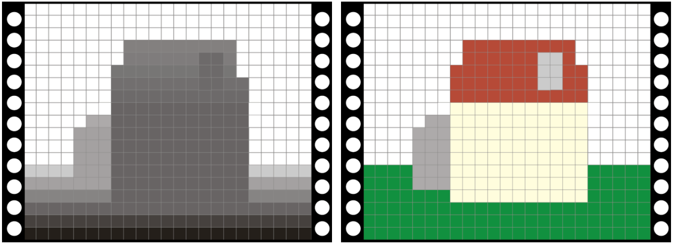
Visibility

- In ray casting, use intersection with closest t
- Now we have swapped the loops (pixel, object)
- What do we do?



Z buffer

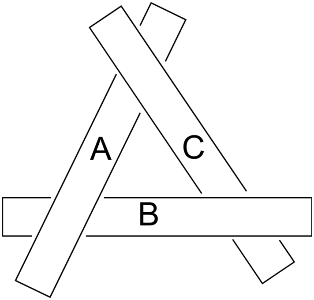
- In addition to frame buffer (R, G, B)
- Store distance to camera (*z*-buffer)
- Pixel is updated only if *newz* is closer than *z*-buffer value

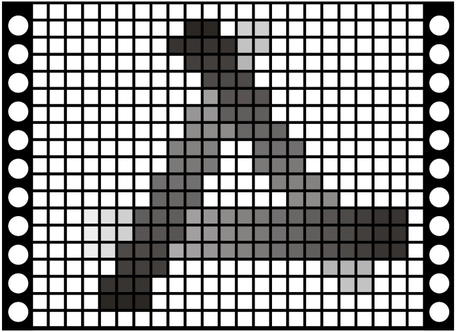


Z-buffer pseudo code

```
For every triangle
   Compute Projection, color at vertices
   Setup line equations
   Compute bbox, clip bbox to screen limits
   For all pixels in bbox
      Increment line equations
      Compute curentZ
      Compute currentColor
      If all line equations>0  //pixel [x,y] in triangle
         If currentZ<zBuffer[x,y]  //pixel is visible
            Framebuffer[x,y]=currentColor
            zBuffer[x,y]=currentZ
```

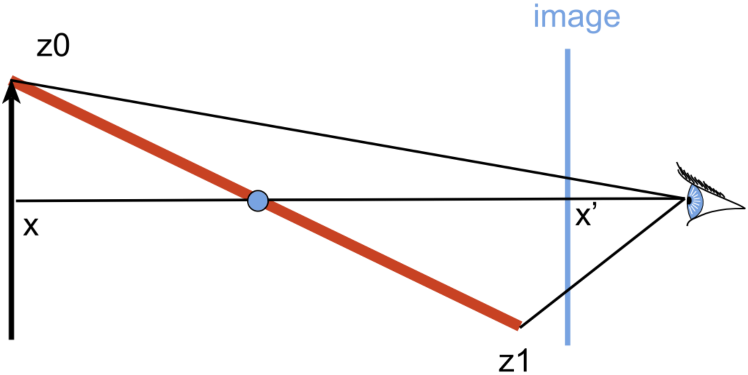
Works for hard cases!





More questions for next time

- How do we get Z?
- Texture Mapping?



TIEA311 - Meanwhile in Jyväskylä…

Our introductory course ends tomorrow, and “TIES471 Reaaliaikainen renderöinti” (Real-time rendering) will most likely pick up from these thoughts, towards being a graphics pro!

Textures, shaders, optimizations, effects … Be there, or be a triangle mesh…

One more computer class session today 16-18 is available for this course.

Those who wish to continue graphics without taking TIES471 may want to examine the rest of the MIT course material (“Lectures 21-22”) and also the parts we have skipped.

Now, we’ll have a break, and have a **final discussion** right after that!

TIEA311 - YPE-hype

During the break, think about the following questions:

- Name the most important thing that you learned during this course.
- Name one thing that you wanted to learn on this course, but for some reason did not.

TIEA311 - Personal plea

Take part in our research, and make the academic world
better! Win a Suffeli if lucky.

Detailed information will be sent on the course mailing list.

TIEA311 - last slide

THANK YOU!