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Introduction to Multiobjective Optimization: Noninteractive Approaches

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Abstract. We give an introduction to nonlinear multiobjective optimization by covering some basic concepts as well as outlines of some methods. Because Pareto optimal solutions cannot be ordered completely, we need extra preference information coming from a decision maker to be able to select the most preferred solution for a problem involving multiple conflicting objectives. Multiobjective optimization methods are often classified according to the role of a decision maker in the solution process. In this chapter, we concentrate on noninteractive methods where the decision maker either is not involved or specifies preference information before or after the actual solution process. In other words, the decision maker is not assumed to devote too much time in the solution process.

1.1 Introduction

Many decision and planning problems involve multiple conflicting objectives that should be considered simultaneously (alternatively, we can talk about multiple conflicting criteria). Such problems are generally known as multiple criteria decision making (MCDM) problems. We can classify MCDM problems in many ways depending on the characteristics of the problem in question. For example, we talk about multiattribute decision analysis if we have a discrete, predefined set of alternatives to be considered. Here we study multiobjective optimization (also known as multiobjective mathematical programming) where the set of feasible solutions is not explicitly known in advance but it is restricted by constraint functions. Because of the aims and scope of this book, we concentrate on nonlinear multiobjective optimization (where at least one function in the problem formulation is nonlinear) and ignore approaches designed only for multiobjective linear programming (MOLP) problems (where all the functions are linear).

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Introduction to Multiobjective Optimization: Interactive Approaches

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Abstract. We give an overview of interactive methods developed for solving nonlinear multiobjective optimization problems. In interactive methods, a decision maker plays an important part and the idea is to support her/him in the search for the most preferred solution. In interactive methods, steps of an iterative solution algorithm are repeated and the decision maker progressively provides preference information so that the most preferred solution can be found. We identify three types of specifying preference information in interactive methods and give some examples of methods representing each type. The types are methods based on trade-off information, reference points and classification of objective functions.

2.1 Introduction

Solving multiobjective optimization problems typically means helping a human decision maker (DM) in finding the most preferred solution as the final one. By the *most preferred solution* we refer to a Pareto optimal solution which the DM is convinced to be her/his best option. Naturally, finding the most preferred solution necessitates the participation of the DM who is supposed to have insight into the problem and be able to specify preference information related to the objectives considered and different solution alternatives, as discussed in Chapter 1. There we presented four classes for multiobjective optimization methods according to the role of the DM in the solution process.

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Introduction to Evolutionary Multiobjective Optimization

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Abstract. In its current state, evolutionary multiobjective optimization (EMO) is an established field of research and application with more than 150 PhD theses, more than ten dedicated texts and edited books, commercial softwares and numerous freely downloadable codes, a biannual conference series running successfully since 2001, special sessions and workshops held at all major evolutionary computing conferences, and full-time researchers from universities and industries from all around the globe. In this chapter, we provide a brief introduction to EMO principles, illustrate some EMO algorithms with simulated results, and outline the current research and application potential of EMO. For solving multiobjective optimization problems, EMO procedures attempt to find a set of well-distributed Pareto-optimal points, so that an idea of the extent and shape of the Pareto-optimal front can be obtained. Although this task was the early motivation of EMO research, EMO principles are now being found to be useful in various other problem solving tasks, enabling one to treat problems naturally as they are. One of the major current research thrusts is to combine EMO procedures with other multiple criterion decision making (MCDM) () tools so as to develop hybrid and interactive multiobjective optimization algorithms for finding a set of trade-off optimal solutions and then choose a preferred solution for implementation. This chapter provides the background of EMO principles and their potential to launch such collaborative studies with MCDM researchers in the coming years.

3.1 Introduction

In a short span of about fourteen years since the suggestion of the first set of successful algorithms, evolutionary multiobjective optimization (EMO) has

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Interactive Multiobjective Optimization Using a Set of Additive Value Functions

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Abstract. In this chapter, we present a new interactive procedure for multiobjective optimization, which is based on the use of a set of value functions as a preference model built by an ordinal regression method. The procedure is composed of two alternating stages. In the first stage, a representative sample of solutions from the Pareto optimal set (or from its approximation) is generated. In the second stage, the Decision Maker (DM) is asked to make pairwise comparisons of some solutions from the generated sample. Besides pairwise comparisons, the DM may compare selected pairs from the viewpoint of the intensity of preference, both comprehensively and with respect to a single criterion. This preference information is used to build a preference model composed of all general additive value functions compatible with the obtained information. The set of compatible value functions is then applied on the whole Pareto optimal set, which results in possible and necessary rankings of Pareto optimal solutions. These rankings are used to select a new sample of solutions, which is presented to the DM, and the procedure cycles until a satisfactory solution is selected from the sample or the DM comes to conclusion that there is no satisfactory solution for the current problem setting. Construction of the set of compatible value functions is done using ordinal regression methods called UTA^{GMS} and GRIP. These two methods generalize UTA-like methods and they are competitive to AHP and MACBETH methods. The interactive procedure will be illustrated through an example.

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Dominance-Based Rough Set Approach to Interactive Multiobjective Optimization

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Abstract. In this chapter, we present a new method for interactive multiobjective optimization, which is based on application of a logical preference model built using the Dominance-based Rough Set Approach (DRSA). The method is composed of two main stages that alternate in an interactive procedure. In the first stage, a sample of solutions from the Pareto optimal set (or from its approximation) is generated. In the second stage, the Decision Maker (DM) indicates relatively good solutions in the generated sample. From this information, a preference model expressed in terms of “*if ..., then ...*” decision rules is induced using DRSA. These rules define some new constraints which can be added to original constraints of the problem, cutting-off non-interesting solutions from the currently considered Pareto optimal set. A new sample of solutions is generated in the next iteration from the reduced Pareto optimal set. The interaction continues until the DM finds a satisfactory solution in the generated sample. This procedure permits a progressive exploration of the Pareto optimal set in zones which are interesting from the point of view of DM’s preferences. The “driving model” of this exploration is a set of user-friendly decision rules, such as “if the value of objective i_1 is not smaller than α_{i_1} and the value of objective i_2 is not smaller than α_{i_2} , then the solution is good”. The sampling of the reduced Pareto optimal set becomes finer with the advancement of the procedure and, moreover, a return to previously abandoned zones is possible. Another feature of the method is the possibility of learning about relationships between values of objective functions in the currently considered zone of the Pareto optimal set. These relationships are expressed by DRSA association rules, such as “if objective j_1 is not greater than α_{j_1} and objective j_2 is not greater than α_{j_2} , then objective j_3 is not smaller than β_{j_3} and objective j_4 is not smaller than β_{j_4} ”.

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Consideration of Partial User Preferences in Evolutionary Multiobjective Optimization

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Abstract. Evolutionary multiobjective optimization usually attempts to find a good approximation to the complete Pareto optimal front. However, often the user has at least a vague idea about what kind of solutions might be preferred. If such information is available, it can be used to focus the search, yielding a more fine-grained approximation of the most relevant (from a user's perspective) areas of the Pareto optimal front and/or reducing computation time. This chapter surveys the literature on incorporating partial user preference information in evolutionary multiobjective optimization.

6.1 Introduction

Most research in evolutionary multiobjective optimization (EMO) attempts to approximate the complete Pareto optimal front by a set of well-distributed representatives of Pareto optimal solutions. The underlying reasoning is that in the absence of any preference information, all Pareto optimal solutions have to be considered equivalent.

On the other hand, in most practical applications, the decision maker (DM) is eventually interested in only a single solution. In order to come up with a single solution, at some point during the optimization process, the DM has to reveal his/her preferences to choose between mutually non-dominating solutions. Following a classification by Horn (1997) and Veldhuizen and Lamont (2000), the articulation of preferences may be done either before (a priori), during (progressive), or after (a posteriori) the optimization process, see also Figure 6.1.

A priori approaches aggregate different objectives into a single auxiliary objective in one way or another, which allows to use standard optimization techniques (including single-objective evolutionary algorithms) and usually

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Interactive Multiobjective Evolutionary Algorithms

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Abstract. This chapter describes various approaches to the use of evolutionary algorithms and other metaheuristics in interactive multiobjective optimization. We distinguish the traditional approach to interactive analysis with the use of single objective metaheuristics, the semi-a posteriori approach with interactive selection from a set of solutions generated by a multiobjective metaheuristic, and specialized interactive multiobjective metaheuristics in which the DM's preferences are interactively expressed during the run of the method. We analyze properties of each of the approaches and give examples from the literature.

7.1 Introduction

As already discussed in Chapters 1 and 6, in order to find the best compromise solution of a multiobjective optimization (MOO) problem, or a good approximation of it, MOO methods need to elicit some information about the DM's preferences. Thus, MOO methods may be classified with respect to the time of collecting the preference information as methods with either a priori, a posteriori, or progressive (interactive) articulation of preferences (Hwang *et al.*, 1980; Słowiński, 1984). While the previous Chapter 6 discussed the use of (partial) a priori preference information in evolutionary MOO, here we will focus on interactive approaches. Note, however, that the two issues are closely related, as methods working with (partial) a priori information can be turned into interactive methods simply by allowing the DM to adjust preferences and re-start or continue the optimization interactively.

In general, interactive methods have the following main advantages:

- The preference information requested from the DM is usually much simpler than the preference information required by a priori methods.

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Visualization in the Multiple Objective Decision-Making Framework

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Abstract. In this paper we describe various visualization techniques which have been used or which might be useful in the multiple objective decision making framework. Several of the ideas originate from statistics, especially multivariate statistics. Some techniques are simply for illustrating snapshots of a single solution or a set of solutions. Others are used as an essential part of the human-computer interface.

8.1 Introduction

We describe various visualization techniques which have been proven useful or which we feel might prove useful in the multiple objective decision making framework. We focus on fundamental visualization techniques (see Chapter 9, for more specific techniques). Several of our ideas originate from statistics, especially multivariate statistics. Typically, in the multiple objectives framework, the decision maker (DM) is asked to evaluate a number of alternatives. Each alternative is characterized using an objective vector. From the perspective of visualization, the complexity of the decision problem depends on two dimensions: the number of objectives and the number of alternatives. A problem may be complex due to a large number of alternatives and a small number of objectives, or the other way round, although the nature of the complexity is different. Different visualization techniques are required for each case. The number of alternatives may also be uncountable, such as a subset of a feasible region in an objective space in multiobjective optimization.

In descriptive statistics, computer graphics is widely used to illustrate numerical information by producing standard visual representations (bar charts, line graphs, pie charts, etc.). More advanced visualization techniques, for example, Andrews (1972) curves and Chernoff (1973) faces have also been proposed. Especially Andrews curves and Chernoff faces were developed to illus-

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Visualizing the Pareto Frontier

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Abstract. We describe techniques for visualizing the Pareto optimal set that can be used if the multiobjective optimization problem considered has more than two objective functions. The techniques discussed can be applied in the framework of both MCDM and EMO approaches. First, lessons learned from methods developed for biobjective problems are considered. Then, visualization techniques for convex multiobjective optimization problems based on a polyhedral approximation of the Pareto optimal set are discussed. Finally, some visualization techniques are considered that use a pointwise approximation of the Pareto optimal set.

9.1 Introduction

Visualization of the Pareto optimal set in the objective space, to be called here a *Pareto frontier*, is an important tool for informing the decision maker (DM) about the feasible Pareto optimal solutions in biobjective optimization problems. This chapter is devoted to the visualization techniques that can be used in the case of more than two objective functions with both MCDM and EMO approaches for multiobjective optimization. In practice, we discuss methods for visualizing a large (or infinite) number of Pareto optimal solutions in the case of three and more objectives.

When discussing the visualization of the Pareto frontier, one has to note the difference between the decision space and the objective space (see definitions given in Preface). The techniques covered here are mainly aimed at the visualization of objective vectors (points) which form a part of the feasible objective region. Such an interest in visualizing the Pareto frontier is related mostly to the fact that in multiobjective optimization problems, the preferences of the DM are related to objective values, but not to decision variables

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Meta-Modeling in Multiobjective Optimization

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Abstract. In many practical engineering design and other scientific optimization problems, the objective function is not given in closed form in terms of the design variables. Given the value of the design variables, the value of the objective function is obtained by some numerical analysis, such as structural analysis, fluidmechanic analysis, thermodynamic analysis, and so on. It may even be obtained by conducting a real (physical) experiment and taking direct measurements. Usually, these evaluations are considerably more time-consuming than evaluations of closed-form functions. In order to make the number of evaluations as few as possible, we may combine iterative search with *meta-modeling*. The objective function is modeled during optimization by fitting a function through the evaluated points. This model is then used to help predict the value of future search points, so that high performance regions of design space can be identified more rapidly. In this chapter, a survey of meta-modeling approaches and their suitability to specific problem contexts is given. The aspects of dimensionality, noise, expensiveness of evaluations and others, are related to choice of methods. For the multiobjective version of the meta-modeling problem, further aspects must be considered, such as how to define improvement in a Pareto approximation set, and how to model each objective function. The possibility of interactive methods combining meta-modeling with decision-making is also covered. Two example applications are included. One is a multiobjective biochemistry problem, involving instrument optimization; the other relates to seismic design in the reinforcement of cable-stayed bridges.

10.1 An Introduction to Meta-modeling

In all areas of science and engineering, models of one type or another are used in order to help understand, simulate and predict. Today, numerical methods

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Real-World Applications of Multiobjective Optimization

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Abstract. This chapter presents a number of illustrative case studies of a wide range of applications of multiobjective optimization methods, in areas ranging from engineering design to medical treatments. The methods used include both conventional mathematical programming and evolutionary optimization, and in one case an integration of the two approaches. Although not a comprehensive review, the case studies provide evidence of the extent of the potential for using classical and modern multiobjective optimization in practice, and opens many opportunities for further research.

11.1 Introduction

The intention with this chapter is to provide illustrations of real applications of multiobjective optimization, covering both conventional mathematical programming approaches and evolutionary multiobjective optimization. These illustrations do cover a broad range of application, but do not attempt to provide a comprehensive review of applications.

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Multiobjective Optimization Software

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Abstract. This chapter provides a description of multiobjective optimization software with a general overview of selected few available tools developed in the last decade. This chapter can be considered a revision of previous valid papers and chapters on nonlinear multiobjective optimization software such as the ones written by Weistroffer *et al.* (2005) and Miettinen (1999) that lists existing software packages up to the year 1999. More precisely, this chapter is focused on the tools and features that advisable multiobjective optimization software should contain.

12.1 Introduction

The main topic to be discussed in this chapter is available multiobjective optimization software. The main concern is devoted to software developed for nonlinear problems. Several questions may be raised when discussing multiobjective optimization software, but among the most recurring questions we may list the following:

- What do experts think about multiobjective optimization tools and what are the most important features good software should always possess?
- What is the current state-of-the-art of multiobjective optimization software?
- What are the advantages and gaps of all these optimization tools?

The description of an ideal software is very close to a complex integrated environment such as a “Process Integration and Design Optimization” (PIDO) or a “Problem Solving Environment” (PSE) (Gallooulos *et al.*, 1991; Houstis *et al.*, 1997). PIDO and PSE are integrated computing environments which

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Parallel Approaches for Multiobjective Optimization

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Abstract. This chapter presents a general overview of parallel approaches for multiobjective optimization. For this purpose, we propose a taxonomy for parallel metaheuristics and exact methods. This chapter covers the design aspect of the algorithms as well as the implementation aspects on different parallel and distributed architectures.

Key words: Parallel algorithms, Parallel metaheuristics, Parallel multiobjective optimization, Parallel exact optimization

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Quality Assessment of Pareto Set Approximations

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Abstract. This chapter reviews methods for the assessment and comparison of Pareto set approximations. Existing set quality measures from the literature are critically evaluated based on a number of orthogonal criteria, including invariance to scaling, monotonicity and computational effort. Statistical aspects of quality assessment are also considered in the chapter. Three main methods for the statistical treatment of Pareto set approximations deriving from stochastic generating methods are reviewed. The *dominance ranking method* is a generalization to partially-ordered sets of a standard non-parametric statistical test, allowing collections of Pareto set approximations from two or more stochastic optimizers to be directly compared statistically. The *quality indicator method* — the dominant method in the literature — maps each Pareto set approximation to a number, and performs statistics on the resulting distribution(s) of numbers. The *attainment function method* estimates the probability of attaining each goal in the objective space, and looks for significant differences between these probability density functions for different optimizers. All three methods are valid approaches to quality assessment, but give different information. We explain the scope and drawbacks of each approach and also consider some more advanced topics, including multiple testing issues, and using combinations of indicators. The chapter should be of interest to anyone concerned with generating and analysing Pareto set approximations.

14.1 Introduction

In many application domains, it is useful to approximate the set of Pareto-optimal solutions, cf. (Ehrgott and Gandibleux, 2000; Deb, 2001; Coello Coello *et al.*, 2002). To this end, various approaches have been proposed ranging from exact methods to randomized search algorithms such as evolutionary algorithms, simulated annealing, and tabu search (see Chapters 2 and 3).

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Interactive Multiobjective Optimization from a Learning Perspective

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Abstract. Learning is inherently connected with Interactive Multiobjective Optimization (IMO), therefore, a systematic analysis of IMO from the learning perspective is worthwhile. After an introduction to the nature and the interest of learning within IMO, we consider two complementary aspects of learning: individual learning, i.e., what the decision maker can learn, and model or machine learning, i.e., what the formal model can learn in the course of an IMO procedure. Finally, we discuss how one might investigate learning experimentally, in order to understand how to better support decision makers. Experiments involving a human decision maker or a virtual decision maker are considered.

15.1 Introduction

The aim of this chapter is to explore the notion of learning in the context of Interactive Multiobjective Optimization (IMO) where Classical Multiobjective Optimization (CMO) (see Chapter 2) or Evolutionary Multiobjective Optimization (EMO) (see Chapter 7) are used. This is an important subject because, on one hand, IMO enables the Decision Maker (DM) to learn about the optimization problem, and, on the other hand, it allows the formal model

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Future Challenges

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Abstract. Many important topics in multiobjective optimization and decision making have been studied in this book so far. In this chapter, we wish to discuss some new trends and challenges which the field is facing. For brevity, we here concentrate on three main issues: new problem areas in which multiobjective optimization can be of use, new procedures and algorithms to make efficient and useful applications of multiobjective optimization tools and, finally, new interesting and practically usable optimality concepts. Some research has already been started and some such topics are also mentioned here to encourage further research. Some other topics are just ideas and deserve further attention in the near future.

16.1 Introduction

Handling problems with multiple conflicting objectives has been studied for decades (as discussed, e.g., in Chapters 1 to 3); yet there still exist many interesting topics for future research. There are both theoretical questions as well as challenges set by real applications to be tackled. Some of the questions

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