Analytical and Numerical Modelling of Paper Web Dynamics in Paper Making Process

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Background and Current Status

- Name: Tytti Saksa
- Master's degree in Mathematics, December 2009
- Postgraduate studies begun in January 2010

Status of the Ph. D. Thesis

- One year behind – three years ahead
- Expected dissertation, December 2013
- Format: collection of articles
- One article accepted, one submitted
- 40 ECTS / 60 ECTS completed

Research Collaborators

- Research group members
  - Lead by Prof. Pekka Neittaanmäki (supervisor)
  - Prof. Nikolay Banichuk, RAS (2 one-month visits per year, supervisor)
  - M. Sc. Juha Jeronen
  - M. Sc. Tero Tuovinen
  - Tech. Lic. Matti Kurki
  - Me
- Prof. Raino Mäkinen is also supervising my Ph. D. studies

Outline

Research Problem

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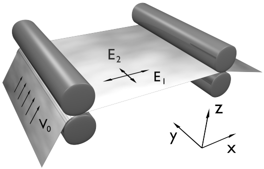
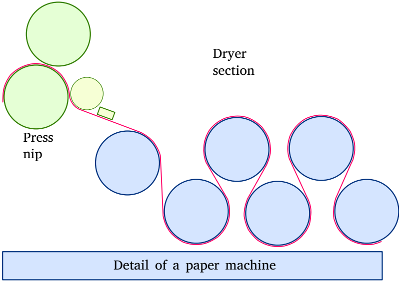
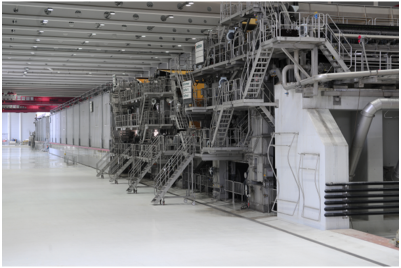


Motivation: Paper industry

- Desire for fast production speed and controlled machine running.



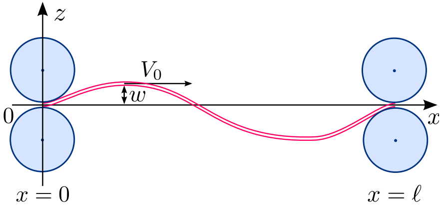


- Open draws.
  - Web vibrations.

Model

The travelling paper web is modelled as a moving plate or panel.

- Term *panel* is used when we assume that the web displacement does not vary in the cross direction to the movement.



- w = deflection, V_0 = web velocity.

Research Objectives

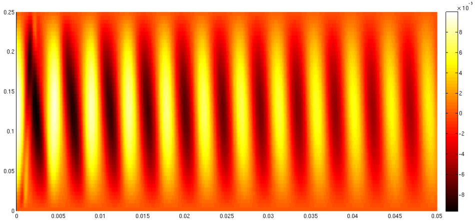
- Stability analysis of the moving web system.
  - Critical conditions.
- Effects of *viscoelasticity* of the web material.
- Effects of nonhomogeneities in the web tension.
- Effects of surrounding fluid.

Previous Work

- Group activities started in 2007.
- Instability of a (paper) web in 2D analysed
  - Banichuk, Jeronen, Neittaanmäki and Tuovinen. *Int. J. of Solids and Structures*. 47:91–99, 2010.
  - Banichuk, Jeronen, Kurki, Neittaanmäki, Saksa and Tuovinen. *Int. J. of Solids and Structures*. Article in Press. 2011.
- Instability of a web interacting with surrounding ideal fluid in 1D analysed
  - Banichuk, Jeronen, Neittaanmäki and Tuovinen. *J. of Fluids and Structures*. 26:274–291, 2010.

Dynamics of a Viscoelastic Panel

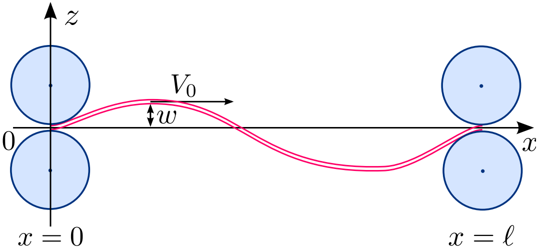
Dynamics of a Viscoelastic Panel



Viscoelasticity?

- Viscoelastic material has both viscous and elastic properties.
  - honey is viscous
  - rubber is elastic
  - chewing gum is viscoelastic
- Viscoelasticity is expected to have a damping effect on the web vibrations.

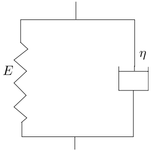
Moving Viscoelastic Panel



- Displacement $w = w(x, t)$.

Basic Model 1/4

- Kirchhoff model for the panel.
  - Web deformations are small.



- Kelvin-Voigt model for viscoelasticity.
  - A constitutive model consisting of a viscous damper η and an elastic string E.

Basic Model 2/4

Kirchhoff model

- Bending forces of the panel must satisfy equilibrium.
- Newton's second law.
- Linear elasticity, $\sigma = C\varepsilon$ (Hooke's law).
- Dynamic equilibrium for the displacement w:

$$m\left(w_{,tt} + 2V_0 w_{,xt} + V_0^2 w_{,xx}\right) = T_0 w_{,xx} - D_e w_{,xxxx} + f\,.$$

Basic Model 3/4

Kelvin-Voigt model

- Linear viscoelasticity, $\sigma = C\varepsilon + \Gamma \frac{\mathrm{d}}{\mathrm{d}t}\varepsilon = C\varepsilon + \Gamma(\varepsilon_{,t} + V_0 \varepsilon_{,x})$.
- In the dynamic equilibrium:

$$D_e \longrightarrow D_\mathrm{e} + D_\mathrm{ve} \cdot \frac{\mathrm{d}}{\mathrm{d}t}$$

- We obtain:

$$m\left(w_{,tt} + 2V_0 w_{,xt} + V_0^2 w_{,xx}\right) = T_0 w_{,xx} - D_e w_{,xxxx} - D_{ve} w_{,xxxxt} - D_{ve} V_0 w_{,xxxxx} + f\,.$$

Basic Model 4/4

$$w_{,tt} + 2\,V_0\,w_{,xt} + \frac{D_{\mathrm{ve}}}{m}w_{,xxxxt}$$
$$+ \left(V_0^2 - \frac{T_0}{m}\right)w_{,xx} + \frac{D_{\mathrm{e}}}{m}w_{,xxxx} + V_0\frac{D_{\mathrm{ve}}}{m}w_{,xxxxx} = \frac{f}{m}\,.$$

- Material derivative $\frac{\mathrm{d}}{\mathrm{d}t}(\cdot) = (\cdot)_{,t} + V_0(\cdot)_{,x}$ used in linear viscoelasticity.
  - Kurki and Lehtinen. In *Papermaking Research Symposium 2009*, PRS 2009.
  - Ding and Chen. *Eur. J. of Mechanics - A/Solids.* 27(6):1108–1120, 2008.

Boundary and initial conditions

- Deflections are zero at the edges $x = 0$ and $x = \ell$:

$$w(0,t) = 0\,, \quad w(\ell,t) = 0\,.$$

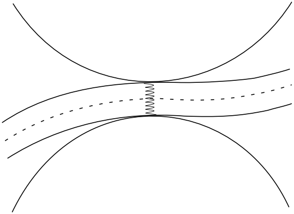
- Edges can rotate freely:

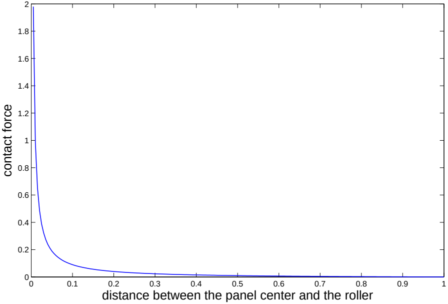
$$w_{,xx}(0,t) = 0\,, \quad w_{,xx}(\ell,t) = 0\,.$$

- Initial conditions:

$$w(x,0) = g_1(x)\,, \quad w_{,t}(x,0) = g_2(x)\,.$$

Contact with Rollers: Spring Model





Contact with Rollers: Numerical Approach

- Finite difference space discretisation.
  - Advantages:
    - Applicaple for the 5th order PDE.
    - Fast and easy to implement.
  - Disadvantages:
    - Poor results if the solution has large gradients.
- Fourth order Runge-Kutta for the time discretisation.
  - Trusted and widely used method.

Contact with Rollers: Numerical Results 1/4

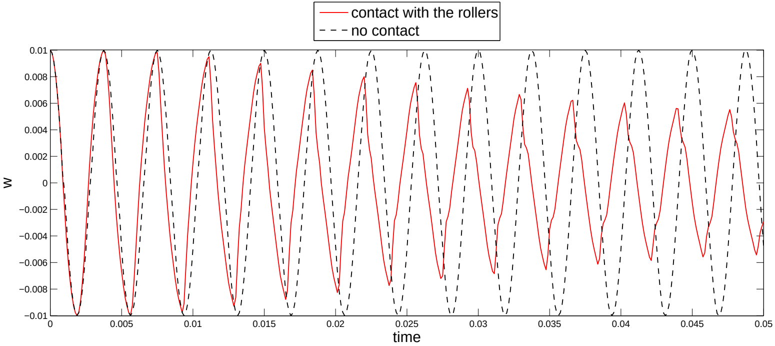


Figure: Nonviscous material. Panel velocity is zero.

Contact with Rollers: Numerical Results 2/4

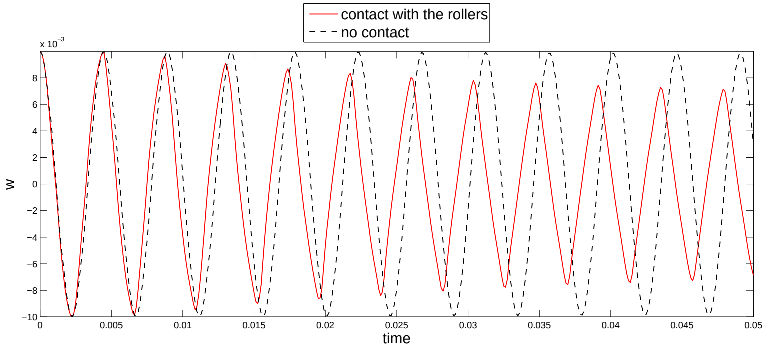


Figure: Nonviscous material. Panel velocity is 20 m/s.

Contact with Rollers: Numerical Results 3/4

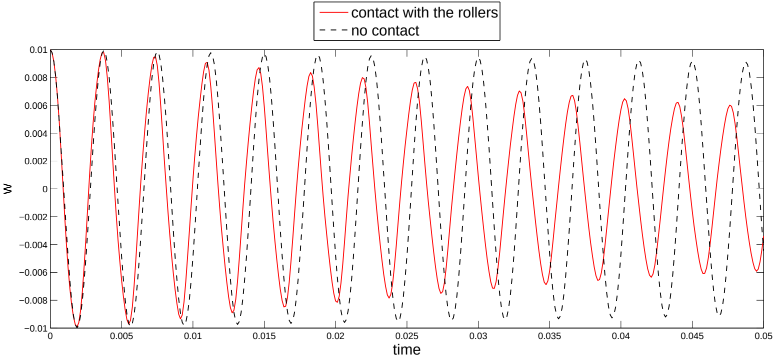


Figure: Viscous material. Panel velocity is zero.

Contact with Rollers: Numerical Results 4/4

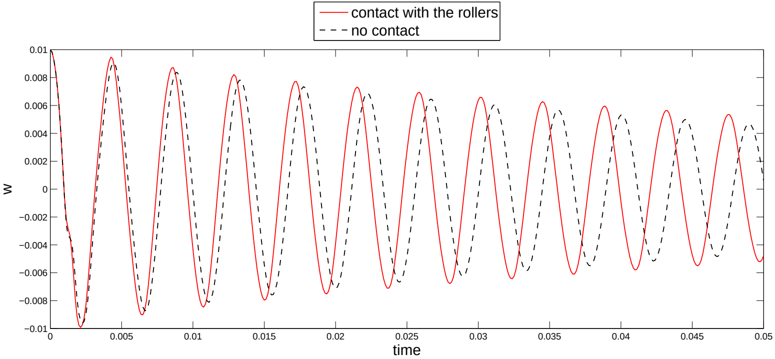


Figure: Viscous material. Panel velocity is 20 m/s.

Contact with Rollers: Conclusions

- Contact with the rollers is
  - *decreasing the amplitude* of the vibrations, and
  - *increasing the frequency* of the vibrations

compared to the case with no roller contact.

- Except: in the case of viscous moving material the *amplitude* of the vibrations is *larger* in the case with contact compared to the case with no contact.

Future Plans

- To perform stability analysis for the 1D viscoelastic model (without contact).
- To derive a 2D model for a viscoelastic moving plate.

Expected Results

- Basic research.
  - Increase the understanding of the physical phenomenon.
- Real time simulations.
- Direct application in paper making process.
- About 5 journal papers.

Summary

- Ph. D. thesis is nicely started with the help of active research group.
    - One accepted journal paper.
- The model including viscoelasticity and roller contact is studied.
    - Manuscript almost ready for journal submission.

Thank You for Your Attention!

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For Further Reading I

N. Banichuk, J. Jeronen, M. Kurki, P. Neittaanmäki, T. Saksa and T. Tuovinen.
On the limit velocity and buckling phenomena of axially moving orthotropic membranes and plates.
International Journal of Solids and Structures. Article in Press. 2011. *doi:10.1016/j.ijsolstr.2011.03.010.*

N. Banichuk, J. Jeronen, P. Neittaanmäki and T. Tuovinen.
On the instability of an axially moving elastic plate.
International Journal of Solids and Structures. 47:91–99, 2010. *doi:10.1016/j.ijsolstr.2009.09.020.*

For Further Reading II

N. Banichuk, J. Jeronen, P. Neittaanmäki and T. Tuovinen.
Statical instability analysis for travelling membranes and plates interacting with axially moving ideal fluid.
Journal of Fluids and Structures. 26:274–291, 2010.
doi:10.1016/j.juidstructs.2009.09.006.

H. Ding and L.-Q. Chen.
Stability of axially accelerating viscoelastic beams: multi-scale analysis with numerical confirmations.
European Journal of Mechanics - A/Solids. 27(6):1108–1120, 2008.