Multiobjective Optimization Applied to Industrial Plant Design

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Outline

Industrial Plant Design

- Simultaneous design of
  - Production system structure
  - Process control system

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- Potential benefits
  - Better controllability
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  - Production system structure
  - Process control system
- Potential benefits
  - Better controllability
  - Lowered capital cost
  - Improved production efficiency
- Difficulties involved
  - Computational complexity
  - Limited modeling accuracy
  - Uncertainty about future events

Multiobjective Optimization

- General multiobjective optimization problem:

$$\begin{aligned} \min \quad & F(x) = [f_1(x), \ldots, f_k(x)]^{\mathsf{T}} \\ \text{subject to} \quad & x \in A, \end{aligned}$$

where $F\colon A \longrightarrow \mathbb{R}^k$ is the objective function

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  where $F\colon A \longrightarrow \mathbb{R}^k$ is the objective function
- Solution x is called *Pareto optimal*, if

$$\begin{aligned} \nexists\, x', j\colon \; & f_j(x') < f_j(x) \wedge \\ & f_i(x') \leq f_i(x) \;\forall\; i = 1, \ldots, k \end{aligned}$$

Bilevel Optimization

- General bilevel optimization problem:

$$\begin{aligned} \min_{x} \quad & f_U(x, y^*) \\ \text{subject to} \quad & f_L(x, y^*) = \min_{y} f_L(x, y), \\ & x \in A, \quad y,\ y^* \in B \end{aligned}$$

- Functions $f_U : A \times B \to \mathbb{R}$ and $f_L : A \times B \to \mathbb{R}$ are the objective functions of the *upper-level* and *lower-level* problems, respectively

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- Lower-level feasible set B may depend on x, i.e. $B = B(x)\ \forall\ x \in A$

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- Solutions $x \in A$ to the upper-level problem can be compared based on
  - $F_U(x, y^*)$, where y^* is the most preferred by decision maker
  - $F_U(x, P_y)$, where P_y is the Pareto set of lower-level problem

Problem Formulation

- Bilevel design optimization problem

$$\begin{aligned} \min_{d} \quad & F_U(w, x, d, u^*) \\ \text{subject to} \quad & F_L(w, x, d, u^*) = \min_{u} F_L(w, x, d, u), \\ & d \in D, \quad u,\ u^* \in U \end{aligned}$$

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- Multiobjective optimization problem at both levels
  - Decomposition of cost function
  - Trade-off between risk and expectation

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- State variable x governed by differential equation

Future Challenges

- Interaction with decision maker
  - Informative representation of the problem
  - Generalization of elicited preferences
- Computational complexity
  - Limiting the number of simulator calls
  - Avoiding wasted effort in lower-level problem