Shape Optimization
(activities 1983-2010)

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What is (mathematical) shape optimization ?

- In general, any optimization problem in which parameter to be optimized has some geometric interpretation (thickness, location, etc)
- In what follows, only those activities at JY related to "***PDE constrained shape optimization***" problems are considered

Abstract setting

Min $\mathcal{J}(\boldsymbol{a})=\boldsymbol{J(a,u(a))}$

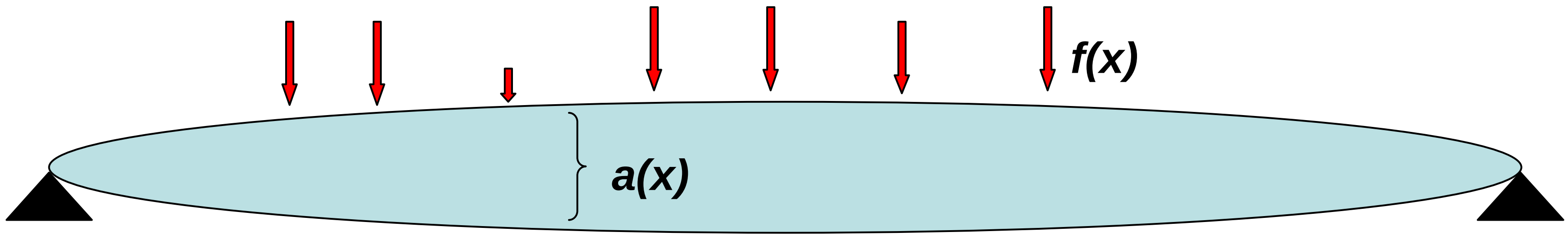
$\boldsymbol{a} \in \boldsymbol{D}$

$\boldsymbol{A(a)u(a)=f(a)}$ *(state problem PDE)*

$\boldsymbol{u(a)} \in \boldsymbol{S}$

- ***D*** technically admissible ”shapes”
- ***A(a)*** differential operator
- ***S*** admissible states of the system
- $\boldsymbol{J: D \times S \to R}$ cost function

Example: thickness optimization of a slender beam



Min $\mathcal{J}(a) = \max_x |a(x)y''(x)|$ (*maximum bending stress*)

s.e. $\int a(x)dx = V$ ***ja*** $a(x) \geq a_0 > 0$

$([a(x)]^3 \, y''(x))'' = f(x) \qquad 0 < x < L$

$y(0) = y''(0) = y(L) = y''(L) = 0$

Control parameter is the function $\boldsymbol{a: [0,L] \to R}$

Multiphysics and/or multicriteria shape optimization

$$\min_{a \in D} \{ J_1(a,u(a)),\ J_2(a,w(a)) \}$$

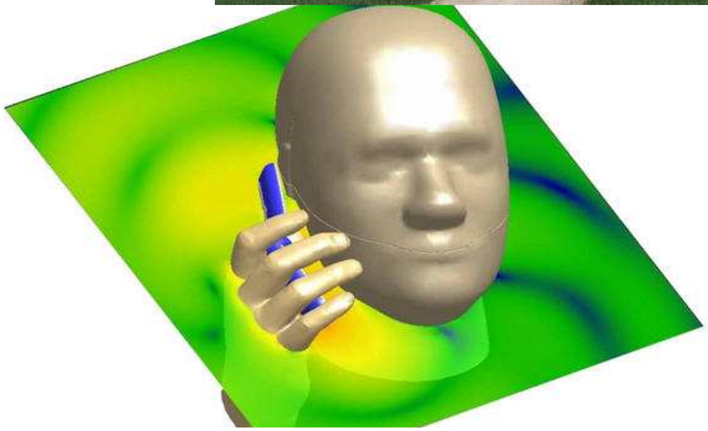
$$A_1(a)u(a)=f_1(a)$$

$$A_2(a)w(a)=f_2(a)$$

$$u(a) \in S_1,\ w(a) \in S_2$$

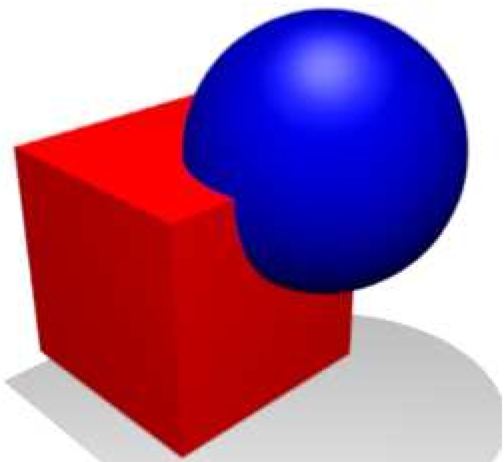
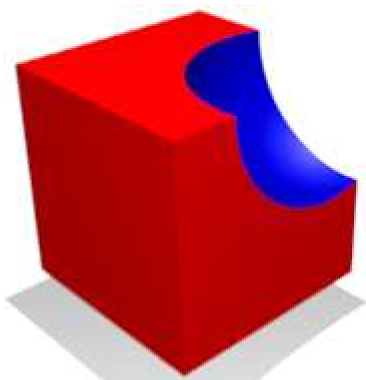
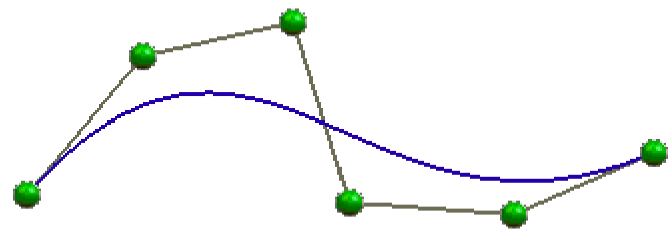
Examples:

- Aeroelasticity (fluid + structure)
- Stealth (fluid + EM / acoustics)
- Mobile technology (structure + EM + heat + acoustics)



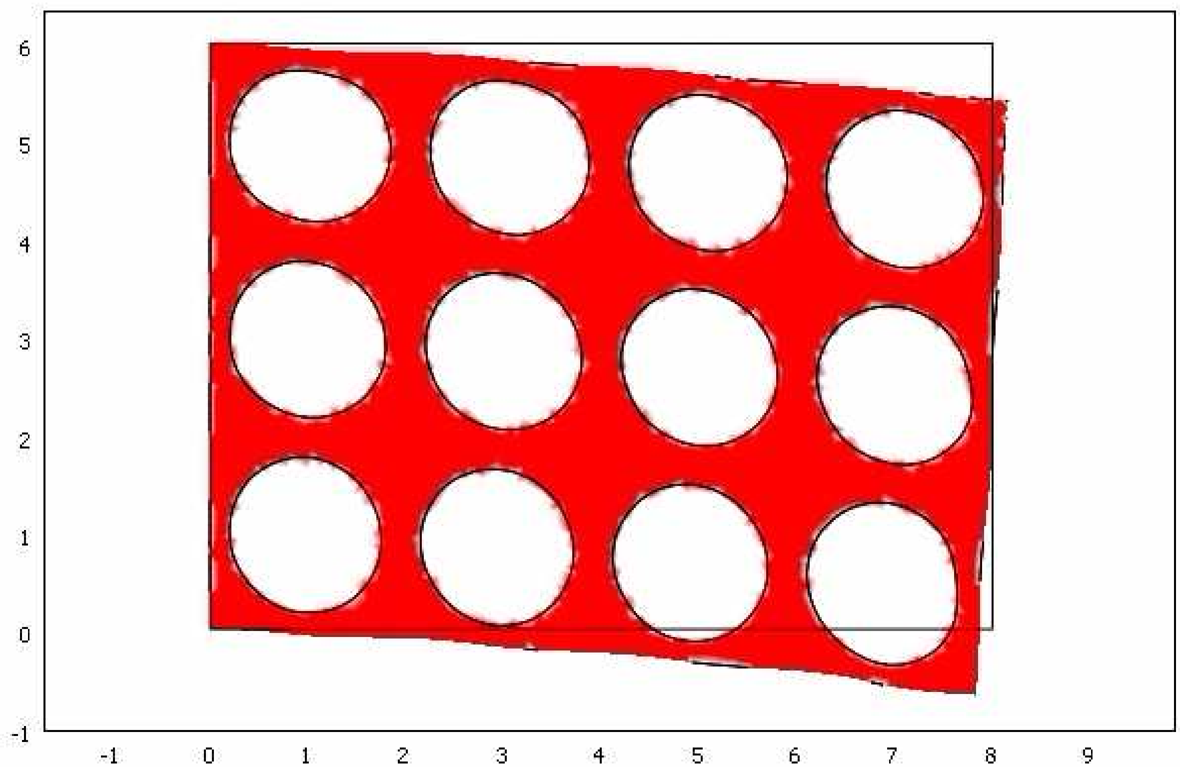
Shape parameterization

- CAD-parameterization
  - Eg. CSG, length, thickness, radius, Spline curve control points,…

- "CAD-free" parameterization
  - "Move" computational grid
- "Level-set function" parameterization:
  $\varphi: R^d \rightarrow R$, $\Omega := \{ x: \ \varphi(x) > 0 \}$, $\varphi(x) = \sum a_i \varphi_i(x)$

Example:Topology optimization of a cantilever using "level set" method

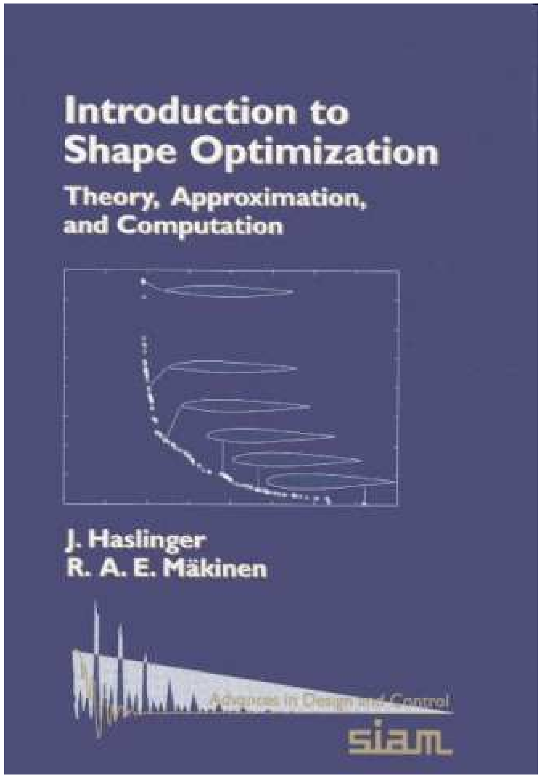


Past and recent activities in Shape Optmization

- **First journal paper**: J. Haslinger, P. Neittaanmäki: *Penalty method in design optimization of systems governed by mixed Dirichlet - Signorini boundary value problem*, Ann. Fac. Sci. Tolouse, 199-216, **1983.**

- **First PhD Thesis**: by T. Tiihonen, **1987**
  - Others: Mäkinen, Salmenjoki, Jari Toivanen, (Kari Kärkkäinen)

- **Monographs:**
  - Haslinger & Neittaanmäki, 1988
  - Haslinger & Neittaanmäki, 1996
  - Neittaanmäki, Rudnicki & Savini, 1996
  - Haslinger & Mäkinen, 2003
  - Banichuk & Neittaanmäki, 2009
- **Papers**:
  - Over 25 journal articles in ISI Web of Science
- **Collaboration with industry**:
  - Technology transfer



Recent journal papers by RM et al.

- Toivanen, Mäkinen, Rahola, Järvenpää & Ylä-Oijala (2010). Gradient-based shape optimisation of ultra-wideband antennas parameterised using splines. IET Microwaves, Antennas & Propagation.

- Toivanen & Mäkinen (2010). Implementation of sparse forward mode automatic differentiation with application to electromagnetic shape optimization. Optimization Methods & Software

- Toivanen, Mäkinen, Järvenpää, Ylä-Oijala & Rahola (2009). Electromagnetic Sensitivity Analysis and Shape Optimization Using Method of Moments and Automatic Differentiation. IEEE Antennas and Propagation.

- Toivanen, Haslinger & Mäkinen (2008). Shape optimization of systems governed by Bernoulli free boundary problems. Computer Methods in Applied Mechanic and Engineering.

- Stebel, Mäkinen & Toivanen (2007). Optimal shape design in a fibre orientation model. Applications of Mathematics.

Adjoint equation method

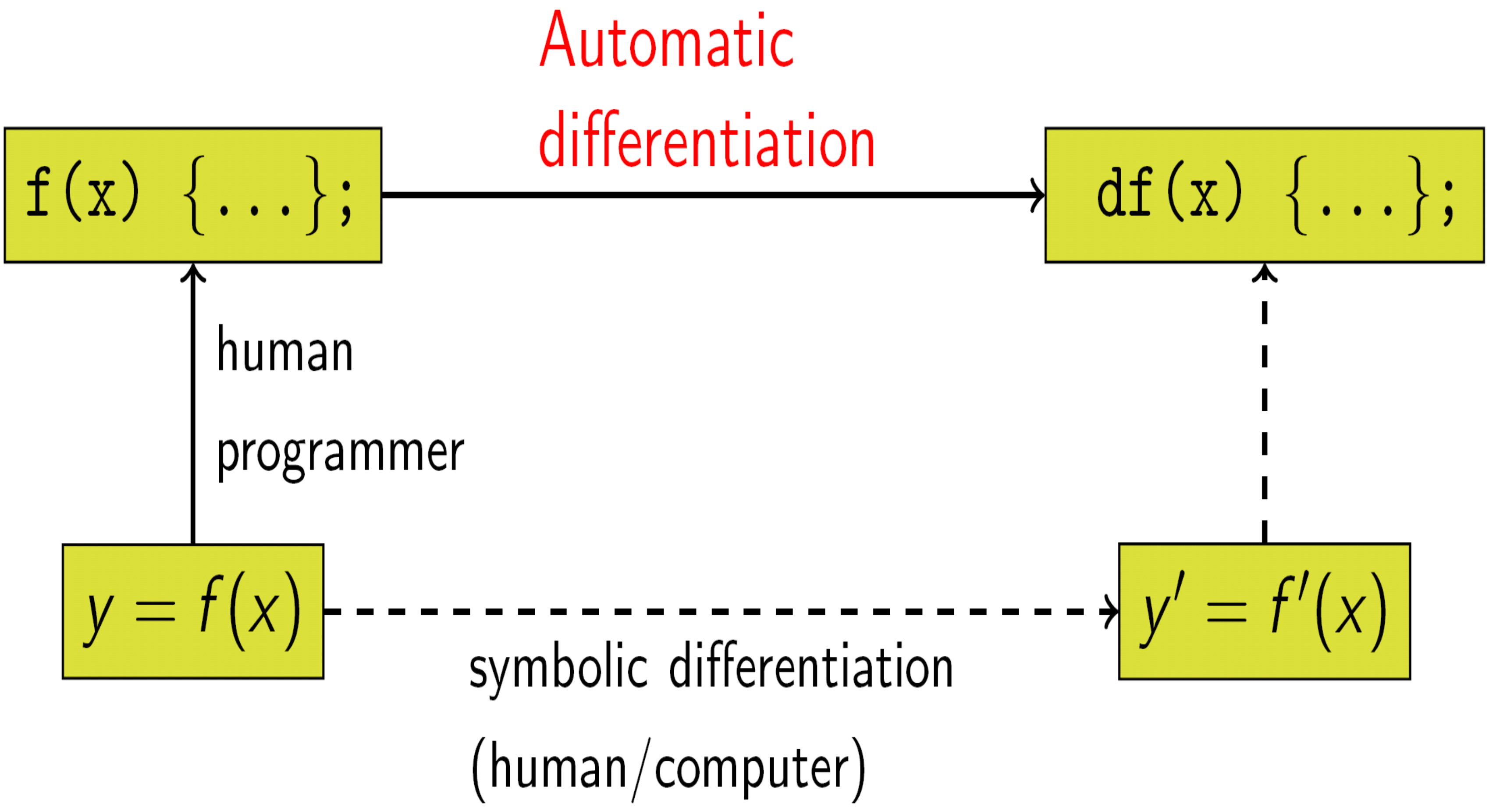
Min $\mathcal{J}(\boldsymbol{a})=J(\boldsymbol{a},\boldsymbol{u}(\boldsymbol{a}))$

$\boldsymbol{A}(\boldsymbol{a})\boldsymbol{u}(\boldsymbol{a})=\boldsymbol{f}(\boldsymbol{a})$

How to compute $\partial\mathcal{J}(\boldsymbol{a})/\partial \boldsymbol{a}_k$ at $\boldsymbol{a}$?

1) Solve state problem: $\boldsymbol{K}(\boldsymbol{a})\boldsymbol{u} = \boldsymbol{f}(\boldsymbol{a})$
2) Solve <u>adjoint equation</u>:
   $[\boldsymbol{A}(\boldsymbol{a})]^T\boldsymbol{p} = \partial J(\boldsymbol{a},\boldsymbol{u})/\partial \boldsymbol{u}$
3) Compute partial derivative:
   $\partial\mathcal{J}(\boldsymbol{a})/\partial \boldsymbol{a}_k = \boldsymbol{p}^T[\,\partial \boldsymbol{f}(\boldsymbol{a})/\partial \boldsymbol{a} - (\partial \boldsymbol{K}(\boldsymbol{a})/\partial \boldsymbol{a})\boldsymbol{u}\,]$

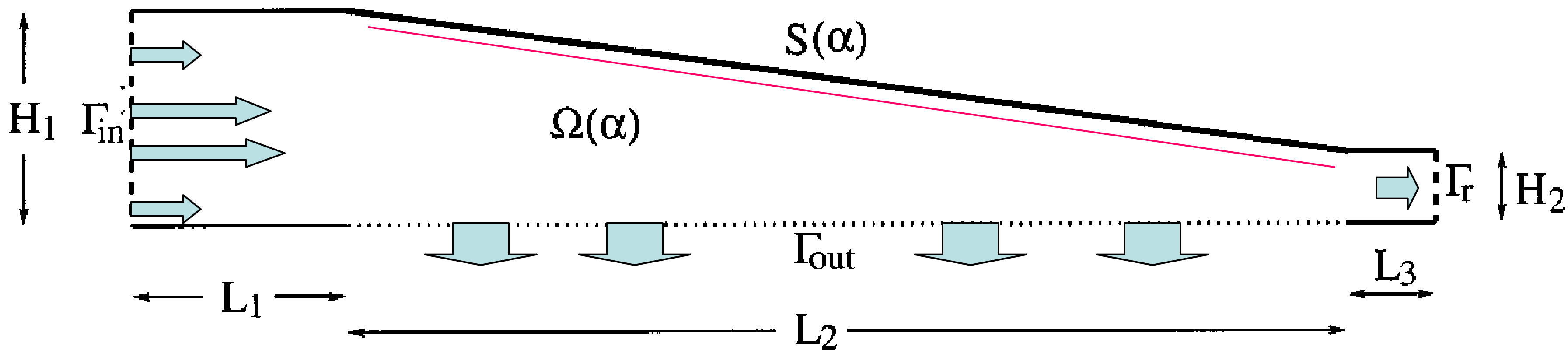
Automatic vs. symbolic differentiation



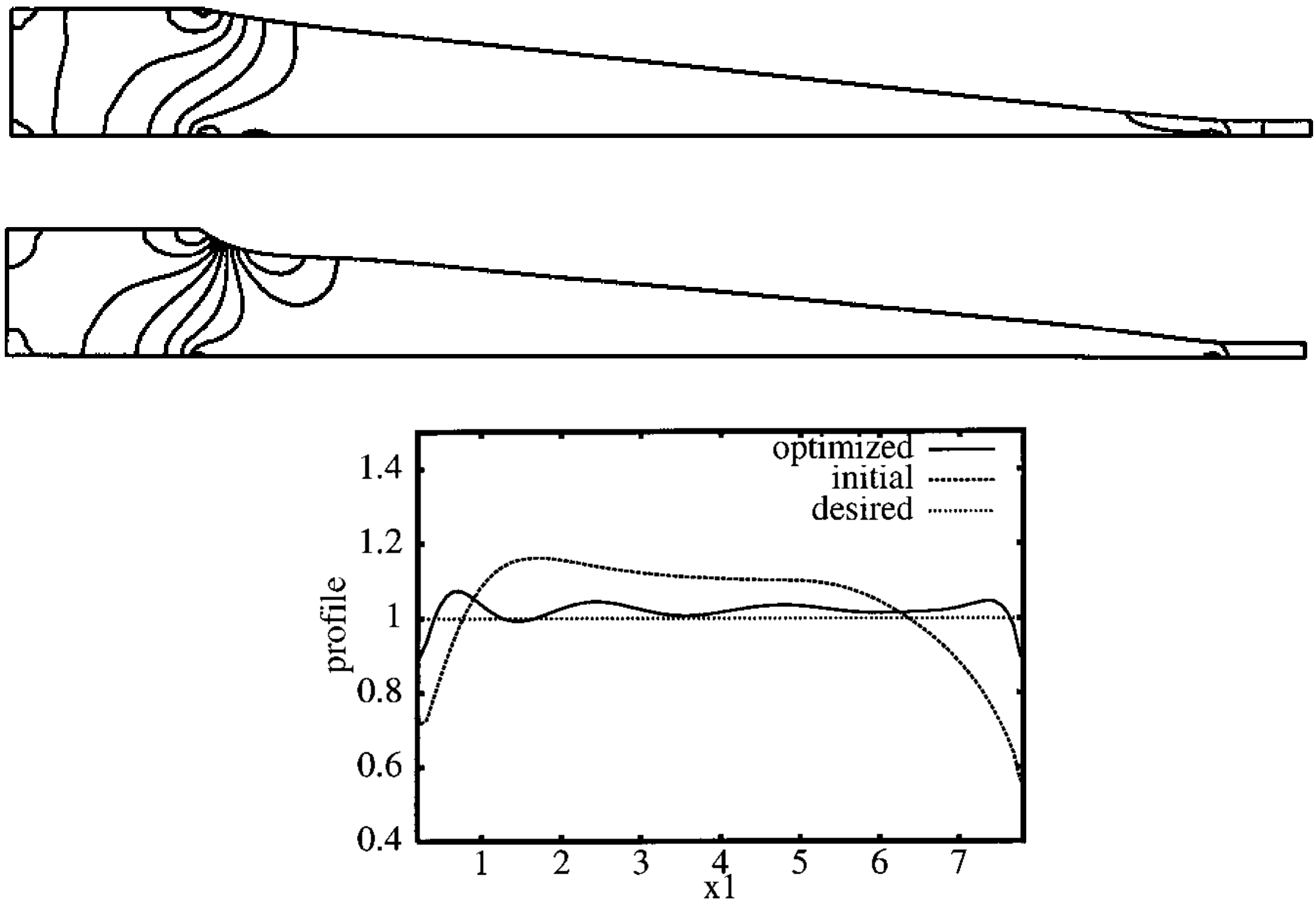
AD in shape optimization

- In principle, one could apply AD to whole (FEM) simulation program
- In practice, often only certain sensitivities (e.g. $\partial K(a)/\partial a$ and $\partial f(a)/\partial a$) are computed using AD.
- Derivatives computed using AD very useful for the simulation model alone (Jacobian for Newton method in case of nonlinear state problem)!!

Example: Shape optimization of a dividing tube



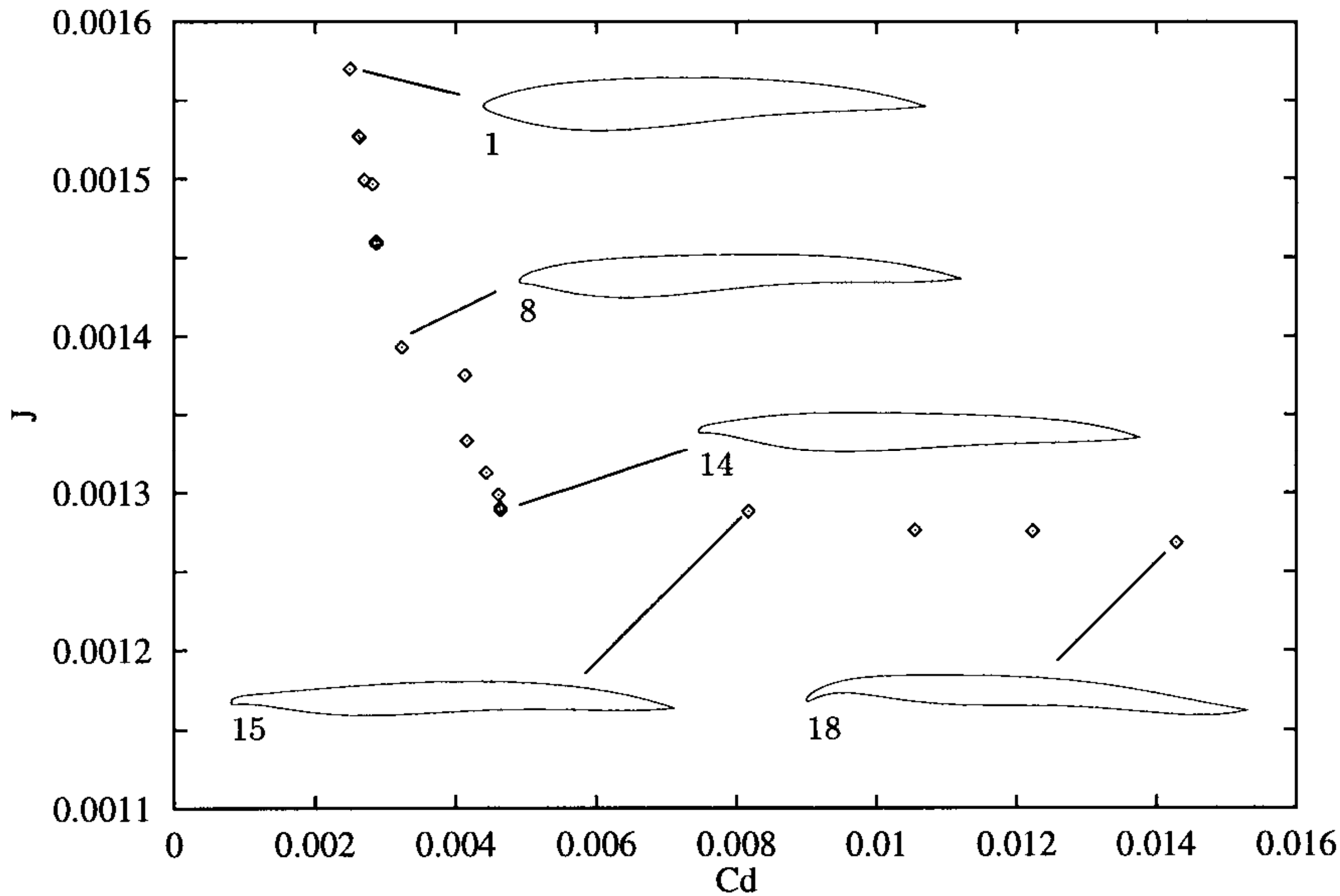
- Navier-Stokes equation, algebraic turbulence model
- FEM, analytic sensitivity analysis for discretized state problem using hybrid AD/hand coded derivatives
- Shape optimization using gradient method (SQP) needs ca. 20 function evaluations (= FEM simulations).



optimized
initial
desired
1.4
1.2
1
0.8
0.6
0.4
profile
1
2
3
4
5
6
7
x1

Example: Multiphysics multicriteria shape optimization of an airfoil using a Genetic Algorithm

Optimization of Radar Cross Section (RCS) and drag coefficient (C_D) subject to constraint for lift coefficient (C_L) . CFD-model: Euler, CEM-model: Helmholtz. Optimization takes ca. 6000 function evaluations ☹



Example: Shape optimization of patch dipole antenna

Surface integral equation formulation of 3D time-harmonic Maxwell's equations:

$$\left(-\frac{1}{i\omega\epsilon_0}\nabla\mathbf{S}(\nabla_S\cdot\mathbf{J})(\mathbf{r}) + i\omega\mu_0\mathbf{S}(\mathbf{J})(\mathbf{r})\right)_{\tan} = -\mathbf{E}^p_{\tan}(\mathbf{r})$$

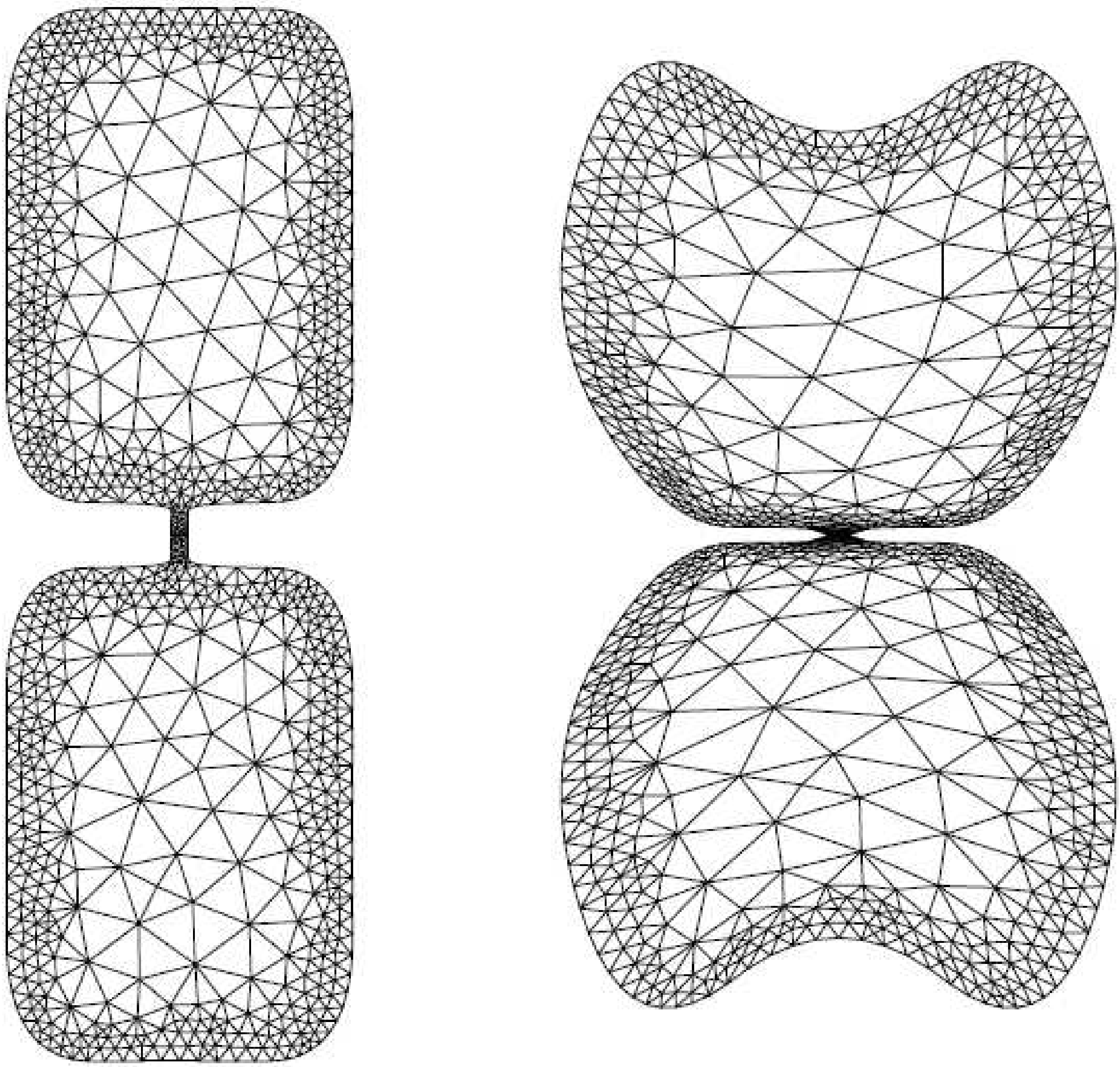
$$\mathbf{S}(\mathbf{F})(\mathbf{r}) = \int_S G_0(\mathbf{r},\mathbf{r}')\mathbf{F}(\mathbf{r})\,dS'$$

Method of Moments discretization leads to a large **dense** linear system:

$$A\mathbf{x} = \mathbf{b}$$

We wish to design an antenna that performs well over wide frequency range (multipoint design) by minimizing ”S-parameter”.

Initial versus optimal shape (and compuational mesh)



Performance over frequency range