An Approximation Approach to Computationally Expensive Multiobjective Optimization Problems

Markus Hartikainen

April 12, 2010

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Me

- Name: Markus Hartikainen

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- Ph. Lic. in mathematics (mathematical analysis)

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- Ph. Lic. in mathematics (mathematical analysis)
- Interested in optimization, scientific computing, statistics, pedagogy

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- The plan is to defend my thesis in 2011.

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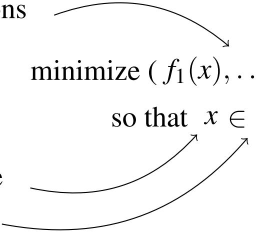

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- A decision variable $x \in S$ is called a Pareto optimal (PO) solution if there does not exist an outcome $z \in f(S)$ that dominates $f(x)$ and the outcome $f(x)$ given by a PO solution x is called a PO outcome.

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  3. Find the PO solution that is closest to the preferred point by means of an achievement scalarizing function [Wierzbicki, 1986].

A Pareto front approximation ⇒ inherent nondominance

Definition

A set $A \subset \mathbb{R}^k$ is called *inherently nondominated* (IND) if there does not exist $z^1, z^2 \in A$ so that z^1 dominates z^2. An IND set A is called an IND PF approximation based on P, if $P \subset \mathbb{R}^k$ is a finite set of PO outcomes and $P \subset A$.

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- Also the (actual) PF is by definition an IND PF approximation based on any set of PO outcomes P.

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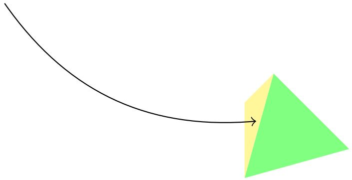

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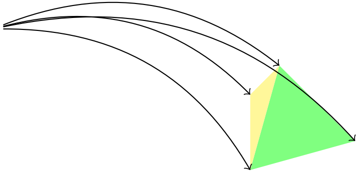

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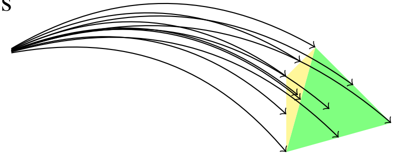

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- A complex $\mathcal{D}$ is a Delaunay triangulation of a finite set $P \subset \mathbb{R}^k$ if $\cup_{K \in \mathcal{D}} K = \text{conv}(P)$ and the complex only contains "polytopes with neighbouring vertices" (see [Edelsbrunner, 1987]).

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- With minor extra conventions, this can be seen as a shape reconstruction problem as defined in [Edelsbrunner, 1998].

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- We want to remove polytopes from the Delaunay triangulation so that there are no polytopes in the resulting collection of polytopes (which can be shown to be a complex) that dominate each other.
- In a paper that we aim to submit "any day now" we propose a set of rules for doing the above.

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- A preferred point on the approximation can be searched with interactive multiobjective optimization methods, because the approximation is IND and the approximation can be parametrized.

Further Research

- A Delaunay triangulation of m points in k dimensions contains at most $O(m^{\lceil k/2 \rceil})$ polytopes. This means that in order to find out all the dominations between polytopes, one may have to solve $O(m^k)$ optimization problems as described above. For this reason the first research question is to reduce the computational effort.

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- Any further applications are welcome. We are ready to try our approach with real world problems.

References

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Thank You!

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