On the Mechanics of Axially Moving Materials

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TIEJ601 Postgraduate seminar (11th Jan 2010)

Contents

About the Thesis 1/2

Title and content

- *On Numerical Simulation of Papermaking Processes*
- Content related to dynamics and instability analysis of axially moving materials

Supervisors

- Professor Raino Mäkinen
- Professor Pekka Neittaanmäki
- Visiting professor Nikolay Banichuk

About the Thesis 2/2

Status

- Started in spring term 2008
- Research underway: two papers published, two more being finished; more planned
- Will be completed in 2011

This talk

- Is focused on a general introduction for a postgraduate audience coming from various backgrounds.
- Some examples of results obtained so far will be shown, and
- Some possible directions for future research will be listed.

Research Status

N. Banichuk, J. Jeronen, P. Neittaanmäki, T. Tuovinen:

- On the Instability of an Axially Moving Elastic Plate. [1]
  International Journal of Solids and Structures 47 (2010), pp. 91-99. DOI: 10.1016/j.ijsolstr.2009.09.020.
- On the Instability of an Axially Moving Orthotropic Plate (working title). *Manuscript in preparation.*
- Static Instability Analysis for Travelling Membranes and Plates Interacting with Axially Moving Ideal Fluid. [2]
  Journal of Fluids and Structures, accepted (to appear in 2010) doi:10.1016/j.jfluidstructs.2009.09.006.
- Dynamical Behaviour of an Axially Moving Plate Undergoing Small Cylindrical Deformation Submerged in Axially Flowing Ideal Fluid.
  Manuscript in preparation.

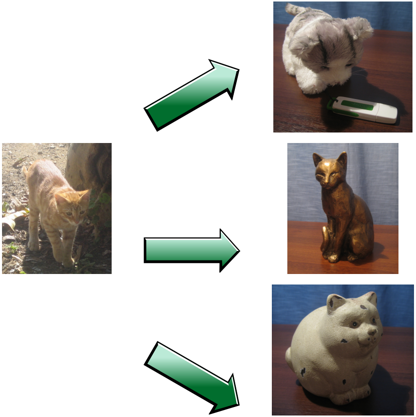
What is a cat? 1/2

Photo taken by author.

How to model?

- Four paws, a set of claws, two ears, and a tail?
- An entire library filled with detailed observations of etology and biology?
- The only complete description of a cat is the cat itself. (I. Ekeland [3]; same book also in English [4])

What is a cat? 2/2

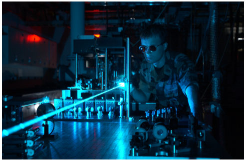


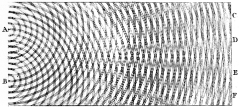
Modeling

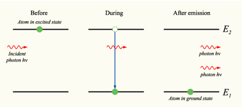
- Simplifies in the desired way,
- While (hopefully) retaining the relevant aspects of the original system.
- The definition of “relevant” depends on the purpose of the model.

All images by author.

Models and Reality







Images from http://en.wikipedia.org/wiki/Photon

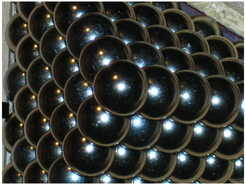
Models and reality

- Reality is fundamental
- Models are constructed to describe reality
- Comparison with reality determines model viability

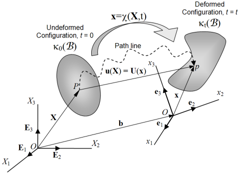
Example

- Light: waves or particles?
- Correct answer: Neither!
- But can be modeled by both.

Why Continuum? 1/2



? ? ⇓ ? ?



Why indeed?

- Isn't quantum mechanics a better model?

Well, it is true that

- Matter is, roughly speaking, made of discrete particles
- Counterintuitive QM effects such as tunneling and entanglement have been experimentally verified

Images from http://en.wikipedia.org/wiki/Solid and http://en.wikipedia.org/wiki/Continuum_mechanics

Why Continuum? 2/2

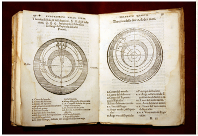
However, continuum models are fine, because

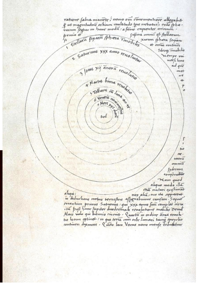
- QM phenomena are subtle
  - Macroscale implications possible, but require very precise engineering
    - E.g. only certain materials will "lase"
  - QM effects went undetected for centuries of physics!

Continuum models were built in the first place, because

- Reality does not care whether a model gets every detail "right"
- The continuum is a pretty good approximation under most circumstances.

Something to Keep in Mind





Getting the right idea is not always that simple

- What would a geocentric universe look like to the observers?
  (R. Dawkins [5], original version attributed to L. Wittgenstein)

Images PD, from http://en.wikipedia.org/wiki/Geocentric_system

and http://en.wikipedia.org/wiki/De_revolutionibus_orbium_coelestium

Mathematical Modeling!

$$\begin{aligned}
\frac{\frac{\partial^2 w}{\partial x^2} T}{1+\left(\frac{\partial w}{\partial x}\right)^2} &= \Delta p \sqrt{1+\left(\frac{\partial w}{\partial x}\right)^2} + mg + \frac{Eh^3}{12} \frac{\partial^4 w}{\partial x^4} \sqrt{1+\left(\frac{\partial w}{\partial x}\right)^2} \\
&+ \frac{m}{1+\left(\frac{\partial w}{\partial x}\right)^2} \left\{ \left[1+\left(\frac{\partial w}{\partial x}\right)^2\right] \frac{\partial^2 w}{\partial t^2} + 2\left[v\sqrt{1+\left(\frac{\partial w}{\partial x}\right)^2} - \frac{\partial w}{\partial x}\frac{\partial w}{\partial t}\right] \frac{\partial^2 w}{\partial x \partial t} \right. \\
&+ \left. \left[v^2 - \frac{2v \frac{\partial w}{\partial x}\frac{\partial w}{\partial t}}{\sqrt{1+\left(\frac{\partial w}{\partial x}\right)^2}} + \frac{\left(\frac{\partial w}{\partial x}\right)^2 \left(\frac{\partial w}{\partial t}\right)^2}{1+\left(\frac{\partial w}{\partial x}\right)^2}\right] \frac{\partial^2 w}{\partial x^2} \right\} \\
&- \frac{1}{2} \frac{C_d \rho_a}{\left[1+\left(\frac{\partial w}{\partial x}\right)^2\right]^3} \left\{ 2v \frac{\partial w}{\partial x}\frac{\partial w}{\partial t} \left[1+\left(\frac{\partial w}{\partial x}\right)^2\right]^{5/2} + \left(\frac{\partial w}{\partial t}\right)^2 \left[1-\left(\frac{\partial w}{\partial x}\right)^4\right] \right\}
\end{aligned}$$

Source: [6]

About Mathematical Modeling

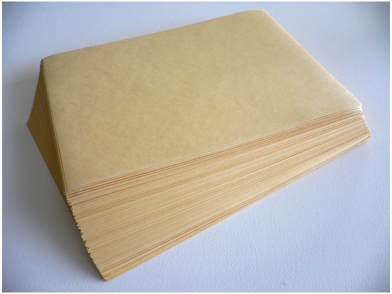
Mathematical Modeling

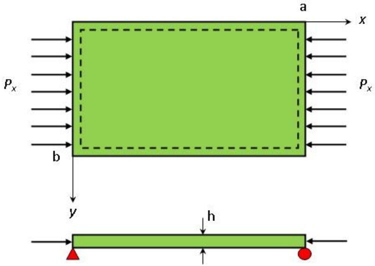
- Some phenomena are inherently more complex than others
  - Extreme example: turbulent flow vs. harmonic oscillator
  - Much of the simple physics has been done over the last few centuries
  - Complex problems can be solved numerically
    $\Rightarrow$ scientific computing
- Modeling of complex phenomena involves a tradeoff between analyzability and accuracy
  - Even then, analytical approaches may be somewhat useful
  - Design decisions made here determine the domain of applicability of the model

Plates?

Image from http://en.wikipedia.org/wiki/Plate_(dishware)

Plates!





Images from http://en.wikipedia.org/wiki/File:ManilaPaper.jpg
and http://commons.wikimedia.org/wiki/File:Rec_plate.jpg

What is an Elastic Plate? 1/2

Elastic plate

- An elastic, relatively flat piece of solid material of constant thickness; usually thin
- Idea: can abstract away the third dimension, handling it via special mathematical considerations
- Elastic behavior = reversible deformation
- Usually investigated phenomenon: **bending**

What is an Elastic Plate? 2/2

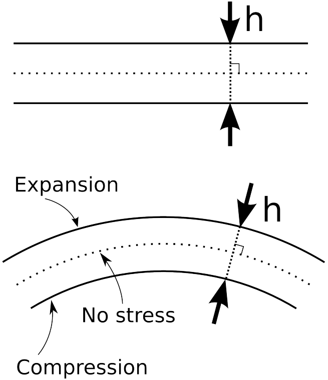


Image by author.

Kirchhoff (classical) plate theory (see e.g. [7])

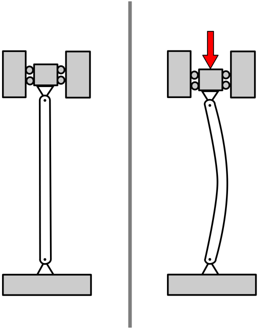
- No stresses in the middle plane of the plate
- Normals remain straight, and normal, during plate bending
- Normals are inextensible during plate bending (normal stresses can be disregarded)

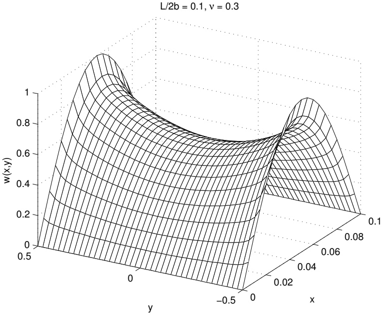
Elastic Instability 1/2

Elastic instability

- *Buckling* i.e. static instability
- First analyzed for compressive loading of beam columns by L. Euler in the 18th century
- As load is slowly increased from zero, at first no transverse displacement
- At some critical load, the structure suddenly gives in
- May be relatively harmless or completely destructive
  - Buckled columns can carry load, if not increased further
  - Buckled paper web rips itself apart

Elastic Instability 2/2





First image PD, from http://en.wikipedia.org/wiki/Buckling
Second image by author.

Stationary Structures vs. Moving Materials 1/2

Classical structural mechanics (and classical aeroelastic analysis)

- Concentrates on stationary, elastic structures (subjected to wind)
- Focuses on finding out how the structure deforms
- E.g. structural safety analysis for design of bridges

Note

- Airplanes are stationary in this sense!
- An airplane in flight behaves the same as a stationary one placed in a wind tunnel.

Stationary Structures vs. Moving Materials 2/2

Moving materials

- For phenomena where the material itself flows
- No clearly delimited physical object; instead, a continuous stream of material
- New material constantly enters one end of the analysis *domain*, and exits it at the other end
- Thus: different case; we cannot "ride" the material particles $\Rightarrow$ Coriolis and centrifugal effects seen in a stationary coordinate system

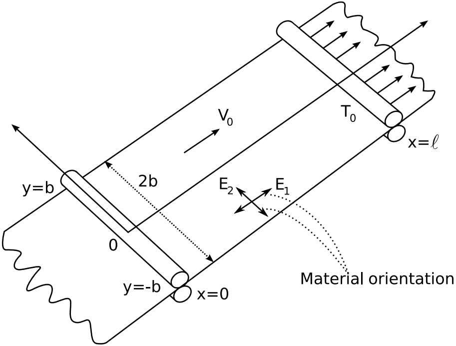
Motivation 1/2

Figure: A paper web being driven through a paper machine.

Motivation 2/2

Open draws in a paper machine

- Intervals, where the paper web travels with no mechanical support



Physical Models

Simple model

- Thin, elastic plate
  - Isotropic: elastic properties the same in every direction
  - Fully elastic: no irreversible (permanent) deformations

More accurate models

- Orthotropic elastic plate
  - Due to the fiber structure, the elastic properties of paper are different in different directions
- Viscoelasticity
  - Paper has also viscous properties $\Rightarrow$ creep and relaxation
- For more, see the review article by Alava and Niskanen [8]

Elastics 1/2

Elastics

- Even linear elastics is geometrically nonlinear
- Can the nonlinearity be approximated away? The solution is...

Small deformation theory / Linearized theory

- Small out-of-plane (transverse) displacement
- Small angles between the plate and the coordinate axes
- One linear partial differential equation (PDE) is enough

And the point is?

- Why use a linear theory?

Elastics 2/2

Small deformation theory / Linearized theory

- Small out-of-plane (transverse) displacement
- Small angles between the plate and the coordinate axes
- One linear partial differential equation (PDE) is enough

This simplified model is

- Analyzable
  - Allows finding fundamental explanations for phenomena
- Understandable
  - The effects of the different parts of the theory can be known in detail
- Accurate enough in certain cases (instability analysis)
- A standard academic research topic

The Physical Model Used

Linear model (3D)

- Small deformation theory for thin, elastic plates
- Belongs to classical mechanics
- Based on linear PDEs

$$\begin{aligned} & m\frac{\partial^2 w}{\partial t^2} + 2mV_0\frac{\partial^2 w}{\partial x \partial t} + mV_0^2\frac{\partial^2 w}{\partial x^2} \\ =& T_{xx}\frac{\partial^2 w}{\partial x^2} + 2T_{xy}\frac{\partial^2 w}{\partial x \partial y} + T_{yy}\frac{\partial^2 w}{\partial y^2} \\ & - D_1\frac{\partial^4 w}{\partial x^4} - 2D_3\frac{\partial^4 w}{\partial x^2 \partial y^2} - D_2\frac{\partial^4 w}{\partial y^4} + q_f \quad \text{in } \Omega = [0,\ell] \times [-b,b] \end{aligned}$$

Physical Setup (3D)

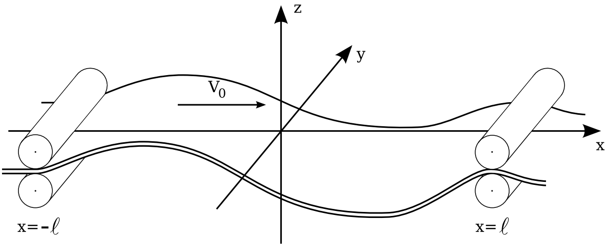


Figure: A model of an open draw (3D).

Phenomena (3D)

Investigated physical process

- Elastic deformations of a thin, axially moving plate (in vacuum)

Elastic Problem

- The investigated physical system is inherently unstable.
- Linear theory is applicable up to the first (critical) instability [9].
- The critical instability is of the static type [1].

Physical Setup (2D)

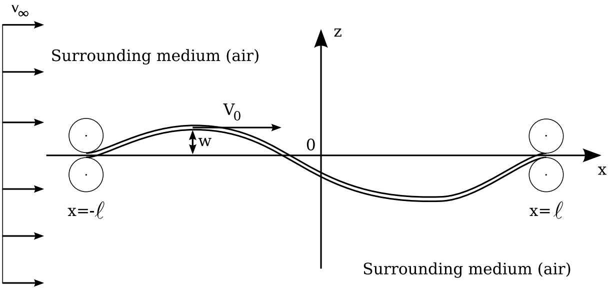


Figure: A model of an open draw (2D).

- The two-dimensional model can be used to investigate cylindrical deformations.

Phenomena (2D) 1/3

Investigated physical process

- Elastic, cylindrical deformations of a thin, axially moving plate, accounting for the effect of the surrounding air

Typically investigated cases

- Moving material surrounded by stationary air
- Stationary structure subjected to axial flow

- In the paper [2] we combined these two cases.

Phenomena (2D) 2/3

Aeroelastic problem

- Vibrations coupled with the effect of the surrounding air
  - Contains the elastic problem
  - The presence of a surrounding medium changes the eigenfrequencies drastically [10], [11, 12].
- The instability of the system is still of the static type, but...
- By applying a dynamic analysis one can extract the eigenfrequencies, too.

Phenomena (2D) 3/3

Aeroelastic problem

- Closely connected to the problems of flow inside a pipe, and of a stationary structure subjected to axial flow [13].
- Results from stationary structures are not necessarily directly applicable [14], [15].
  - We found [2] that the assumption of cylindrical deformation only works well in case of narrow strips; cf. the stationary case where this approximation works best for wide plates [7].
- Ignoring viscosity may have a drastic effect on the critical velocities, but the correct behavior is obtained for the eigenfrequencies [15].

Numerical Results 1/8

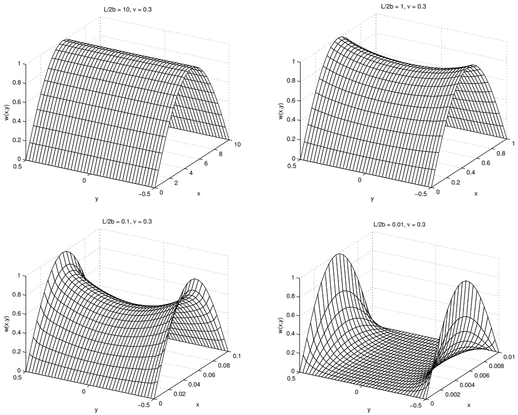
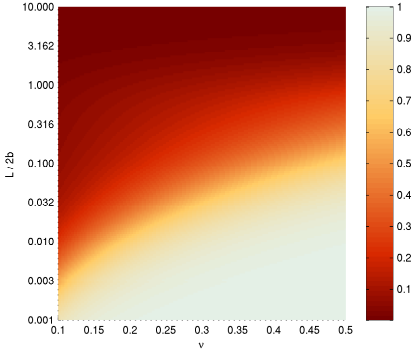


Figure: Buckling modes of isotropic plates.

Numerical Results 2/8



The degree of localization

of the buckling mode for isotropic plates, as a function of the aspect ratio $L/2b$ and the Poisson ratio ν. In the localized modes, most of the deformation occurs near the free boundaries.

In the figure, the color represents the relative degree of localization.

Numerical Results 3/8

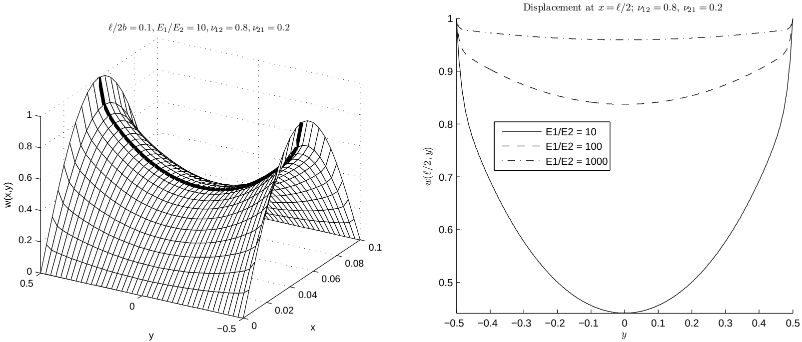
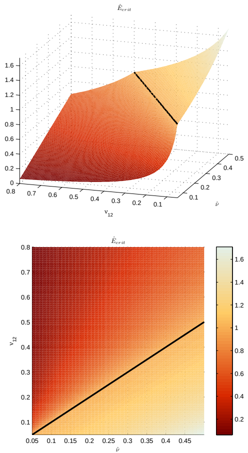


Figure: Buckling mode for dry paper. Orthotropic model, fibers along the machine direction.

Numerical Results 4/8



Parameter range where

bending force stabilization is guaranteed for orthotropic plates. If a given parameter triplet $\hat{E}$, $\hat{\nu}$ and ν_{12} indicates a point on or below the surface in the figure, the bending forces will have a stabilizing effect regardless of the transverse displacement w. The parameters

$$\hat{E} \equiv \sqrt{E_2/E_1} \quad \text{and}$$

$$\hat{\nu} \equiv \sqrt{\nu_{12}\nu_{21}} \, .$$

Analytical Result

Aerodyn. pressure difference q_f as a functional of displacement w

$$q_f(x,t) = -\rho_f \left(\frac{1}{\tau}\frac{\partial}{\partial t} + \frac{1}{\ell}(v_\infty - V_0)\frac{\partial}{\partial x} \right) \ldots$$

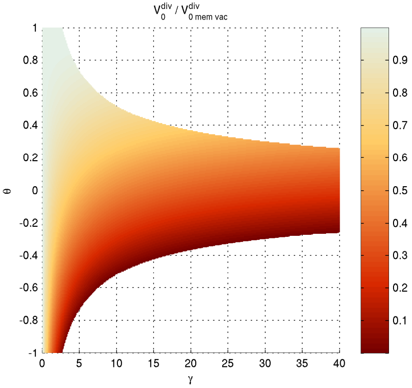
$$\ldots \int_{-1}^{1} N(\xi,x) \left(\frac{\ell}{\tau}\frac{\partial w}{\partial t} + (v_\infty - V_0)\frac{\partial w}{\partial x} \right) \mathrm{d}\xi \qquad (1)$$

where

$$N(\xi,x) \equiv \frac{1}{\pi} \ln \left| \frac{1+\Lambda}{1-\Lambda} \right| , \qquad (2)$$

$$\Lambda(\xi,x) \equiv \left[\frac{(1-x)(1+\xi)}{(1-\xi)(1+x)} \right]^{1/2} . \qquad (3)$$

Numerical Results 5/8



Dimensionless critical velocity

for an ideal membrane ($D = 0$), as a function of dimensionless fluid density

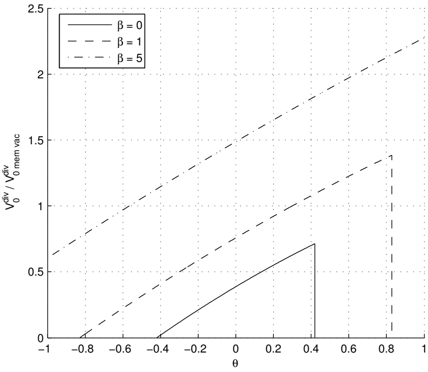
$$\gamma \equiv \frac{\ell \rho_f}{m}$$

and dimensionless fluid velocity

$$\theta \equiv \frac{v_\infty}{V_{0\ \text{mem vac}}^{\text{div}}} ,$$

where $V_{0\ \text{mem vac}}^{\text{div}} \equiv \sqrt{T/m}$.

Numerical Results 6/8



The effect of fluid velocity

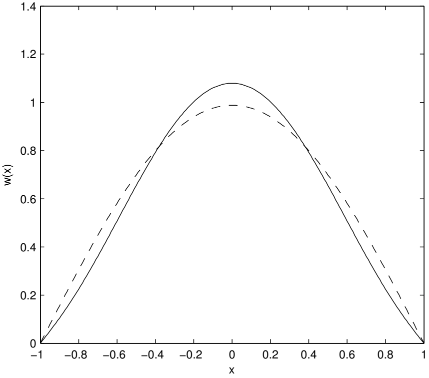
on the critical velocity of a plate, parametrized by dimensionless bending rigidity

$$\beta \equiv \frac{D}{\ell^2 T} .$$

In all cases in the figure, the dimensionless fluid density is

$$\gamma \equiv \frac{\ell \rho_f}{m} = 15.625 .$$

Numerical Results 7/8



Buckling mode

for the parameter values

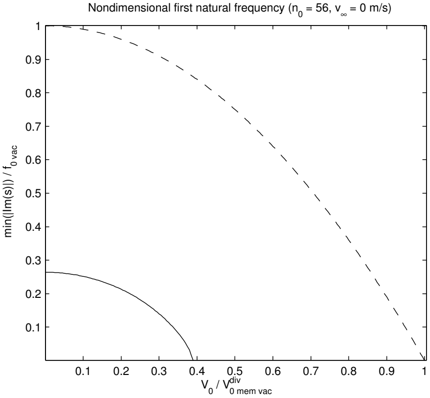
$$\beta \equiv \frac{D}{\ell^2 T} = 0.01 \,,$$

$$\gamma \equiv \frac{\ell \rho_f}{m} = 15.625 \,, \quad \text{and}$$

$$\theta \equiv \frac{V_\infty}{V_{0\,\text{mem vac}}^{\text{div}}} = 0.43238 \,.$$

Dashed line: vacuum case for comparison.

Numerical Results 8/8



Lowest nondim. eigenfrequency

for an ideal membrane as a function of dimensionless membrane velocity

$$\lambda \equiv \frac{V_0}{V^{\text{div}}_{0\text{ mem vac}}} ,$$

where

$$V^{\text{div}}_{0\text{ mem vac}} \equiv \sqrt{T/m} .$$

Dashed line: vacuum case.
Solid line: in stationary ideal fluid.

Directions for Future Research 1/2

Directions for future research

- Currently planned:
  - Non-homogeneous tension distribution at supports
    - Linear tension profile (preliminary work begun)
    - Nonlinear tension profile
  - Effects of viscoelasticity on the dynamics and instability
  - Finding a balance between structural integrity and stability
    - Small cracks may cause the whole web to break if the tension is too high

Directions for Future Research 2/2

Directions for future research

- Possible additional topics:
    - A posteriori error estimates for the simplest cases?
    - Numerical fluid-structure interaction using more realistic fluid models (Navier–Stokes)?

Summary

- When modeling complex phenomena, there is a compromise between analyzability and completeness.
- Sometimes simple models are accurate enough, but on the other hand their applicability is limited.
- In the thesis, linearized Kirchhoff plate theory has been used as a basis for analytical and numerical research on mechanical instability phenomena in papermaking.

Thank you for your attention!

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