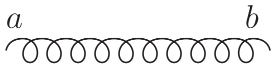
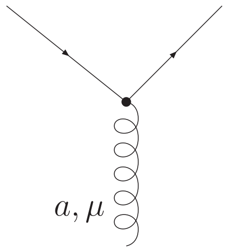
Exercise 6 Return by 12.00 on Wednesday 23.10

1. Compute the QCD correction to the quark self-energy for massless quarks at 1-loop order. The relevant Feynman rules are



$$= \frac{-i\delta^{ab}}{k^2+i\epsilon}\left(g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}\right)$$

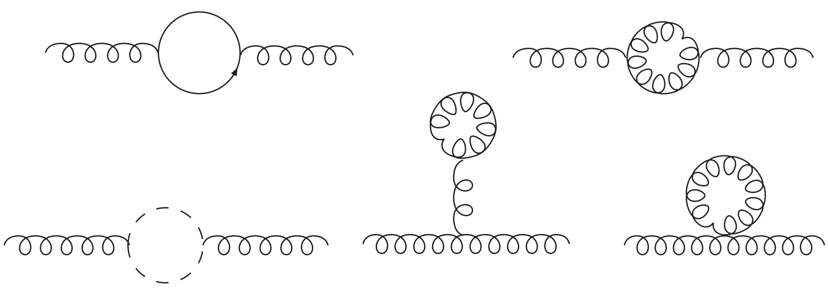


$$= ig\gamma^\mu t^a$$

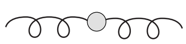
$$= \frac{i\delta^{ab}\not{p}}{p^2+i\epsilon}$$

where t^a's are the QCD-generators. Do the calculation in the dimensional regularization scheme, using this time the Landau gauge $\xi = 0$.

2. Compute the gluon self-energy to 1-loop order, using the dimensional regularization and the Feynman gauge. For the Feynman rules and algebra for $SU(3)$, see the Appendix of Peskin & Schroeder. If you are unfamiliar with the color algebra, you may want to also consult the section 15.4 from P&S. Note that there are now several diagrams at 1-loop:



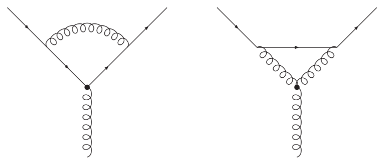
After the smoke has cleared, you should find the current-conserving strucrure $(q^2 g^{\mu\nu} - q^\mu q^\nu)$. The Feynman rule for the counter term is given by



$$= -i\left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)\delta^{ab}\delta_3$$

Perform the renormalization in the $\overline{MS}$-scheme. In this particular scheme one subtracts only the pole $\sim 2/\epsilon - \gamma_E + \log(4\pi)$. The finite part should now explicitly depend on μ^2, the mass dimensional parameter introduced by the dimensional regularization.

3. Compute the UV divergent parts of the 1-loop corrections to the QCD quark-quark-gluon 1PI-function. Take the quarks to be massless. There is now two diagrams to consider:



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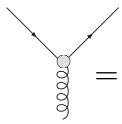

$= igt^a\gamma^\mu\delta_1$

Perform the renormalization in the $\overline{MS}$-scheme by subtracting the pole $\sim 2/\epsilon - \gamma_E + \log(4\pi)$. Note that in order to extract the counter term δ_1 you need only the UV-divergent terms. There is also IR-divergences present in the diagrams above which can as well show up as a $1/\epsilon$ -type poles. So be careful not to mix up these terms with UV-poles!

4. Show that the BRST transformation.

$$\begin{aligned} QA^a_\mu &= D^{ab}c^b \\ Q\psi &= igc^a t^a \psi \\ Qc^a &= -\frac{1}{2}gf^{abc}c^b c^c \\ Q\bar{c}^a &= B^a \\ QB^a &= 0\,, \end{aligned}$$

where $D^{ab}_\mu = \partial_\mu \delta^{ab} + gf^{abc}A^c_\mu$, is nilpotent, *ie.* $Q^2\phi = 0$ for all fields $\phi = A^a_\mu, \psi, c^a, \bar{c}^a, B^a$. Show also in detail that the transformation $\phi \to \phi + \epsilon Q\phi$ (where ϵ is an anticommuting number), is a continuous symmetry of the QCD Lagrangean

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \frac{\xi}{2}(B^a)^2 + B^a(\partial_\mu A^a_\mu) + \bar{c}^a(-\partial^\mu D^{ac}_\mu)c^c.$$