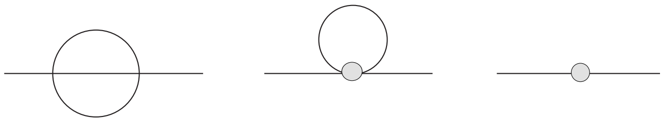
Exercise 5 Return by 12.00 on Wednesday 16.10

**1.** The 2-loop contributions to the propagator in the $\phi^4$-theory involves effectively only the three diagrams shown below (why only these three? Sure enough, there are other 2-loop diagrams.).



Compute the first of these diagrams *in the limit of zero mass* for the scalar field, using dimensional regularization. Show that, near $d = 4$, this diagram takes the form

$$-ip^2 \cdot \frac{\lambda^2}{12(4\pi)^4} \left[ -\frac{1}{\epsilon} + \log p^2 + \cdots \right],$$

with $d = 4 - \epsilon$. The coefficient in this equation involves a Feynman parameter integral that can be evaluated by setting $d = 4$. Verify that the second diagram vanishes near $d = 4$. Thus the first diagram is the only singular one, with a pole at $\epsilon = 0$ that can be canceled by a field-strength renormalization counterterm. (This is Peskin & Schroeder Problem 10.3.)

**2.** Show that the result of excercise 1 implies the anomalous dimension

$$\gamma = \frac{\lambda^2}{12(16\pi^2)^2}.$$

Compute the value of this anomalous dimension in the WF-fixed point up to $\epsilon^2$.

**3.** In the pseudoscalar Yukawa theory, with masses set to zero

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \overline{\psi}(i\partial\!\!\!/)\psi - ig\overline{\psi}\gamma^5\psi\phi - \frac{\lambda}{4!}\phi^4.$$

Compute the Callan-Symanzik $\beta$ functions for $\lambda$ and $g$:

$$\beta_\lambda(\lambda, g) \qquad \beta_g(\lambda, g),$$

to leading order in coupling constants, assuming that $\lambda$ and $g^2$ are of the same order. Sketch the coupling constant flows in the $\lambda - g$ plane. (This is Peskin & Schroeder Problem 12.1.) Use the $\overline{MS}$-scheme.

**4.** Except for the Yang-Mills theories there is no consistent theory which is asymptotically free in four dimensions. In other space-time dimensions, however, such theories exist and $\phi^3$-theory in six dimensions is one of those. Starting from the Lagrangian

$$\mathcal{L}_{\phi^3} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi^3,$$

your job is to verify this statement to 1-loop order. You should obtain

$$\beta(g) = -\frac{3g^3}{4(4\pi)^3}$$

for the beta-function. Its negative value implies that the running coupling gets weaker as the momentum scale grows. Verify this by solving the corresponding renormalization group equation. Do the computation in the $\overline{MS}$-scheme.