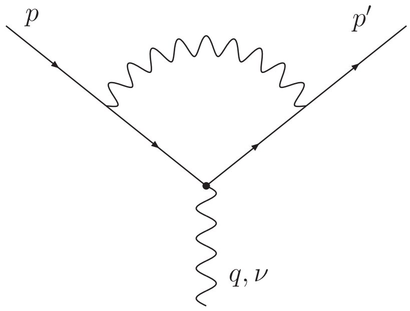
1. Compute the photon-electron 3-point function in the limit $q^2 = 0$ in the dimensional regularization. That is work out the functions $F_1(0)$ and $F_2(0)$ explicitly. Note that in this case the two electron propagators combine to a single bosonic propagator, so that the integral can again be expressed in terms of the two-point functions of the problem 1. Show that Ward identity $\delta_2 = \delta_1 = -\delta F_1(0)$ holds.



2. Exotic contributions to $g - 2$. (Peskin & Schröder 6.3 a-b). Any particle that couples to the electron can produce a correction to the electron-photon form factors and, in particular, a correction to $a \equiv (g-2)/2$. Because a factors for electron and for muon agree with QED to high accuracy, these corrections allow us to constrain the properties of hypothetical new particles. The standard model of weak interactions contains a higgs scalar that couples to fermions through interaction

$$\mathcal{L}_{\rm int} = y_f h \bar{\psi}_f \psi_f \,,$$

where the coupling constant y_f is related to the mass of the fermion f by $y_f v = \sqrt{2} m_f$, where $v = 246$ GeV is the vacuum expectation value of the higgs condensate. Compute the contribution of a virtual higgs boson to electron and muon anomalous magnetic moments a_e and a_μ. Use these results to put bounds on the higgs particle mass, given the following observational constraints:

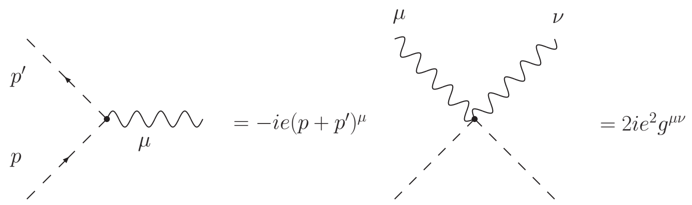
$$\begin{aligned} |a^{(e)}_{\rm expt.} - a^{(e)}_{\rm QED}| &< 1 \times 10^{-10} \\ |a^{(\mu)}_{\rm expt.} - a^{(\mu)}_{\rm QED}| &< 8 \times 10^{-8} \,. \end{aligned} \tag{1}$$

Hint: Do not attempt to do full renormalization calculation with the higgs contribution; we are not interested on that here. Instead, just work out the contribution proportional to $\bar{u}(p+q) i\sigma^{\mu\nu} q_\nu u(p)$ coming from the higgs diagram. It is not affected by renormalization and gives the entire contribution to anomalous magnetic moment just like in QED.

3. The scalar QED is described by the Lagrangian

$$\mathcal{L}_{\rm SED} = |D_\mu \Phi|^2 - m^2 |\Phi|^2 - \frac{1}{4} F_{\mu\nu}^2,$$

where $D_\mu = \partial_\mu + ieA_\mu$ and $\Phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$. Convince yourself that this implies the following Feynman rules for Φ-photon interactions:



Draw the two diagrams contributing to the photon self-energy correction to 1-loop order in the scalar QED, and compute them using dimensional regularization. Compute the running coupling constant $\alpha_{\rm eff}(q^2)$ in scalar QED.