Exercise 3 Return by 12.00 on Tuesday 24.9.

1. We continue to compute some basic 1-loop integral functions. First show that

$$A_{\mu\nu}(m) = \mu^{4-d}\int \frac{d^dk}{(2\pi)^d}\frac{k_\mu k_\nu}{k^2-m^2+i\epsilon} = A_2(m)g_{\mu\nu},$$

where

$$A_2(m) = \frac{m^2}{4}A_0(m) + \frac{im^4}{8(4\pi)^2}.$$

where A_0 is the one-point scalar function computed in the first excercise. Explain why any tensor with odd number of k^μ-factors in the nominator has to vanish.

Next consider vector and tensor B functions using (Passarino-Veltman) reduction to a linear combinations of scalar functions:

$$\begin{aligned} B_\mu(p^2;m_1^2,m_2^2) &\equiv \mu^{4-d}\int \frac{\mathrm{d}^dk}{(2\pi)^d}\frac{k_\mu}{(k^2-m_1^2)((k-p)^2-m_2^2)}\,, \\ B_{\mu\nu}(p^2;m_1^2,m_2^2) &\equiv \mu^{4-d}\int \frac{d^dk}{(2\pi)^d}\frac{k_\mu k_\nu}{(k^2-m_1^2)((k-p)^2-m_2^2)}\,. \end{aligned}$$

First show that $B_\mu(p^2;m_1^2,m_2^2) = p^\mu B_1(p^2;m_1^2,m_2^2)$, where

$$B_1(p^2;m_1^2,m_2^2) \equiv \frac{1}{2p^2}\Big[A_0(m_2^2) - A_0(m_1^2) + (p^2+m_1^2-m_2^2)B_0(p^2;m_1^2,m_2^2)\Big]$$

and B_0 is the two-point scalar integral computed in the first excercise. Note the symmetry property $B_1(p^2;m_2,m_1) + B_1(p^2,m_1,m_2) = B_0(p^2;m_1,m_2)$. Next consider the rank two integral. Explain why we must be able to make a reduction

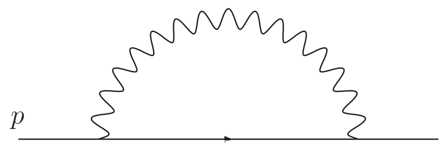
$$B_{\mu\nu}(p^2;m_1^2,m_2^2) = B_{2g}(p^2;m_1^2,m_2^2)\,g_{\mu\nu} + B_{2p}(p^2;m_1^2,m_2^2)\,\frac{p_\mu p_\nu}{p^2}\,,$$

where B_{2g} and B_{2p} are two new scalar functions. Show that this relation leads to a system of two linear equations for B_{2p} and B_{2g}:

$$\begin{aligned} dB_{2g}(p^2;m_1^2,m_2^2) + B_{2p}(p^2;m_1^2,m_2^2) &= A_0(m_2^2) + m_1^2B_0(p^2;m_1^2,m_2^2) \\ B_{2g}(p^2;m_1^2,m_2^2) + B_{2p}(p^2;m_1^2,m_2^2) &= \frac{1}{2}\big[A_0(m_2^2) + (p^2+m_1^2-m_2^2)B_1(p^2;m_1^2,m_2^2)\big]. \end{aligned}$$

Finally, solve the above linear equation for B_{21}, B_{22} and express your results in terms of the basic A_0- and B_0-functions.

2. Use the results from the problem 1 to compute the electron self-energy diagram in the dimensional regularization scheme using the Feynman gauge. Regulate the photon propagator by a finite mass μ_p as was done in the lectures. Compute counter terms explicitly and show that δ_m is IR-finite, but δ_2 is IR- as well as UV-divergent.



3. Compute the photon self-energy diagram within the dimensional regularization scheme, again using the results obtained above for the vector and tensor 2-point integrals. Compute in particular the counter term δ_3 explicitly.