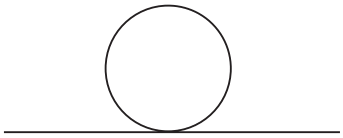
Quantum Field Theory Aplications Fall 2024

Exercise 2 Return by 12.00 on Tuesday 17.9.

1. Consider *massless* ϕ^4-theory. Show that in the dimensional regularization scheme, the 1-loop graph shown below is zero.



Hint: The loop integral diverges both in the infrared and in the ultraviolet limit. To regularize it, you need to break the integral in two pieces and choose the dimensions $D_{\rm IR}$ and $D_{\rm UV}$ in such a way that the both integrals converge. The trick is that by analytical continuation it is possible to take the limit $D_{\rm IR} \to D_{\rm UV}$, and magically the infrared and ultraviolet poles cancel! (Hint: Taizo Muta, Quantum chromodynamics)

2. Peskin & Schroeder Problem 10.2: Renormalization of Yukawa theory. Consider the pseudoscalar Yukawa Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \overline{\psi}(i\partial\!\!\!/ - M)\psi - ig\overline{\psi}\gamma^5\psi\phi,$$

where ϕ is a real scalar field and ψ is a Dirac fermion. Notice that this Lagrangian is invariant under the parity transformation $\psi(t, \mathbf{x}) \to \gamma^0\psi(t, -\mathbf{x})$, $\quad \phi(t, \mathbf{x}) \to -\phi(t, -\mathbf{x})$, in which the field ϕ carries odd parity.

(a) Determine the superficially divergent amplitudes and work out the Feynman rules for renormalized perturbation theory for this Lagrangian. Include all necessary counterterm vertices. Show that the theory contains a superficially divergent 4ϕ amplitude. This means that the theory cannot be renormalized unless one includes a scalar self-interaction,

$$\delta\mathcal{L} = \frac{\lambda}{4!}\phi^4,$$

and a counterterm of the same form. It is of course possible to set the renormalized value of this coupling to zero *at some scale*, but the coupling would be non-vanishing in other scales. Are any further interactions required?

(b) Compute the divergent part (the pole as $d \to 4$) of each counterterm, to the one-loop order of perturbation theory, implementing a sufficient set of renormalization conditions. You need not worry about finite parts of the counterterms. Since the divergent parts must have a fixed dependence on the external momenta, you can simplify this calculation by choosing the momenta in the simplest possible way.