STANDARD MODEL OF ELECTROWEAK INTERACTIONS

The most essential part of the SM is the spontaneous symmetry breaking through Higgs mechanism:

$$SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_{em} \qquad (5.1)$$

Let us see how this takes place. Introduce a complex scalar field

value of the hyperchange

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = (\underset{\sim}{2}, 1) \qquad (5.2)$$

SU(2)-representation

The kinetic term $|\partial_\mu \Phi|^2$ can be made invariant under <u>local</u> gauge-transformation

$$\Phi \to e^{i\theta^a t^a} e^{i\beta \frac{Y}{2}} \Phi \qquad (5.3)$$

At the expense of introducing SU(2)- and $U(1)_Y$-gauge-fields W^a_μ and B_μ in the covariant derivative $|D_\mu\phi|^2$:

wrong sign

$$\boxed{\mathcal{L}_{SM} = |D_\mu \Phi|^2 \underbrace{+ \mu^2 |\Phi|^2 - \lambda |\Phi|^4}_{\text{SSB-potential}} \underbrace{- \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{SU(2)} \underbrace{- \frac{1}{2} f_{\mu\nu} f^{\mu\nu}}_{U(1)_Y} + \cdots} \qquad (5.4)$$

where

$$D_\mu \equiv \partial_\mu - i g t^a W^a_\mu - i g' \frac{Y}{2} B_\mu \qquad (5.5)$$

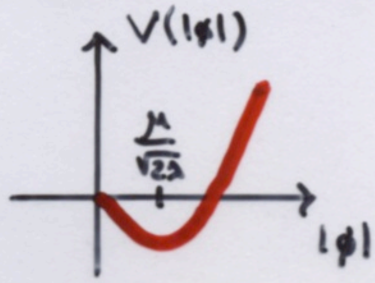
and

$$F_{\mu\nu}^{a} \equiv \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc} W_\mu^b W_\nu^c$$

$$f_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu . \qquad (5,6)$$

The Lagrangian (5,4) is clearly fully $SU(2)\times U(1)$-symmetric. However, given that $\mu^2>0$, the minimum configuration is again not at $|\phi|=0$, but instead at

$$|\Phi_0|^2 = |\langle 0|\phi|0\rangle|^2 \equiv \frac{v}{2} = \frac{\mu^2}{2\lambda} \qquad (5,7)$$



$V(|\phi|) = -\mu^2|\phi|^2 + \lambda|\phi|^4$

Of course, there is a full SU(2)-symmetry of equivalent asymmetric vacua. We "break" the symmetry by a choice

$$\Phi_0 \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad (5,8)$$

The fact that both $SU(2)$- and $U(1)_Y$-symmetries are broken by this choice is seen from the fact that

$$t^a \Phi_0 = \frac{1}{2}\sigma^a \Phi_0 \neq 0$$

$$\frac{Y}{2}\Phi_0 = \frac{1}{2}\Phi_0 \neq 0 \qquad (5,9)$$

This is just a compact way of saying that $U_{SU2}\Phi_0 \neq \Phi_0$ and $U_Y\Phi_0 \neq \Phi_0$, while of course $|U\Phi_0| = |\Phi_0|$ for any choice.

However, the ground state still <u>is</u> destroyed by the sum-operator $Q \equiv t_3 + \frac{Y}{2}$:

$$\hat{Q}\,\Phi_0 = \left(t_3 + \frac{Y}{2}\right)\Phi_0 = \frac{1}{2}\left(\begin{pmatrix}1&0\\0&-1\end{pmatrix} + \begin{pmatrix}1&0\\0&1\end{pmatrix}\right)\Phi_0 = \begin{pmatrix}1&0\\0&0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix} = 0! \qquad (5,10)$$

That is, any transformation of the form

$$U_{SU(2)\times U(1)} = e^{i\alpha\hat{Q}} \qquad (5,10)$$

leaves not only $\mathcal{L}$, but also the SSB-ground state (5,8) invariant. Thus (5,10) is a continuous symmetry of the low-energy theory and Q is a conserved charge. We identify (5,10) as the $U(1)_{em}$ corresponding to electromagnetic theory and $\hat{Q}$ as the electric charge! The equation

$$\boxed{Q = t_3 + \frac{Y}{2}} \qquad (5,11)$$

is called Gell-Mann – Nishijima relation. When applied for the Φ-doublet (5,2) we find that ϕ_1 has a charge $+1$ and ϕ_2 a charge 0:

$$\Phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \qquad (5.12)$$

(In fact one can say that physically the vacuum must be chargeless, and fixing that and the choice (5.8) <u>defines</u> $Y_\phi = 1$)

Higgs mechanism; unitary gauge

Choose the parametrization

$$\Phi \equiv \exp\left(i\, t^a \xi^a / v\right)\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+\eta\end{pmatrix} \qquad (5,13)$$

And make an SU(2)-gauge transformation:

$$\Phi \longrightarrow e^{i\theta^a t^a}\,\Phi \qquad (5.14)$$

with $\theta^a = \frac{1}{v}\xi^a$, such that

$$\Phi \longrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\eta \end{pmatrix}. \qquad (5.15)$$

The field η is again the only scalar d.o.f. left in the Lagrange function in the Unitary gauge. It is called Higgs field and denoted by H_0. To see the spectrum we write

$$|D_\mu \Phi|^2 = \frac{1}{2}\left|\begin{pmatrix} \partial_\mu - \frac{i}{2}(gW_\mu^3 + Yg'B_\mu) & -\frac{i}{2}g(W_\mu^1 - iW_\mu^2) \\ -\frac{i}{2}g(W_\mu^1 + iW_\mu^2) & \partial_\mu + \frac{i}{2}(gW_\mu^3 - Yg'B_\mu) \end{pmatrix}\begin{pmatrix} 0 \\ v+\eta \end{pmatrix}\right|^2$$

$$= \frac{1}{2}\left|\begin{pmatrix} -\frac{i}{2}g(W_\mu^1 - iW_\mu^2)(v+\eta) \\ \partial_\mu\eta + \frac{i}{2}(gW_\mu^3 - g'YB_\mu)(v+\eta) \end{pmatrix}\right|^2$$

$Y=1$!

$$= \frac{1}{2}(\partial_\mu\eta)^2 + \frac{1}{4}(v+\eta)^2\left(g^2(W_1^2 + W_2^2) + (gW_3 - g'YB)^2\right) \qquad (5.16)$$

This clearly contains a mass term for W- and B-bosons of the form:

$$A^T M A = \frac{v^2}{4}\cdot(W_1, W_2, W_3, B)\begin{pmatrix} g^2 & 0 & & 0 \\ 0 & g^2 & & \\ & & g^2 & -gg' \\ 0 & & -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix} \qquad (5.17)$$

It is now clear that SSB mixes W_3 an B !

We can identify the mass-eigenvalues

$$M_W^2 = \frac{g^2 v^2}{4} \qquad (5.18)$$

corresponding to the charged gauge bosons

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_{1\mu} \mp i\, W_{2\mu}\right) \qquad (5.19)$$

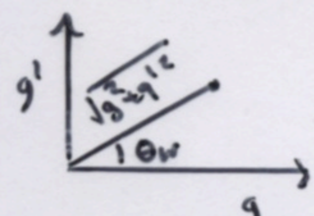
The subsystem (W_3, B) gives rise to:

$$\det\begin{pmatrix} g^2-\lambda & -gg' \\ -gg' & g'^2-\lambda \end{pmatrix} = (g^2-\lambda)(g'^2-\lambda) - g^2 g'^2 = \lambda(\lambda - (g^2+g'^2)) = 0$$

$$\Rightarrow \quad \lambda = 0 \quad \vee \quad \lambda = g^2+g'^2 \qquad (5.20)$$

One massless field corresponding to the symmetry charge Q

Neutral weak gauge boson



Diagonalizing (W_3, B) with the rotation

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv U_{\theta_W}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \qquad (5.21)$$

From equation

$$U_{\theta_W}^+ \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} U_{\theta_W} \equiv \begin{pmatrix} g^2+g'^2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad \tan\theta_W = \frac{g'}{g} \quad ; \quad \left(\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \; ; \; \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}\right)$$

(5.22)

Thus the photon

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g' W^3_\mu + g B_\mu\right) \qquad (5.23)$$

remains massless, while the Z-boson

$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g W^3_\mu - g' B_\mu\right) \qquad (5.24)$$

gets the mass

$$M_Z^2 = \frac{g^2+g'^2}{4} v^2. \qquad (5.25)$$

Note that at tree level the ρ-parameter:

$$\rho \equiv \frac{M_W^2}{\cos^2\theta_W M_Z^2} = 1 \qquad (5.26)$$

This will deviate from 1 at loop level $\Rightarrow$ precision EW-constraints.

Now, going back to (5.16) and (5.4) we can rewrite the Lagrangian as

$$\begin{aligned}\mathcal{L}_{SM} = & \frac{1}{2}\left[(\partial_\mu \eta)^2 - 2\mu^2\eta^2\right] + \frac{1}{2}\lambda v \eta^3 - \frac{1}{4}\lambda(\eta+v)^4 \\ & + M_W^2 W^+_\mu W^{\mu-} + M_Z^2 Z_\mu Z^\mu \\ & + \frac{g^2}{4}\left(\eta^2 + 2v\eta\right) W^+_\mu W^{-\mu} + \frac{g^2+g'^2}{4}\left(\eta^2+2v\eta\right) Z_\mu Z^\mu \\ & \underbrace{- \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}}_{\mathcal{L}_{gauge}} + \text{fermions}\end{aligned} \qquad (5.27)$$

After a reasonable amount of tedium, the gauge-Lagrangian can be rewritten as

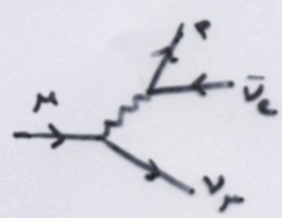
$$
\begin{aligned}
\mathcal{L}_{gauge} = & -\frac{1}{4}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu - ig\cos\theta_W \left(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu\right)\right)^2 \\
& -\frac{1}{4}\left(\partial_\nu A_\mu - \partial_\mu A_\nu - ie\left(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu\right)\right)^2 \\
& -\frac{1}{2}\Big| \partial_\nu W^+_\mu - \partial_\mu W^+_\nu + ig\cos\theta_W\left(W^+_\mu Z_\nu - W^+_\nu Z_\mu\right) \\
& \qquad + ie\left(W^+_\mu A_\nu - W^+_\nu A_\mu\right) \Big|^2
\end{aligned}
\qquad (5.28)
$$

The matter fields

Quarks and leptons have been observed to have different Lorenz-structure under Weak interactions than under the electro-magnetic ones. While the EM-field always coupled to the vector current; eg:

$$\bar{\psi}_e \gamma^\mu \psi_e \equiv \bar{e}\gamma^\mu e = j^\mu_e \qquad (5.29)$$

The weak SU(2)-gauge fields have been observed to couple only to the left chiral currents. Eg. the effective theory for muon decay was found to be:



$$\sim \bar{\nu}_\mu \gamma_\lambda (1-\gamma_5)\mu \; \underbrace{\bar{\nu}_e \gamma^\lambda (1-\gamma_5) e}_{\text{V-A-current.}}$$

$$\sim j^\mu_{L\lambda} j^{e,\lambda}_L \qquad (5.30)$$

Remember the notions of chirality and helicity. The chiral projections of any given spinor ψ are

$$\psi_L \equiv \frac{1}{2}(1-\gamma_5)\psi \equiv P_L\psi = \begin{pmatrix}\psi_L\\0\end{pmatrix}$$
$$\psi_R \equiv \frac{1}{2}(1+\gamma_5)\psi \equiv P_R\psi = \begin{pmatrix}0\\\psi_R\end{pmatrix} \qquad (5.31)$$

where the last equalities refer to the Weyl basis. From section 2 on the fall course we remember the helicity eigenstates on the Weyl basis:

$$u(p,h) = \begin{pmatrix}\sqrt{E-h|p|}\\\sqrt{E+h|p|}\end{pmatrix}\otimes\xi_{h,\hat{p}} \xrightarrow{E\gg m} \begin{cases}\begin{pmatrix}1\\0\end{pmatrix}\xi_{-1} ; & h=-1\\ \begin{pmatrix}0\\1\end{pmatrix}\xi_{1} ; & h=+1\end{cases}$$

$$v(p,h) = \begin{pmatrix}\sqrt{E+h|p|}\\-\sqrt{E-h|p|}\end{pmatrix}\otimes\xi_{-h,\hat{p}} \longrightarrow \begin{cases}-\begin{pmatrix}0\\1\end{pmatrix}\xi_{-1} ; & h=-1\\ -\begin{pmatrix}1\\0\end{pmatrix}\xi_{+1} & h=+1\end{cases} \qquad (5.32)$$

Weak interactions only feel ψ_L, and they thus couple dominantly to left helicity particles and right helicity antiparticles.

There is no problem constructing such a chiral theory at the classical level. We simply require that under SU(2)

$$\psi \xrightarrow{SU(2)} \psi + i\theta^a t^a P_L\psi = \psi + \delta\psi_L \qquad (5.33)$$

and $\delta\psi_R = 0$, while under the $U(1)_Y$ both fields transform:

$$\psi_L \xrightarrow{U(1)_Y} \psi_L + i\alpha\frac{Y_L}{2}\psi_L$$
$$\psi_R \rightarrow \psi_R + i\alpha\frac{Y_R}{2}\psi_R \qquad (5.34)$$

All in all, this means just that the different chiral components are put into different representations of the $SU(2)_L \otimes U(1)_Y$ as follows:

SU(2)-dublet SU(2)-singlet

$$\boxed{\begin{array}{lll} \begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L = (\underset{\sim}{2}, -1) & \ell^-_R = (\underset{\sim}{1}, -2) & \nu_R = (\underset{\sim}{1}, 0) \\ \\ \begin{pmatrix} u \\ d \end{pmatrix}_L = (\underset{\sim}{2}, \tfrac{1}{3}) & u_R = (\underset{\sim}{1}, \tfrac{4}{3}) & d_R = (\underset{\sim}{1}, -\tfrac{2}{3}) \end{array}} \quad (5.35)$$

hyperchange quantum number

where $\ell = e, \mu$ or τ and "u" = u, c or t and "d" = d, s or b.

The hypercharge assignments were computed from the Gell-Mann-Nishijima relation

$$Y = 2(Q - t_3)$$

and the known charges: $Q_\ell = -1$, $Q_\nu = 0$, $Q_u = \frac{2}{3}$ and $Q_d = -\frac{1}{3}$, together with the isospin-values as is obvious from (5.35).

The kinetic terms then become

$$i\bar{\Psi}_{\substack{iL\\(R)}} \not{\partial} \Psi_{\substack{iL\\(R)}} \longrightarrow i\bar{\Psi}_{\substack{iL\\(R)}} \not{D}_{iL} \Psi_{\substack{iL\\(R)}} \quad (5.36)$$

where

$$\boxed{\begin{aligned} D^i_{\mu_L} &= \partial_\mu - igT^a W^a_\mu - \frac{ig'}{2} Y_{Li} B_\mu \\ D^i_{\mu_R} &= \partial_\mu - \frac{ig'}{2} Y_{Ri} B_\mu \end{aligned}} \quad (5.37)$$

When (5.37) is plugged into (5.36), one finds the following fermion-gauge-field interaction terms:

$$= -i \sum_{f_L} \bar{f}_L \gamma^\mu \left(g T^3 W^3_\mu + \frac{1}{2} g' Y_{f_L} B_\mu \right) f_L - i \sum_{f_R} \bar{f}_R \frac{\gamma^\mu}{2} g' Y_{f_R} B_\mu f_R$$

$$- i \sum_{f_L} (\bar{f}_{u_L}, \bar{f}_{d_L}) \gamma^\mu \begin{pmatrix} 0 & \frac{g}{2}(W^1_\mu - i W^2_\mu) \\ \frac{g}{2}(W^1_\mu + i W^2_\mu) & 0 \end{pmatrix} \begin{pmatrix} f_{u_L} \\ f_{d_L} \end{pmatrix}$$

$$\equiv \mathcal{L}_{nc} + \mathcal{L}_{cc} \qquad (5.38)$$

Using $W^1_\mu \pm i W^2_\mu = \sqrt{2}\, W^\mp_\mu$ the charged current Lagrangian becomes

$$\mathcal{L}_{cc} = \frac{-ig}{\sqrt{2}} \underbrace{J^+_\mu}_{\text{weak charged current}} W^{\mu -} + h.c., \qquad (5.39)$$

where

$$J^-_\mu \equiv \sum_f \bar{f}_{d_L} \gamma^\mu f_{u_L} = \frac{1}{2} \bar{\nu}_e \gamma^\mu (1-\gamma_5) e + \ldots \qquad (5.40)$$

Neutral current part requires a little more work. Remembering from (5.21) that $B_\mu = -\sin\theta_W Z_\mu + \cos\theta_W A_\mu$ and $W^3_\mu = \cos\theta_W Z_\mu + \sin\theta_W A_\mu$ one can rewrite

$$\mathcal{L}_{nc} = -ig J^\lambda_3 W_{3\lambda} - ig' J^\lambda_Y B_\lambda \qquad (5.41)$$

with $J^\lambda_3 = \sum_f \bar{f} \gamma^\lambda T_3 f$ & $J^\lambda_Y = \sum_f \bar{f} \gamma^\lambda \frac{Y_f}{2} f$ (5.42)

note; $T_3 = 0$ for f_R.

as follows:

$$i\mathcal{L}_{nc} = \left(g\cos\theta_W J_3^\mu - g'\sin\theta_W J_Y^\mu\right) Z_\mu + \left(g\sin\theta_W J_3^\mu + g'\cos\theta_W J_Y^\mu\right) A_\mu. \qquad (5,43)$$

Now

$$\boxed{g\sin\theta_W = g'\cos\theta_W \equiv e} \qquad (5,44)$$

and

$$J_3^\mu + J_Y^\mu = \sum_f \bar{f}\gamma^\mu\left(T_3 + \frac{Y}{2}\right) f = \sum_f Q_f \bar{f}\gamma^\mu f \equiv J_{em}. \qquad (5,45)$$

And furthermore

$$g\cos\theta_W J_3^\mu - g'\sin\theta_W J_Y^\mu = \frac{g}{\cos\theta_W}\left(\overbrace{\cos^2\theta_W}^{1-\sin^2\theta_W} J_3^\mu - \sin^2\theta_W J_Y^\mu\right)$$

$$= \frac{g}{\cos\theta_W}\sum_f \bar{f}\gamma^\mu\left(T_3 - \sin^2\theta_W\left(T_3 + \frac{Y}{2}\right)\right) f$$

$$= \frac{g}{\cos\theta_W}\sum_f \bar{f}\gamma^\mu\left(T_3 - \sin^2\theta_W Q\right) f \equiv \frac{g}{\cos\theta_W} J_Z^\mu \qquad (5,46)$$

So we have

$$\mathcal{L}_{nc} = -i e J_{em}^\mu A_\mu - i\frac{g}{\cos\theta_W} J_Z^\mu Z_\mu \qquad (5,47)$$

the usual em-current!

weak neutral current!

One customarily rewrites

$$\boxed{J_Z^\mu \equiv \sum_f \bar{f}\gamma^\mu(v_f - a_f\gamma^5)f} \qquad (5,48)$$

where v_f and a_f can be read from (5,46):

	v_f	a_f
ν	$\frac{1}{4}$	$\frac{1}{4}$
ℓ^-	$-\frac{1}{4}+x_W$	$-\frac{1}{4}$
u	$\frac{1}{4}-\frac{2}{3}x_W$	$\frac{1}{4}$
d	$-\frac{1}{4}+\frac{1}{3}x_W$	$-\frac{1}{4}$

(5,49)

; $x_W \equiv \sin^2\theta_W$

Neutral current structure is thus, unlike the charged current, pure V−A, as a result of gauge-boson mixing.

The Weinberg angle have been measured from many processes that depend on v_f and a_f in different ways. These measurements are now so accurate that radiative corrections need to be accounted for. Through these the results become sensitive to the higgs mass and to α_s, and also on the renormalization scheme. Currently

$$\sin^2\theta_W(M_Z)_{\overline{MS}} \approx 0,231 \qquad (5,50)$$

At tree level one can always use $\sin^2\theta_W \approx 0,23$.

From muon life-time one can measure

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}} \approx \frac{1}{\sqrt{2}}(1.166 \times 10^{-5}\ \mathrm{GeV}^{-2})$$

Now, since

$$\underline{M_W = \frac{gv}{2}} \quad \Rightarrow \quad v = 2\frac{M_W}{g} = 2\sqrt{\frac{\sqrt{2}}{8G_F}} = (\sqrt{2}G_F)^{-1/2}$$

we get

$$\underline{v \simeq 246\ \mathrm{GeV}}$$

Furthermore $\quad e = g\sin\theta_W \approx 0.303 \Rightarrow \underline{g = \frac{e}{\sin\theta_W} \approx 0.63}$

Not small!

$SU(2)\times U(1)$-model has (apart from fermion masses) only four independent parameters.

$$g,\ g',\ \mu\ \&\ \lambda,$$

or alternatively G_F, $\sin^2\theta_W$, v and M_H. All others quantities like M_W and M_Z follow from these. Currently

$$M_W \simeq 80{,}42(4)\ \mathrm{GeV}$$

$$M_Z \simeq 91{,}188(2)\ \mathrm{GeV}$$

Radiative corrections / Precision tests.

In the GWS-model we have a very large number of observables that can be / must be explained by only a few (essentially 3-5) underlying parameters.

For example, we can consider observables $\hat{m}_W, \hat{m}_Z$, eg the pole masses of the gauge bosons, $\hat{\alpha}$ (from Thomson limit of $\gamma^* \to e^+e^-$ - scattering), G_F (from muon decay), $\hat{\Gamma}_{\ell^+\ell^-}$ (the leptonic partial width of the Z-boson, and s_*^2, ie the effective Weinberg angle defined from requiring that to all orders

$$\hat{A}_{LR}^{e} = \frac{(\frac{1}{2}-s_*^2)^2 - s_*^4}{(\frac{1}{2}-s_*^2)^2 + s_*^4}$$

The measured values of these observables are

$$\hat{\alpha}^{-1} = 137{,}0359895(61)$$

$$G_F = 1.16639(1) \times 10^{-5} \text{ GeV}^{-2}$$

$$\hat{m}_Z = 91{,}187 \pm 0{,}0021 \text{ GeV}$$

$$\hat{m}_W = 80{,}385 \pm 0{,}015 \text{ GeV}$$

$$\hat{s}_*^2 = 0{,}23150 \pm 0{,}00016$$

$$\hat{\Gamma}_{\ell\ell^-} = 83{,}984 \pm 0{,}086 \text{ MeV}$$

At the tree level we can compute all these in terms of g, g' & v:

some parameters

- $\hat{\alpha} = \frac{e^2}{4\pi}$; $e^2 = \frac{g^2 g'^2}{g^2+g'^2} = g^2 \sin^2\theta_w = g'^2 \cos^2\theta_w$; $e = \frac{gg'}{\sqrt{g^2+g'^2}}$

- $\hat{G}_F = \frac{g^2}{4\sqrt{2} M_W^2} = \frac{1}{\sqrt{2} v}$

- $m_Z^2 = \frac{g^2+g'^2}{4} v$; $m_W^2 = \frac{g^2 v^2}{4}$

- $s_*^2 = s^2 = \frac{g'^2}{g^2+g'^2}$

- $\hat{\Gamma}_{ee} = \frac{(g^2+g'^2)^{3/2} v}{96\pi} \left(\left(-\frac{1}{2}+2s^2\right)^2 + \frac{1}{4} \right)$

$$G_F m_Z^2 = \frac{g^2}{\sqrt{2}} \frac{m_Z^2}{m_W^2} = \frac{1}{\sqrt{2}} (g^2+g'^2)$$

$$e^2 = gg' \sin\theta_W \cos\theta_W$$
$$= \frac{1}{2} \frac{gg'}{\sqrt{\;}} \cdot \sqrt{\;} \cdot \sin 2\theta_W$$
$$= \frac{1}{2} \cancel{e} \left(\sqrt{2} G_F m_Z^2 \right)^{1/2} \sin 2\theta_W$$
$$= (4\pi\hat{\alpha})^{1/2}$$
$$\Rightarrow \sin 2\theta_W = \left(\frac{4\pi\hat{\alpha}}{\sqrt{2} G_F m_Z^2} \right)^{1/2}$$

It is now simple to show that <u>at tree level</u> (taking $\hat{\alpha}, \hat{G}_F \& \hat{M}_Z$ as input)

- $v = \left(\frac{1}{\sqrt{2}\hat{G}_F} \right)^{1/2} \simeq 246.22$
- $e = (4\pi\hat{\alpha})^{1/2} \simeq 0.302822$
- $\sin 2\theta_0 = \left(\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F \hat{M}_Z^2} \right)^{1/2} \simeq 0.81766(2)$

not directly observables

This leads to <u>predictions</u>:

$\sin^2\theta_0 \simeq 0.21215 \pm 0.00015 \Rightarrow g \simeq 0.65747 \pm 0.00002$

$s_*^2 = 0.23150 \pm 0.00016$; $g' \simeq 0.341166 \pm 0.00003$

discrepancy: 128σ.

$\Rightarrow M_W \simeq \frac{gv}{2} \simeq 80.9383 \pm 0.0025$

80.385 ± 0.015 : $\sim 37\sigma$

and

$\hat{\Gamma}_{ee} \simeq (84.84 \pm 0.01)$ MeV

83.984 ± 0.086 : $\sim 10\sigma$

So, clearly at tree level SM is not in a good agreement with the existing precise data !!

So, one must compute radiative corrections at least to 1-loop order to check the consistency better.

Radiative corrections / precision tests

* In GWS-model neutral currents have large number of manifestations. All these have different dependences on fundamental parameters, and in particular $\sin\theta_W$. $\Rightarrow$ Many independent measurements for $\sin\theta_W$.
* Some of these also probe different energy ranges, whereby we obtain tests for running of SM-parameters.
* If we can discard fermion masses (in these observables) GWS model essentially depends on only 4 parameters, which can be chosen in many different ways: Eg. (λ is essentially m_H)

$$g,\ g',\ v \ \Big|\ \lambda$$

Or

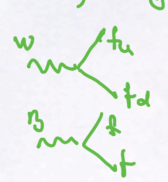
$$\alpha = \frac{e^2}{4\pi^2}\ ; \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}\ ; \quad \sin\theta_W \ \Big|\ \lambda$$

For example, one could measure e at $q^2=0$ (Thomson scattering) and μ-decay, which gives G_F, essentially at $q^2=0$, and one might set

$$\boxed{\sin^2\theta_W \equiv 1 - \frac{M_W^2}{M_Z^2}}$$

Or, one could just derive $g_{\overline{MS}}$, $g'_{\overline{MS}}$ and define

One-loop, say



$$\boxed{\sin^2\theta_{\overline{MS}} \equiv \frac{g_{\overline{MS}}}{\sqrt{g^2_{\overline{MS}} + g'^2_{\overline{MS}}}}}$$

All different choices agree at tree level, but differ at radiative level. This only refers to parametrization of the model, not to physical results.
ambiguity

$\nearrow e, g$; $e = \frac{g g'}{\sqrt{g^2+g'^2}} \Rightarrow g' \Rightarrow \sin\theta_W$

For example, if one takes α & G_F as fixed parameters, each different observable independently sets s_W and $s\theta_W^{\overline{MS}}$, and they should agree!

Hmm. e, g + tree level relation $\Rightarrow g'$ $\sin\theta_W$
or + observable $\Rightarrow g'$ $\sin\theta_W$

From Peskin & Schroeder, table 20.1

	s_W^2	$s\theta_W^{\overline{MS}}$
m_Z	0,2247	0,2320
m_W	0,2264	0,2338
Γ_Z	0,2250	0,2322
A_{LR}^e	0,2221	0,2292
		(larger)

$(G_F$ & α fixed.)

Consistent.

In particular the differences between different definitions of $\sin\theta_W$ should be finite and calculable from theory. They (their expressions) of course differ only at loop-level, and how much they do, depends on the particle content (particles in the loop). Thus precise determination of these differences & their comparison to SM-prediction can lead to interesting constraints on new physics.

Let us see how this works. We have already two different definitions for $\sin\theta_W$. Let us now define a third:

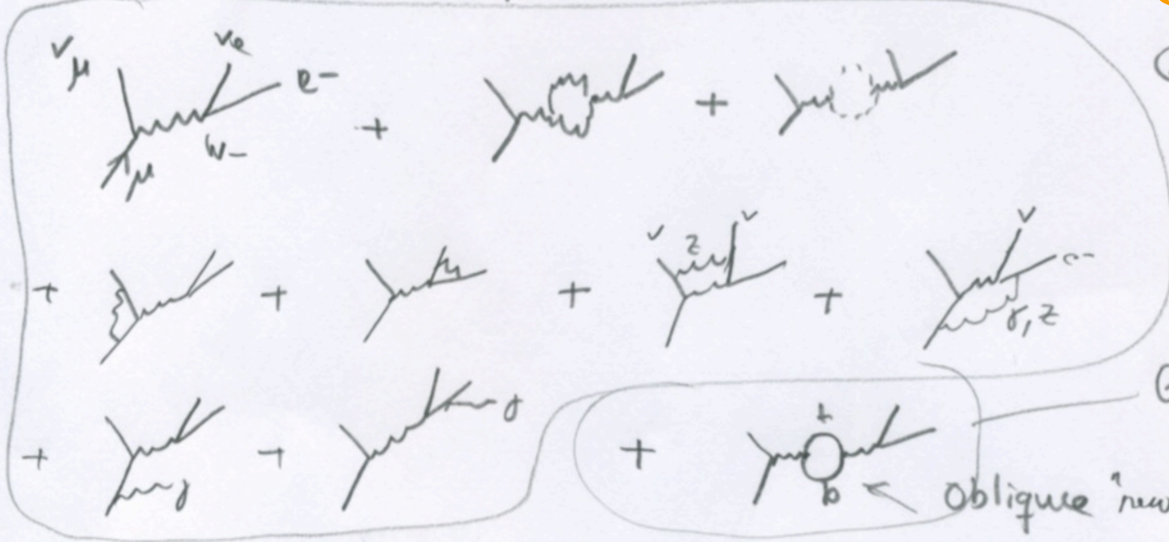
$$\boxed{\sin 2\theta_0 \equiv \left(\frac{4\pi\alpha_*}{\sqrt{2}\,G_F\, m_Z^2}\right)^{1/2}}$$

Where $\alpha_* \equiv \alpha(M_Z)$, is the running QED-coupling at $Q^2 = M_Z^2$ (from RGE). This is very precisely measured (Peskin p. 759)

$$\sin^2\theta_0 = 0{,}2307 \pm 0{,}0005 \qquad \alpha^* \simeq \frac{1}{127}$$

The complete Renormalization program is very complicated. For example corrections to μ-decay involve

(gauge invariant



GI all combined

GI alone

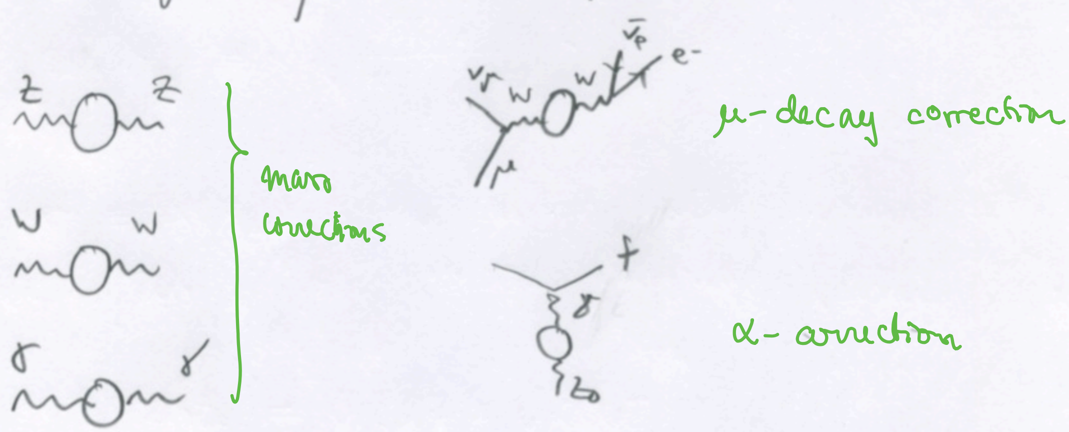
oblique 'new' correction

The oblique corrections to gauge boson propagators do not couple to light fields at external legs ⇒ they must alone give gauge-invariant contribution, which is easy to compute.

This applies to (t,b)-contributions within SM as well as all new physics corrections due to new particles from beyond SM.

└ which does not couple to light fields directly.

In particular the corrections to Z & W-masses, α, μ-decay and LR-asymmetry* involve diagrams



- - -

LR-asymmetry is defined as:

$$A_{LR}^{f} = \frac{\Gamma(Z^0\to f_L\bar f_R) - \Gamma(Z^0\to f_R\bar f_L)}{\Gamma(Z^0\to f_L\bar f_R) + \Gamma(Z^0\to f_R\bar f_L)} \overset{\text{tree level}}{=} \frac{\left(\frac{1}{2}-|Q_f|\sin^2\theta_W\right)^2 - \left(Q_f\sin^2\theta_W\right)^2}{\left(\frac{1}{2}-|Q_f|\sin^2\theta_W\right)^2 + \left(Q_f\sin^2\theta_W\right)^2}$$

At tree level A comes from graph

measured at Z-resonance.

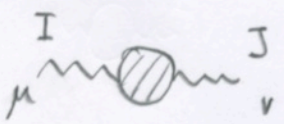
All these boil down to computing loop-corrections to gauge boson polarization tensors (at different energy scales). Moreover, since in all observables these tensors are coupled to light fermions, we can ignore the $k^\mu k^\nu$-parts of these tensors:

$$\Pi^{\mu\nu} j_\nu \sim \left(a g^{\mu\nu} + \frac{b k^\mu k^\nu}{k^2} \right) j_\nu \sim a j^\mu + b \frac{k^\mu m}{k^2}$$

$$\sim a j^\mu \quad \text{since} \quad a \sim b \quad \text{and} \quad m \ll \sqrt{k^2} \sim M_{Z,W}$$

Asse $\qquad$ μ-decay

So we can concentrate on a generic self-energy



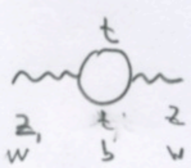
$$= i \Pi^{\mu\nu}_{IJ}(q)$$

$$= i g^{\mu\nu} \Pi_{IJ}(q) + \ldots$$

where $I, J = W, Z$, or γ.

1 Mass corrections due to top and bottom (or any other new heavy degrees of freedom)

$$i\Pi^{\mu\nu}_{\substack{ZZ\\WW}} =$$



$$= i g^{\mu\nu} \Pi_{\substack{ZZ\\WW}}(q^2)$$

$$\Rightarrow (D^{-1}_{\substack{Z\\W}})^{\mu\nu} = -i \left[g_{\mu\nu} \left(q^2 - M^2_{\substack{Z,\\W_0}} - \Pi_{\substack{ZZ\\WW}}(q^2) \right) + \ldots \right]$$

with $m^2_{Z_0} = \frac{g^2 + g'^2}{4} v^2$, $\quad m^2_{W_0} = \frac{g^2 v^2}{4}$ $\qquad$ ("0" refers to tree level) whereby

- $m_Z^2 = \frac{g^2+g'^2}{4} v^2 + \Pi_{ZZ}(M_Z^2)$
- $m_W^2 = \frac{g^2 v^2}{4} + \Pi_{WW}(M_W^2)$

These give corrections between the tree-level & pole masses. If we fix α, G_F and $\sin\theta_W$ from some other constraints Π_W are in general nonzero. (They are finite however, since Π's contain also counter terms)

* The zero-mass of photon requires that

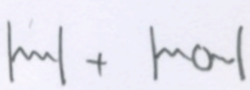
$$\Pi_{\gamma\gamma}(0) = \Pi_{\gamma Z}(0) \equiv 0$$

However, there is a w.f.r.-correction given by

$$\Pi'_{\gamma\gamma}(0) = \left.\frac{d\Pi_{\gamma\gamma}}{dp^2}\right|_{p^2=0}$$

(Note that $\Pi'_{\gamma\gamma}(0)$ is the quantity called $\hat{\Pi}(q^2)$ in (11.9c) of chapter 2, where we discussed renormalization of QED. This is because we used $\Pi_{\gamma\gamma} \equiv q^2\hat{\Pi}$ there, and $\Pi'_{\gamma\gamma}|_{q^2\to 0} = \hat{\Pi}(q^2\to 0)$.)

2) One-loop correction to Coulomb potential in

 $\Rightarrow$ $\mathcal{M} \propto -\frac{ie_0^2}{q^2}\left(1 + i\Pi_{\gamma\gamma}(q^2)\frac{-i}{q^2}\right)$

$$\Rightarrow \simeq -\frac{e_0^2}{q^2}\left(1+\Pi'_{\gamma\gamma}(0)\right) \equiv -\frac{e^2}{q^2}$$

$$\Rightarrow \overset{e^2}{\overset{\shortparallel}{4\pi\alpha}} = \overbrace{\frac{g^2 g'^2}{g^2+g'^2}}^{e^2_{tree}}\left(1+\overbrace{\Pi'_{\gamma\gamma}(0)}^{\delta\alpha/\alpha}\right) \quad ; \quad \text{or } 4\pi\alpha(M_Z) = \frac{g^2 g'^2}{g^2+g'^2}\left(1+\frac{\Pi_{\gamma\gamma}(M_Z^2)}{M_Z^2}\right)$$

$\alpha(0)$

Using this in place of the tree-level $\sin\theta_W$ accounts for the total change to A_{LR} due to oblique corrections. It is thus an alternative way to define $\sin\theta_W$. We have defined two others:

$$\sin^2\theta_W \equiv 1 - \frac{M_W^2}{M_Z^2} = 1 - \frac{\frac{g^2v^2}{4} + \Pi_{WW}(M_W^2)}{\frac{g^2+g'^2}{4}v^2 + \Pi_{ZZ}(M_Z^2)}$$

$$= \frac{g'^2}{g^2+g'^2} - \frac{1}{M_Z^2}\left(\Pi_{WW}(M_W^2) - \frac{M_W^2}{M_Z^2}\Pi_{ZZ}(M_Z^2)\right) \qquad (S2)$$

Finally, we may return to our first definition, and write it as

$$\sin 2\theta_0 = \sin 2\theta_{tree} + 2\cos 2\theta_{tree}\,\delta\theta_0 = \sin 2\theta_{tree}\left(1 + 2\cot 2\theta_{tree}\,\delta\theta_0\right)$$

$$= \left(\frac{4\pi\alpha_{tree}\left(1+\frac{\delta\alpha}{\alpha}\right)}{\sqrt{2}\,G_F^{tree}\left(1+\frac{\delta G_F}{G_F}\right)M^2_{Z_{tree}}\left(1+\frac{\delta M_Z^2}{M^2_{Z_{tree}}}\right)}\right)^{1/2}$$

$$\sin 2\theta_0 = \left(\frac{4\pi\alpha(M_Z)}{\sqrt{2}\,G_F M_Z^2}\right)^{1/2}$$

$$= \sin 2\theta_{tree}\left(1 + \frac{1}{2}\frac{\delta\alpha}{\alpha} - \frac{1}{2}\frac{\delta G_F}{G_F} - \frac{\delta M_Z^2}{2M_Z^2}\right)$$

$$\Rightarrow \underbrace{4\cot 2\theta_{tree}}_{\frac{4\cos 2\theta}{\sin 2\theta}}\,\delta\theta_0 = \frac{\delta\alpha}{\alpha} - \frac{\delta G_F}{G_F} - \frac{\delta M_Z^2}{M_Z^2}$$

$$G_F \sim \left|\cancel{\#}\ \frac{1}{M_W^2 + \Pi_W(0)}\right|^2$$

$$\Rightarrow \frac{\delta G_F}{G_F} \sim -\frac{\Pi_W(0)}{M_W^2}$$

$$\Rightarrow \sin^2\theta_0 = \sin^2\theta_{tree} + \overbrace{2\sin\theta\cos\theta}^{\sin 2\theta}\,\delta\theta$$

$$= \frac{g'^2}{g^2+g'^2} + \frac{\sin^2\theta_{tr}\cos^2\theta_{tr}}{\cos^2\theta_{tr} - \sin^2\theta_{tr}}\left[\Pi'_{\gamma\gamma}(0) - \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2}\right] \qquad (S3)$$

$$\frac{\Pi_{\gamma\gamma}(M_Z^2)}{M_Z^2}$$

Yes, should be. Where's the problem (P&S ?)

3) Similarly one obtains the heavy-oblique corrections to μ-decay:
W-propagator there gets (one insertion here; equivalent to expanding resummed propagator to $\underset{\sim}{G_{F_{tree}}}$ first order,

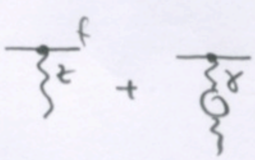
$$\mathcal{M} \propto \frac{-g_0^2}{q^2-M_W^2}\left(1+i\Pi_{WW}(q^2)\,\frac{-i}{q^2-M_W^2}\right) \overset{\propto}{\approx} \frac{g_0^2}{M_W^2}\left(1-\frac{\Pi_{WW}(0)}{M_W^2}\right) \propto G_F$$

$$\Rightarrow \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}\Big(1-\overbrace{\frac{\Pi_{WW}(0)}{M_W^2}}^{\delta G_F/G_F}\Big) \qquad \Rightarrow v = \left[\frac{1}{\sqrt{2}G_F}\left(1-\frac{\Pi_W}{M_W^2}\right)\right]^{1/2}$$

where we used: $\frac{G_{F_{tree}}}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2}{8\,\frac{g^2v^2}{4}} = \frac{1}{2v^2}$.

Note that this formula says that in general v depends on the order we compute. (G_F is a fixed number, just as M_W).

4) Finally there is the correction to asymmetry. Note that there is no diagram with Z. That diagram is resummed to Z-propagator and its effect would be the same on both $Z\to f_L\bar{f}_R$ and $Z\to\bar{f}_L f_R$ and hence it would not affect A_{LR}.



$$\propto \sqrt{g^2+g'^2}\Big(T_f^3-\overbrace{\frac{g'^2}{g^2+g'^2}}^{\sin^2\theta_W^{tree}}Q_f\Big)+i\Pi_{Z\gamma}\,\frac{-i}{q^2}(ieQ_f)$$

$$\equiv \sqrt{g^2+g'^2}\left(T_f^3-\sin^2\theta_* Q_f\right)$$

$\sin^2\theta_*$ is the proportionality factor between f-couplings to isospin and to charge

(S1)

where

$$\sin^2\theta_* \equiv \frac{g'^2}{g^2+g'^2}-\overbrace{\frac{e}{\sqrt{g^2+g'^2}}}^{=\sin\theta_W\cos\theta_W}\frac{\Pi_{\gamma Z}(m_Z^2)}{M_Z^2}$$

assuming that the asymmetry is measured at Z-pole $q^2=M_Z^2$.

While unrenormalized loop-contributions to S1 – S3 contain UV-divergences, the differences between various $\sin^2\theta_W$-definitions must be finite without renormalization. That is quantities

$$\sin^2\theta_* - \sin^2\theta_0 = \frac{\sin^2\theta_W\cos^2\theta_W}{\cos^2\theta_W - \sin^2\theta_W}\left(\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2} - \Pi'_{\gamma\gamma}(0) - \frac{\cos^2\theta_W - \sin^2\theta_W}{\sin\theta_W\cos\theta_W}\frac{\Pi_{\gamma Z}(m_Z^2)}{M_Z^2}\right) \equiv \delta\sin^2\theta_{0*}(m_t)$$

A_{LR}

$\dfrac{\Pi_{\gamma\gamma}(M_Z^2)}{M_Z^2}$

$$\sin^2\theta_W - \sin^2\theta_* = -\frac{\Pi_{WW}(M_W^2)}{m_Z^2} + \frac{M_W^2}{M_Z^2}\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \sin\theta_W\cos\theta_W\frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} = \delta\sin^2\theta_{W*}(m_t)$$

should be finite. They are also precisely predicted by SM. Any deviation from SM-predictions would be a sign of new physics. (In SM the largest unknown is still top mass

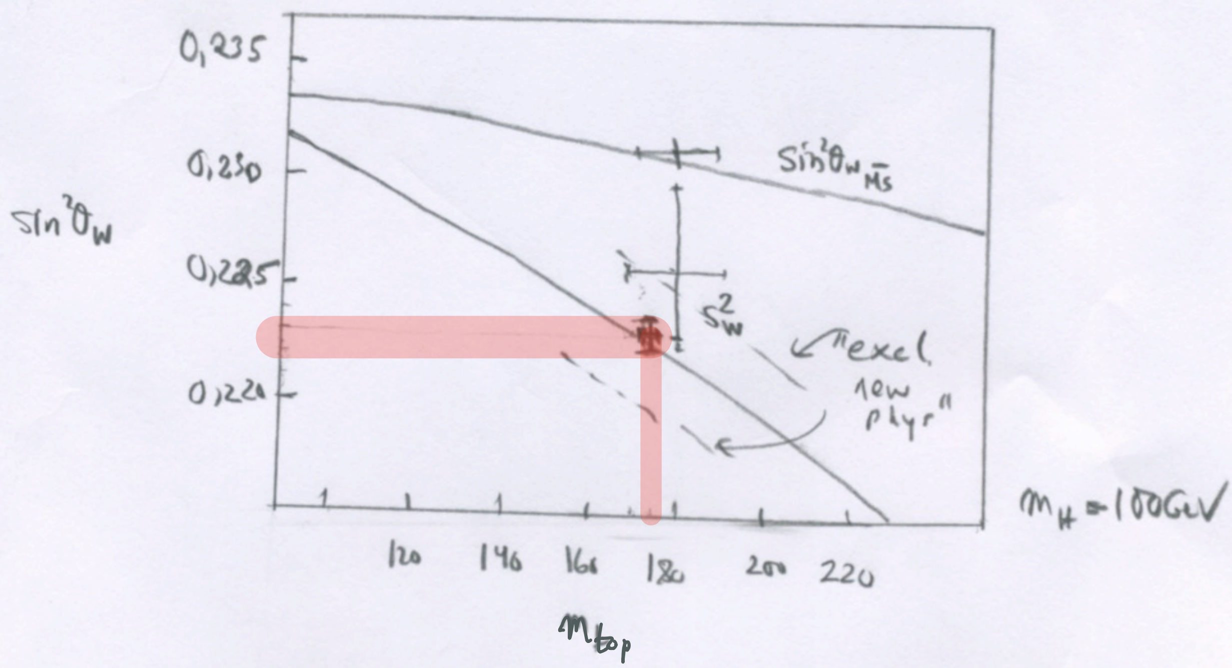
So, measuring eg. $\sin^2\theta_0$ one can plot $\sin\theta_*$ & $\sin\theta_W$ as a function of m_t from

$$\sin^2\theta_* = \sin^2\theta_0 + \delta\sin^2\theta_{0*}(m_t, m_H)$$

$$\sin^2\theta_W = \sin^2\theta_0 + \delta\sin^2\theta_{0*}(m_t, m_H) + \delta\sin^2\theta_{W*}(m_t)$$

$\sin^2\theta_W$ is shown along with $\sin^2\theta_W^{exp}$ in the figure next page

This figure is very old (from -94; page 772 P&S). Present results are much more accurate, but this is enough to show the situation qualitatively.



Larger cross is from P&S. year -94. Smaller comes from current numbers

$$m_Z = 91{,}187 \pm 0{,}0021 \quad \text{GeV}$$
$$m_W = 80{,}385 \pm 0{,}015 \quad \text{GeV}$$
$$m_t = 173{,}21 \pm 0{,}51 \pm 0{,}71 \quad \text{GeV} \quad \hat{=}\ 173{,}2 \pm 0{,}87$$
$$\simeq 171{,}4 - 174{,}9 \quad 2\sigma$$

$$\Rightarrow s_W^2 \approx 0{,}2225 - 0{,}2238 \quad 2\sigma$$

Obviously new physics would move the predicted curve. If it gets moved too far, it is excluded!

Such precision electroweak corrections are usually expressed in terms of Peskin-Takeuchi S,T,U - parameters, or other equivalents.