Quantum field theory applications, fall 2018, **Excercise 3.**

1. We continue to compute some basic 1-loop integral functions. First show that

$$A_{\mu\nu}(m) = \mu^{4-d} \int \frac{d^dk}{(2\pi)^d} \frac{k_\mu k_\nu}{k^2 - m^2 + i\epsilon} = A_2(m) g_{\mu\nu},$$

where

$$A_2(m) = \frac{m^2}{4} A_0(m) + \frac{im^4}{8(4\pi)^2}.$$

where A_0 is the one-point scalar function computed in the first excercise. Explain why any tensor with odd number of k^μ-factors in the nominator has to vanish.

Next consider vector and tensor B functions using (Passarino-Veltman) reduction to a linear combinations of scalar functions:

$$\begin{aligned} B_\mu(p^2; m_1^2, m_2^2) &\equiv \mu^{4-d} \int \frac{\mathrm{d}^dk}{(2\pi)^d} \frac{k_\mu}{(k^2 - m_1^2)((k-p)^2 - m_2^2)}, \\ B_{\mu\nu}(p^2; m_1^2, m_2^2) &\equiv \mu^{4-d} \int \frac{d^dk}{(2\pi)^d} \frac{k_\mu k_\nu}{(k^2 - m_1^2)((k-p)^2 - m_2^2)}. \end{aligned}$$

First show that $B_\mu(p^2; m_1^2, m_2^2) = p^\mu B_1(p^2; m_1^2, m_2^2)$, where

$$B_1(p^2; m_1^2, m_2^2) \equiv \frac{1}{2p^2} \Big[A_0(m_2^2) - A_0(m_1^2) + (p^2 + m_1^2 - m_2^2) B_0(p^2; m_1^2, m_2^2) \Big]$$

and B_0 is the two-point scalar integral computed in the first excercise. Note the symmetry property $B_1(p^2; m_2, m_1) + B_1(p^2, m_1, m_2) = B_0(p^2; m_1, m_2)$. Next consider the rank two integral. Explain why we must be able to make a reduction

$$B_{\mu\nu}(p^2; m_1^2, m_2^2) = B_{2g}(p^2; m_1^2, m_2^2)\, g_{\mu\nu} + B_{2p}(p^2; m_1^2, m_2^2)\, \frac{p_\mu p_\nu}{p^2},$$

where B_{2g} and B_{2p} are two new scalar functions. Show that this relation leads to a system of two linear equations for B_{2p} and B_{2g}:

$$\begin{aligned} dB_{2g}(p^2; m_1^2, m_2^2) + B_{2p}(p^2; m_1^2, m_2^2) &= A_0(m_2^2) + m_1^2 B_0(p^2; m_1^2, m_2^2) \\ B_{2g}(p^2; m_1^2, m_2^2) + B_{2p}(p^2; m_1^2, m_2^2) &= \frac{1}{2} \big[A_0(m_2^2) + (p^2 + m_1^2 - m_2^2) B_1(p^2; m_1^2, m_2^2) \big]. \end{aligned}$$

Finally, solve the above linear equation for B_{21}, B_{22} and express your results in terms of the basic A_0- and B_0-functions.

2. Use the results from the problem 1 to compute the electron self-energy diagram in the dimensional regularization scheme using the Feynman gauge. Regulate the photon propagator by a finite mass μ_p as was done in the lectures. Compute counter terms explicitly and show that δ_m is IR-finite, but δ_2 is IR- as well as UV-divergent.