

1. Let us consider the renormalizability of a scalar theory described by Lagrangian

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{m^2}{2} \phi^2 + \frac{\lambda_m}{m!} \phi^m$$

in  $d$ -dimensions. Prove first that if  $m = 2d/(d-2)$  the theory is renormalizable, that is, only finite number of  $n$ -point functions are divergent. Furthermore, show that the renormalizability depends on the dimension of the coupling constant:

- If  $\lambda_m$  is dimensionless, theory is renormalizable.
- If  $\lambda_m$  has negative dimension of mass,  $[\lambda_m] = M^\alpha$ ,  $\alpha < 0$ , theory is non-renormalizable: infinite number of  $n$ -point functions are divergent.
- If  $\lambda_m$  has positive dimension of mass,  $[\lambda_m] = M^\alpha$ ,  $\alpha > 0$ , theory is super-renormalizable: only finite number of Feynman diagrams are divergent.

Categorize the cases  $(d = 3, m = 6)$ ,  $(d = 6, m = 3)$ ,  $(d = 3, m = 4)$  and  $(d = 4, m = 3)$  to renormalizable, non-renormalizable, super-renormalizable theories. Draw then a diagram in  $(d, m)$ -plane which indicates the regions of different degrees of renormalizability.

2. Compute very carefully the following two integrals in the dimensional regularization method:

$$A_0(m^2) \equiv \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m^2 + i\epsilon}$$

$$B_0(m_1^2, m_2^2, p^2) \equiv \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)}$$

I want you to work out both the divergent parts of the integrals (the poles in  $1/\epsilon$  where  $\epsilon = 4 - d$  and the finite part. Throw out the  $\mathcal{O}(\epsilon)$  corrections. Note that the latter integral needs to be handled differently in the three different domains:  $p^2 < 0$ ,  $0 < p^2 < (m_1 + m_2)^2$  and  $p^2 > (m_1 + m_2)^2$ . In the last domain it obtains a complex part, which is well defined because of the  $\epsilon$ -prescription in the denominator.

(These two integrals suffice for 1-loop renormalization of  $\lambda\phi^4$ -theory and get us a long way into renormalizing QED as well.)