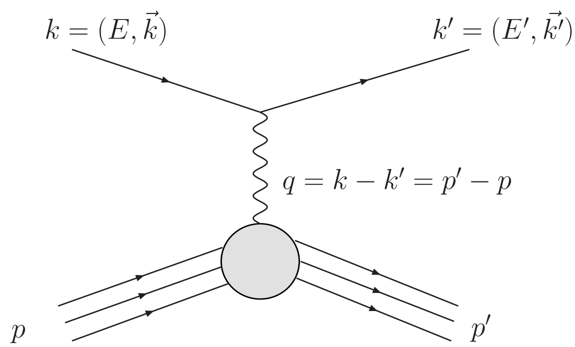
Quantum field theory, fall 2023, **Exercise 9.**

1. Let us now consider the elastic scattering between a proton and an electron: $e^-(k) + p(p) \to e^-(k') + p(p')$:



The blob in the figure above represents all the details of the photon-proton interaction and it is, a priori, unknown. Without further input, we can still express the lower part of the graph in a most general form for the matrix element of the hadronic current between the proton-states $|\mathbf{p}, s\rangle$ and $|\mathbf{p}', s'\rangle$

$$J^\mu = e\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)\right]u(p)$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ and M is the proton mass. F_1 ja F_2 are called *form factors* that parametrize the detailed structure of the proton — a bag that is dominated by the QCD effects.

Treating the electron and photon parts of the graph as usual, but using the general expression above for the hadronic part, derive the *Rosenbluth formula* (LAB-frame)

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{LAB}} = \frac{\alpha^2}{4E^2\sin^4(\theta/2)}\frac{E'}{E}\left[(F_1^2 - \frac{q^2}{4M^2}F_2^2)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}(F_1+F_2)^2\sin^2\frac{\theta}{2}\right].$$

with

$$\frac{E}{E'} = 1 + \frac{2E}{M}\sin^2(\theta/2).$$

Compare this result with the Rutherford scattering formula in the limit $q^2 \ll \mathrm{fm}^{-2}$. One Fermi is a good estimate for the proton radius. Hint: The Gordon decomposition

$$\overline{u}(p')\,\gamma^\mu\,u(p) = \frac{1}{2M}\overline{u}(p')\left[(p'+p)^\mu + i\sigma^{\mu\nu}\,(p'-p)_\nu\right]u(p)$$

might be useful.

2. Derive carefully the path integral representation for a 1-dimensional quantum mechanical transition amplitude starting from its representation in terms of the Heisenberg picture time-evolution operator, and discretizing the time-evolution of the system. (That is, do the inverse of what was done in the lectures.)

3. Derive the free particle transition amplitude $K(b, a)$

$$K(b,a) = \mid \frac{m}{2\pi(t_b - t_a)} \mid^{1/2} e^{\frac{im(x_b - x_a)^2}{2(t_b - t_a)}}$$

directly from its the path integral representation. You have to discretize the path-integral, use the result derived in the lectures for the k-factor in the measure, perform the Gaussian integrals and tak the limit $N \to \infty$.

4. Show that the amplitude found in the previous problem, is just the second quantization propagator, found as a solution of the equation

$$(i\partial_t + \frac{1}{2m}\frac{\partial^2}{\partial x^2})G_0(x, x') = \delta^2(x - x')$$

(Hint. Solve equation by moving to the momentum space and choose the retarded propagator as the boundary condition. (Why this instead of Feynman?)