Quantum field theory, fall 2023, **Excercise 7.**

Lighter load this time, since this is the exam week.

1. Compute the unpolarized cross section for the process $a\bar{b} \to a\bar{b}$ in the Yukawa theory. This is the relevant cross section if the colliding particles have random spins and the apparatus does not measure, or averages out over the final state spins. So, in practice you should form the absolute value of the T-matrix element and sum over the spins in the final state and average over the spins in the initial state. Show first that this renders the matrix element into a trace over a number of Dirac matrices and then perform these traces using the Dirac algebra and trace theorems learned in the past weeks excercises.

2. Assume that the field ϕ in the Yukawa theory considered in the lectures is heavier than the fermion pair coupling to it. (Let us say that those fields are electrons.) Compute the decay rate for the process $\phi \to e^+e^-$.

3. In the region of low energies, the weak interactions can be described by a effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\overline{\nu}_e\gamma_\mu(1-\gamma^5)e\overline{e}\gamma^\mu(1-\gamma^5)\nu_e,$$

where e and ν_e are Dirac spinors for electron and its neutrino. G_F is the Fermi constant $G_F \approx 1.166 \times 10^{-5}\,\text{GeV}^{-2}$.

Using $\mathcal{L}_{\text{eff}}$, write the amplitudes for elastic $e + \nu_e \to e + \nu_e$ and $e + \overline{\nu}_e \to e + \overline{\nu}_e$ scatterings. Derive the differential cross-sections $d\sigma/d\cos\theta$ for these processes in the CMS-frame, and sketch them as a function of θ. Compute the numerical values for the total cross-section (in units cm^2) and estimate the probability for a solar neutrino with energy $E = 10\,\text{MeV}$, to scatter as it traverses the Earth.