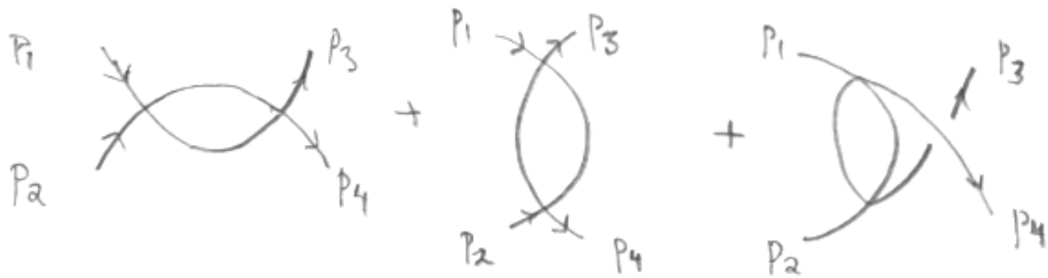
Quantum field theory, fall 2023, **Exercise 6.**

1. Show carefully that the following result given in the lectures holds:

$$\langle 0|T(e^{\int \mathrm{d}^4x\mathcal{L}_\mathcal{I}})|0\rangle = \exp(\sum_i V_i)\,,$$

where $\sum V_i$ is a sum over all different connected vacuum graphs. Show furthermore that the graphs "oo" ja "ooo" are proportional to the factor VT.

2. Show that the only connected 1PI-diagrams for the $\phi\phi \to \phi\phi$ scattering at the second order in perturbation theory are the following:



Starting from the LSZ-reduction formula, derive the T-matrix elements corresponding from these diagrams. (Do not attempt to perform the momentum integrals; that would require other techniques we have not learned yet.) Show that your result agrees with what you get by using directly the Feynman rules given on p.112 in the lectures.

3. Derive the T-matrix for particle-antiparticle scattering $a\bar{a} \to a\bar{a}$ in the Yukawa-theory (eq. (3.131) in the lectures), starting from the LSZ-integral expression for the scattering amplitude. How does the situation change if we consider a scattering off a different type of antiparticle $a\bar{b} \to a\bar{b}$?

4. Derive the expression

$$\Gamma_{i\to f} = \frac{\lambda^{1/2}(m_a^2, m_b^2, m_c^2)}{16\pi m_a^3} \mid T^2 \mid .$$

for the decay rate of an unstable particle a into a two-particle final state: $a \to bc$. Hint. Follow the derivation given in the lectures for the $2 \to 2$ scattering-amplitude. (Do not let yourself be bothered by the fact that strictly speaking an asymptotic in-state cannot be defined for the unstable field a.)