Quantum field theory, fall 2023, **Exercise 2.**

1. The contribution from the source term to the solution of the scalar field equation depends on the choice of the propagator function:

$$\delta\phi_X(x) \equiv \int d^4y \Delta_X(x-y)J(y),$$

where $\Delta_X = \Delta_R, \Delta_A, \Delta_F$ tai $\Delta_{\bar{F}}$. Show that the *differences* $\delta\phi_X(x) - \delta\phi_{X'}(x)$ fulfill the homogenous equation, and can thus be absorbed to the free part $\phi_0(x) \to \phi_{0X}(x)$. From this conclude that the use of different propagators simply corresponds to different boundary conditions at spatial and temporal infinity (we assume that j vanishes at infinity).

2. Verify that the integral representation of the Feynman propagator agrees with the expcetation value of the time-ordered 2-point function:

$$D_{\mathrm{F}}(x-y) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{i}{p^2 - m^2 + i\epsilon} = \langle|T(\phi(x)\phi(y))|\rangle.$$

3. Show that the generator matrices $S^{\mu\nu} = \frac{1}{2}\sigma^{\mu\nu} \equiv \frac{i}{4}[\gamma^\mu, \gamma^\nu]$ fulfill the Lorentz algebra

$$[S^{\mu\nu}, S^{\rho\sigma}] = i(g^{\nu\rho}S^{\mu\sigma} - g^{\mu\rho}S^{\nu\sigma} - g^{\nu\sigma}S^{\mu\rho} + g^{\mu\sigma}S^{\nu\rho}).$$

when γ^μ-matrices obey the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

4. Show that γ^μ transforms as a vector under Lorentz-transform:

$$\Lambda_{\frac{1}{2}}^{-1}\gamma^\mu\Lambda_{\frac{1}{2}} = \Lambda^\mu{}_\nu\gamma^\nu.$$

Hint: Follow Peskin-Schröder on page 42.

5. We can write a unitary representation for an arbitrary 4-vector V as follows

$$V = V^0 + \vec{\sigma}\cdot\vec{V},$$

where σ_i are the familiar Pauli matrices. Let us now require that V-matrix transforms according to the $(\frac{1}{2}, \frac{1}{2})$-representation of the Lorentz-group,

$$V \to \Lambda_{\frac{1}{2}L}^{-1}(\theta,\beta)\, V\, \Lambda_{\frac{1}{2}R}(\theta,\beta)$$

where $\Lambda_{\frac{1}{2}L,R}$ are 2×2-matrices that transform the Weyl spinors under $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations (see the lectures on page 42). Show that this transformation law implies that the components of V transform as a Lorentz-vector. (This demonstrates that $(\frac{1}{2}, \frac{1}{2})$ in fact is a *vector* representation of the Lorenz-group.)