Quantum field theory, fall 2023, **Excercise 12.**

The last set of excercises this fall is here. Thank you for your good effort during the semester! The following first three excercises are just going through the examples given in the lecture notes, and the last one is not that bad either. So, enjoy!

1. Show that in D-dimensional Euclidian space

$$\int \mathrm{d}^D q = \frac{2\pi^{D/2}}{\Gamma(\frac{D}{2})} \int \mathrm{d}q q^{D-1} \,.$$

2. Compute the divergent part of the $\lambda\phi^4$-theory 4-point fuction $\Gamma(0)$ up to one loop order in dimensional regularization:

$$\Gamma(0) \equiv \frac{\lambda^2}{2}\mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q^2 - m^2 + i\epsilon)^2},$$

where μ is an arbitraty constant with mass dimension. You can assume the results

$$\int_0^\infty \mathrm{d}t \frac{t^{m-1}}{(t+a^2)^\alpha} = \frac{\Gamma(m)\Gamma(\alpha - m)}{\Gamma(\alpha)} (a^2)^{m-\alpha}$$

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$$\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma_E + \mathcal{O}(\epsilon)$$

when $\epsilon \to 0$ and $\gamma_E \simeq 0.5772$.

3. Compute the finite part of the $\lambda\phi^4$-theory 4-point fuction up to one loop order:

$$\tilde{\Gamma}(p^2) \equiv \Gamma(p^2) - \Gamma(0).$$

Show explicitly that for $p^2 > 4m^2$ $\Gamma(p^2)$ has a complex part, and that there is a relation: $\mathrm{Im}\,\tilde{\Gamma}(p^2) = \sigma(\phi\phi \to \phi\phi)$, where the cross section has been computed at the tree level. (Note that the cross section has to be integrated over the angles and one must account for the identical particles in the final state (which explains a missing 2).) Follow freely the lectures where you can.

4. Compute the one loop self-energy $\Pi(p^2)$ for the field ϕ in the theory

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{3!}\phi^3$$

in six dimensions. (Use dimensional regularization with $\epsilon \equiv 6 - D$.) Compute $\Pi(p^2)$ explicitly as an integral over one Feynman z. Show that the two point function has a complex part for $p^2 > (2m)^2$ and compute it. Renormalize the propagator at one-loop at $p^2 = 0$, through $\Delta_R^{-1}\,|_{p^2=0} \equiv -m^2$ and $\partial\Delta_R^{-1}/\partial p^2\,|_{p^2=0} \equiv 1$ and compute the corresponding Z_ϕ and δm^2.