Quantum field theory, fall 2023, **Excercise 11.**

1. Let us define the (axial) gauge condition

$$\eta \cdot A(x) - \omega(x) = 0,$$

where η^μ is some constant 4-vector. Show that QED is ghost-free also in this gauge (Faddeev-Popov determinant is independent of the field A), and compute the FP-corrected Lagrange density for the theory. Compute the photon propagator in η-gauge when ξ parameter appearing in the Gaussian weight in the integral over ω's ($\sim \exp \int \omega^2/2\xi$) goes to zero.

3. Show that also Yang-Mills theory is ghost-free in the axial gauge.

4. Show that a gauge-field "mass term" $A_\mu A^\mu$ is not gauge invariant in QED or in Yang-Mills theories. Consider then the theory

$$\mathcal{L} = |D_\mu \phi|^2 + m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,,$$

where ϕ is a complex scalar field, $D_\mu \equiv \partial_\mu - igA_\mu$ and $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. Assume that $m^2 > 0$, so that the mass of the $|\phi|$ field appears to be complex. Show that the energy-minimum of the theory corresponds to the configuration $A_\mu = 0$ and $|\phi| = 2m/\sqrt{\lambda} \equiv \sqrt{2}v$. Redefine the scalar field as $\sqrt{2}\phi \equiv v + \phi_1 + i\phi_2$ and write the $\mathcal{L}$ in terms of these new parameters. Show that the mass of the field ϕ_1 is positive. However, your $\mathcal{L}$ now contains a mixing term $\sim A^\mu \partial_\mu \phi_2$. To get rid of it choose the gauge

$$f(A_\mu, \phi) = \partial^\mu A_\mu + \xi g v \phi_2 = 0$$

and perform the steps involved in the FP-procedure as before. Write the final Lagrange function for the theory and show in particular that as a result of the gauge-fixing the mixing term has vanished and all fields have quadratic, clearly identifiable mass terms. ($M_A = gv$, $m_{\phi_2} = \sqrt{\xi}gv$ and $m_{\phi_1} = \sqrt{2}m$). Did you break the gauge symmetry of the theory?

4. Derive the photon propagator from the final form for the Lagrangian in the previous excercise.

5. Gauge condition can generate, in addition to the ghost fields, also additional interactions between the gauge fields themselves. Consider the following gauge in QED:

$$\partial_\mu A^\mu + eA_\mu A^\mu = 0$$

Show that in addition to ghosts coupling to photons, this theory contains also gauge interactions of the type AAA and $AAAA$! Work out the photon and ghost field propagators, and derive the Feynman rule for the AAA-vertex (up to an overall constant) from a requirement that one cannot allow unphysical transitions of the type $A \to AA$ in a free photon gas. Additional bonus for those who work out the AAA-Feynman rule by direct evaluation.