Redshift Drift in LTB Void Universes

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based on CY, Kai and Nakao PRD 83, 043527

Table of Contents

NOTE

In this talk, I focus on the situation in which the observer is at the exact center of an LTB universe. That is, we consider non-Copernican universe models.

Introduction

Observational Tests and Limitations

◎Type Ia SNe

Determined by only late time geometry. Simple and useful.

It can be fitted by using LTB models with 1 functional d.o.f.

◎Hubble parameter at the center (H_0)

This is not enough alone to prove the inhomogeneity.

We need H in the off-center region.

◎Cosmic Microwave Background

We need initial conditions for perturbations before the decoupling epoch, which could be significantly different from the standard cosmology.

◎Baryon Acoustic Oscillation

Not only the initial condition but also evolution of perturbations in late time is needed.

Loopholes in Observational Tests

◎Bigbang Time Inhomogeneity

Absence of Bigbang time inhomogeneity is often assumed. But, if we do not impose the homogeneity at the early stage of the matter dominated era, we should consider this also.

◎Primordial Spectrum

Primordial spectrum can be adjusted so that the CMB angular power spectrum is well explained.

◎Iso-curvature Inhomogeneities

・The kSZ effect can give a strong constraint on the adiabatic scenario of non-Copernican cosmology. But, the existence of iso-curvature inhomogeneities can be a caveat.

・It also affects the BAO scale. The BAO scale can be significantly different from that is predicted by CMB.

Is there any other powerful discriminator?

Going Back to the Beginning

◎What is the difference? LTB vs LCDM

△Homogeneity and Inhomogeneity

But, It is not directly measurable in cosmological scales.

△Existence of Dark Energy

How can we **directly test** the existence of the dark energy?

Dark energy→Repulsive force

→Acceleration of cosmic expansion

→ "Redshift drift"

[Uzan,Clarkson and Ellis,(arXiv:0801.0068)]

NOTE

Using the distance-redshift relation is not the direct test of the acceleration of the cosmic expansion.
Since the distance and the redshift are physical quantities on the light cone, we need to assume the Copernican principle to conclude the acceleration of the cosmic expansion.

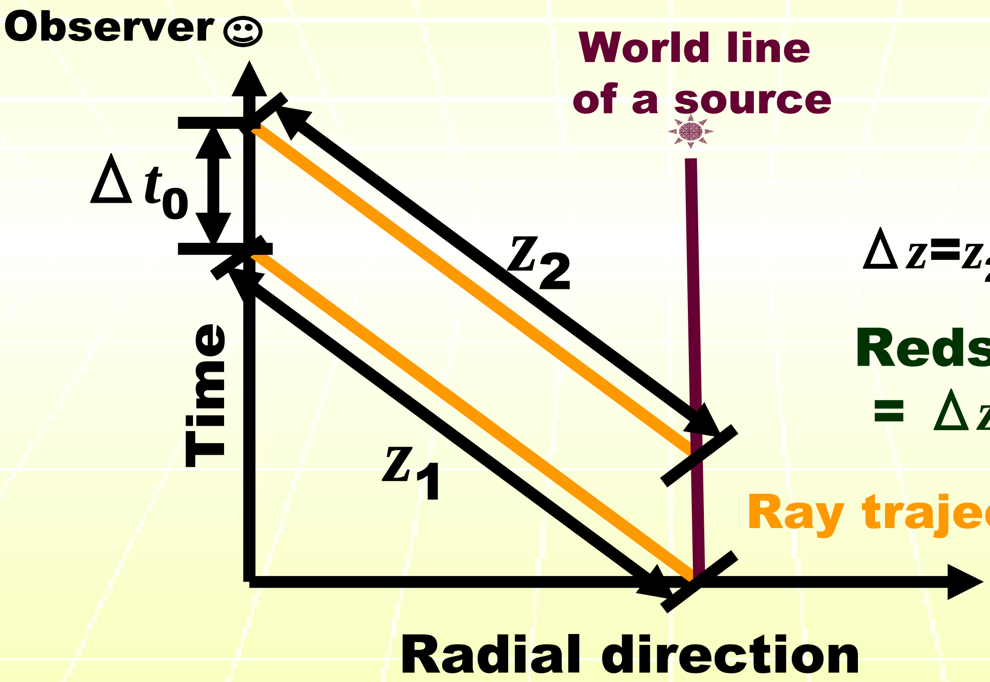
Redshift Drift

Redshift Drift

◎Redshift Drift

Temporal variation of the redshift

[Sandage, ApJ 136, 319 (1962)]



$\Delta z = z_2 - z_1$

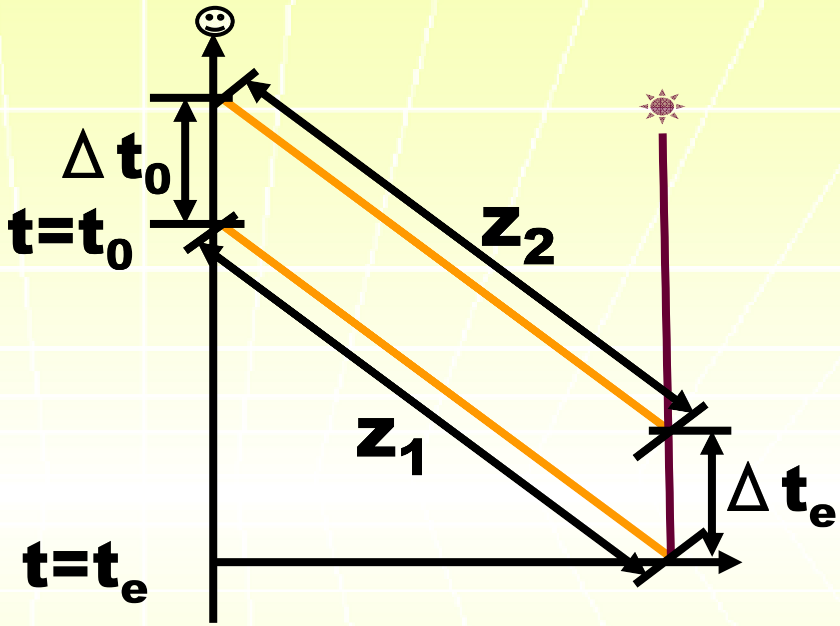
Redshift drift
$= \Delta z / \Delta t_0$

Since $z \sim v/c$, $\Delta z / \Delta t_0 \sim$ acceleration

Homogeneous Case

$$1 + z_1 = \frac{a(t_0)}{a(t_e)}$$

$$1 + z_2 = \frac{a(t_0+\Delta t_0)}{a(t_e+\Delta t_e)}$$

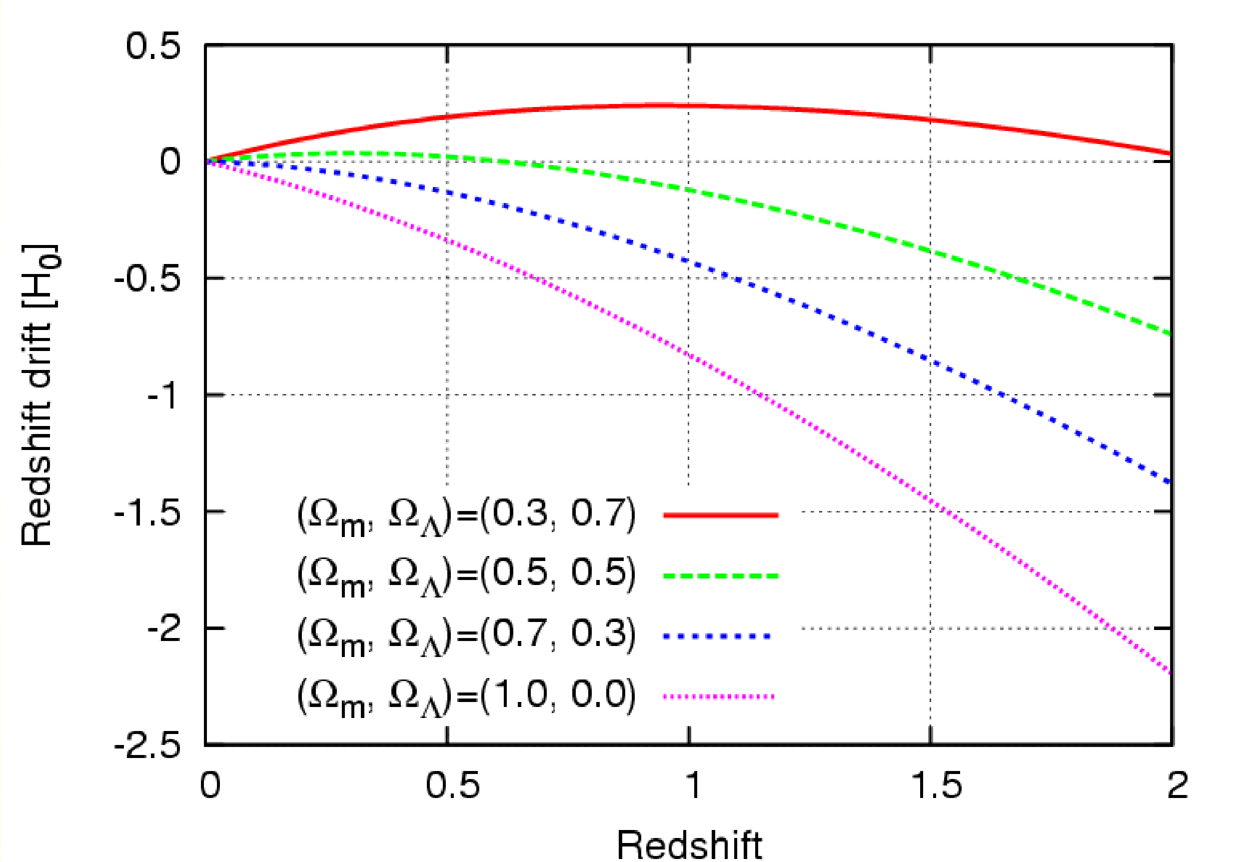


$$\begin{aligned} 1 + z_2 &\simeq \frac{a(t_0)}{a(t_e)} - H(t_e)\frac{a(t_0)}{a(t_e)}\Delta t_e + H_0\frac{a(t_0)}{a(t_e)}\Delta t_0 \\ &= 1 + z_1 - H(t_e)\Delta t_0 + (1+z_1)H_0\Delta t_0 \\ &= 1 + z_1 + H_0\Delta t_0(1 + z - H(t_e)/H_0) \end{aligned}$$

$$\Delta z := z_2 - z_1 = H_0\Delta t_0\left(1 + z - \frac{H(z)}{H_0}\right) \sim \frac{\text{obs. time}}{\text{cosmic age}}$$

Redshift Drift in LCDM

◎ **Δz is positive (accelerating) in $z<2$**



What about LTB models?

Redshift Drift in LTB Models

LTB Universe Models

◎Solutions of a spherically symmetric dust

△Metric

$$ds^2 = -dt^2 + \frac{(\partial_r R(t,r))^2}{1-k(r)r^2} dr^2 + R^2(t,r) d\Omega^2$$

△Energy momentum tensor

$$T^{\mu\nu} = \rho(t,r) u^\mu u^\nu$$

△Einstein equations

$$\partial_t R^2 = -k(r) r^2 + \frac{2M(r)}{R} \qquad 4\pi\rho = \frac{\partial_r M(r)}{R^2 \partial_r R}$$

△Solution

$$R(t,r) = (6M(r))^{1/3} (t - t_{\mathrm{B}}(r))^{2/3} \mathcal{S}(x)$$

$$x = k(r) r^2 \left(t - t_{\mathrm{B}}(r)\right)^{2/3} \left(6M(r)\right)^{-2/3}$$

$$S(x) = \begin{cases} \dfrac{\cosh\sqrt{-\eta} - 1}{6^{1/3}(\sinh\sqrt{-\eta} - \sqrt{-\eta})^{2/3}}\,; & x = \dfrac{-(\sinh\sqrt{-\eta} - \sqrt{-\eta})^{2/3}}{6^{2/3}} \quad \text{for } x \le 0\,, \\ \dfrac{1 - \cos\sqrt{\eta}}{6^{1/3}(\sqrt{\eta} - \sin\sqrt{\eta})^{2/3}}\,; & x = \dfrac{(\sqrt{\eta} - \sin\sqrt{\eta})^{2/3}}{6^{2/3}} \quad \text{for } x > 0\,. \end{cases}$$

Physical Degrees of Freedom

◎3 arbitrary functions

$$k(r),\ M(r) \text{ and } t_{\mathrm{B}}(r)$$

1 of these is a gauge degree of freedom.
For example, we can set

$$M(r) = \frac{4}{3}\pi\rho_0 r^3$$

$$t_{\mathrm{B}}(r) = 0(\mathrm{const.})$$

Inhomogeneity grow with time
k(r) : growing mode

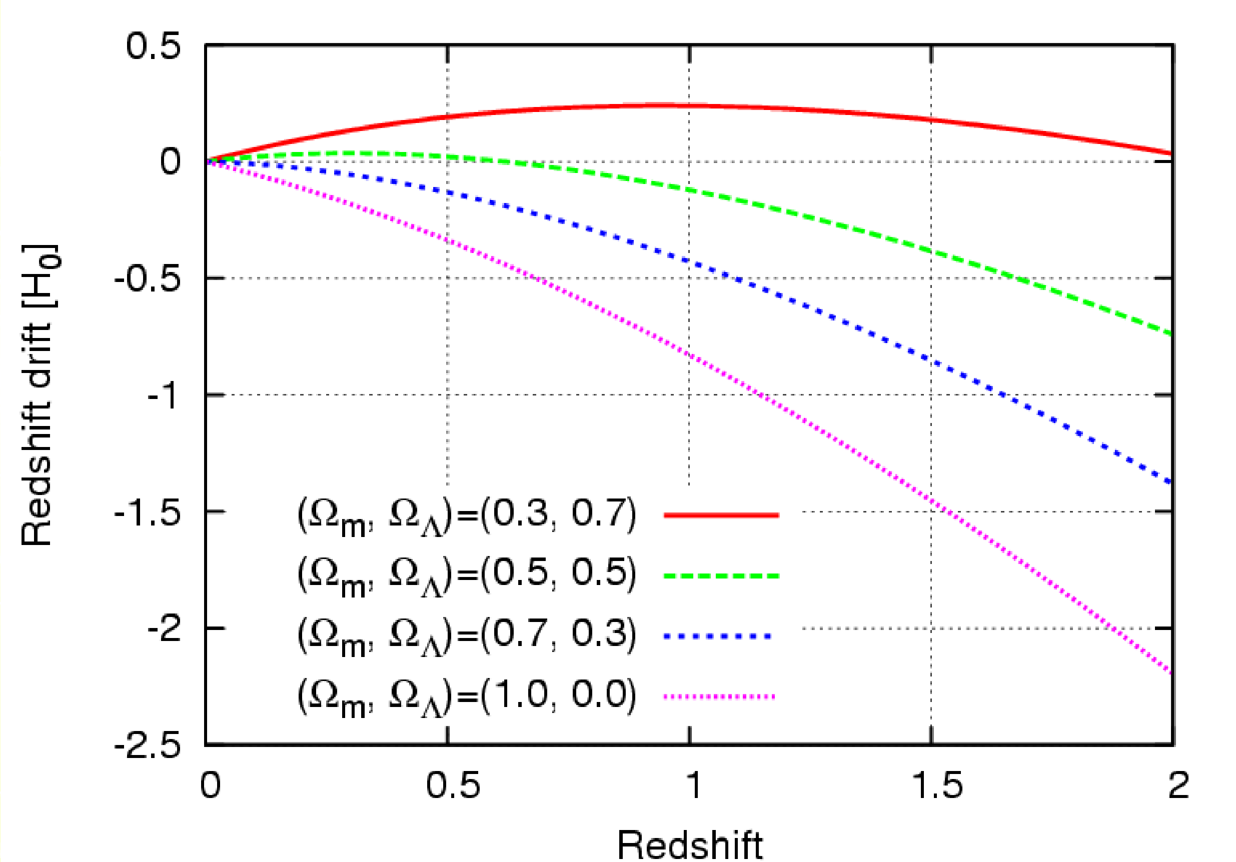
$$k(r) = \mathrm{const.}$$

Inhomogeneity decay with time
t_B(r) : decaying mode

2 physical degrees of freedom correspond to growing modes and decaying modes

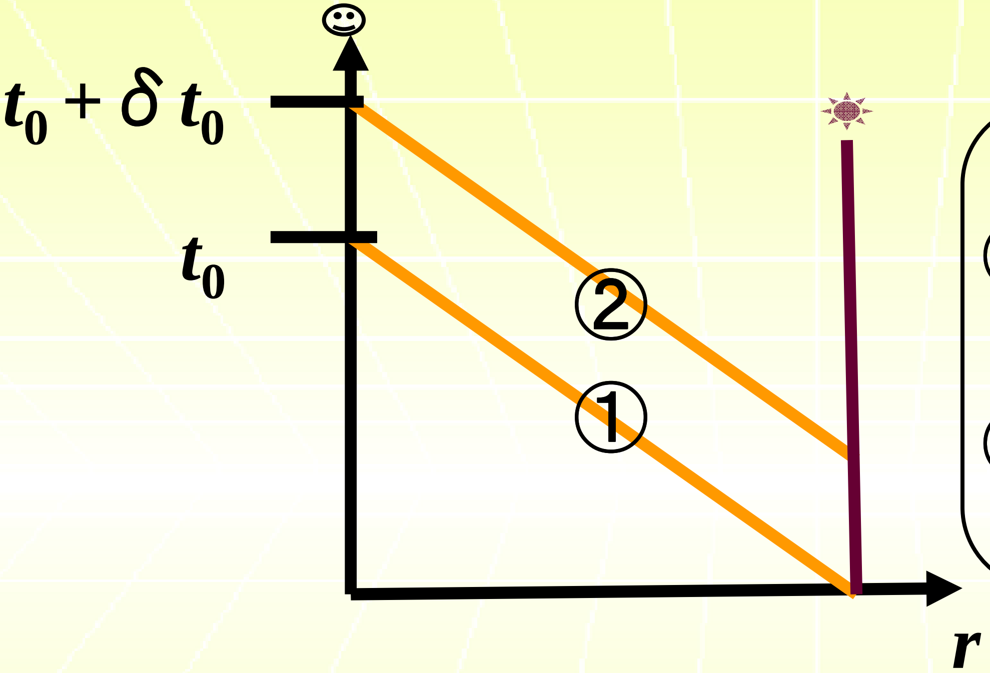
Redshift Drift in LCDM

◎ Δz is positive (accelerating) in $z<2$



We naively expect negative Δz for LTB

Redshift Drift in LTB models



① $z = z_{\rm lc}(r; t_0)$, $t = t_{\rm lc}(r; t_0)$

② $z = z_{\rm lc}(r; t_0) + \delta z(r)$, $t = t_{\rm lc}(r; t_0) + \delta t(r)$

null geodesic equation

$$\frac{dz}{dr} = \frac{(1+z)\partial_t \partial_r R}{\sqrt{1-kr^2}}$$
$$\frac{dt}{dr} = \frac{-\partial_r R}{\sqrt{1-kr^2}}$$

$\longrightarrow$

$$\frac{d\delta z}{dr} = \frac{\partial_t \partial_r R}{\sqrt{1-kr^2}}\delta z + \frac{(1+z)\partial_t^2 \partial_r R}{\sqrt{1-kr^2}}\delta t$$
$$\frac{d\delta t}{dr} = \frac{-\partial_t \partial_r R}{\sqrt{1-kr^2}}\delta t$$

Calculation（1）

$$\frac{d\delta z}{dr} = \frac{\partial_t \partial_r R}{\sqrt{1-kr^2}}\delta z + \frac{(1+z)\partial_t^2 \partial_r R}{\sqrt{1-kr^2}}\delta t$$

$$\frac{d\delta t}{dr} = \frac{-\partial_t \partial_r R}{\sqrt{1-kr^2}}\delta t$$

$$\frac{d}{dr} = \frac{dz}{dr}\frac{d}{dz} = \frac{(1+z)\partial_t \partial_r R}{\sqrt{1-kr^2}}\frac{d}{dz}$$

$$\frac{d\delta z}{dz} = \frac{\delta z}{1+z}\delta z + \frac{\partial_t^2 \partial_r R}{\partial_t \partial_r R}\delta t$$

$$\frac{d\delta t}{dz} = \frac{-1}{1+z}\delta t$$

$$\delta t = \frac{\delta t(0)}{1+z}$$

$$\frac{d\delta z}{dz} = \frac{\boxed{\delta z}}{1+z} + \frac{\boxed{\partial_t^2 \partial_r R}}{\boxed{(1+z)}\boxed{\partial_t \partial_r R}}\boxed{\delta t(0)}$$

$$\delta z(0) = 0$$

$\square$ **> 0**

$$\partial_t^2 \partial_r R < 0 \Rightarrow \delta z < 0 \text{ for } z > 0$$

Calculation（2）

$$\partial_t^2 \partial_r R = -\frac{\partial_r M}{R^2} + \frac{2M\partial_r R}{R^3}$$
$$= \left(-4\pi\rho + \frac{8\pi \int \rho R^2 dR}{R^3}\right) \partial_r R$$
$$= 4\pi \frac{\partial_r R}{R^3} \int \left(-\frac{d\rho}{dR} R^3 - \rho R^2\right) dR$$

where $\frac{d\rho}{dR} = \frac{\partial_r \rho}{\partial_r R}$

Therefore

$$\partial_r \rho > 0 \Leftrightarrow \frac{d\rho}{dR} > 0 \Rightarrow \partial_t^2 \partial_r R < 0 \Rightarrow \delta z < 0$$

Void!

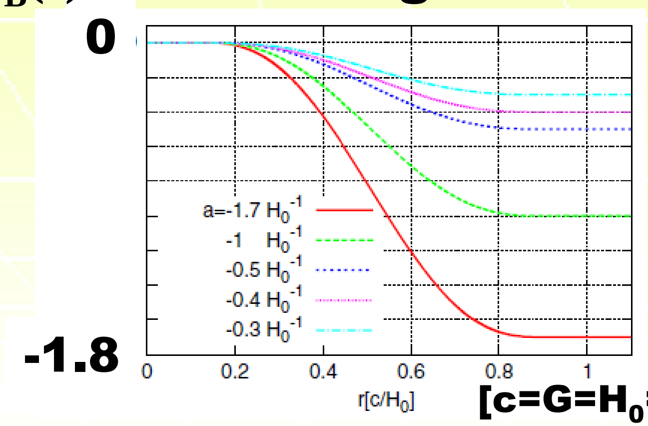
δ z<0 for all void LTB models

Is it possible to get a positive value?

Positive Redshift Drift in LTB Models

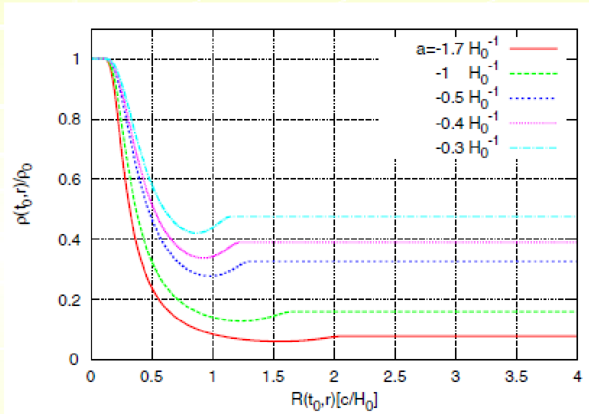
LTB with Large Hump(k=0)

- $k(r)$=0
- $t_B(r)$: decreasing function

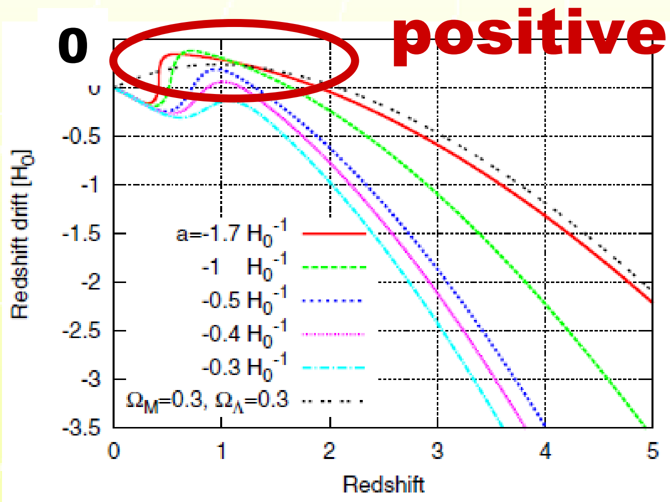


[c=G=H_0=1]

△ Density $\rho(t_0, r)$



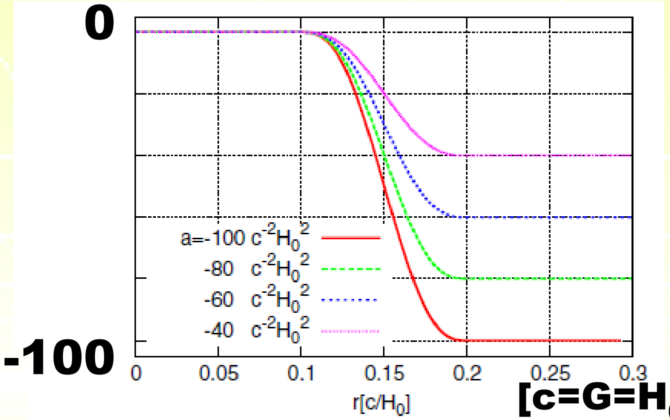
△ Redshift drift



It can be positive for very large hump models with k=0

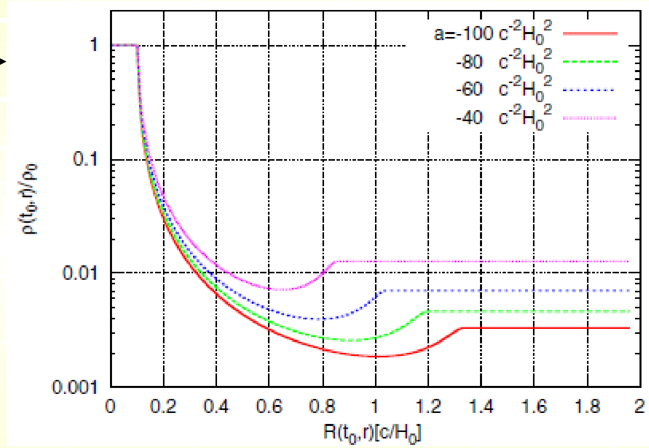
LTB with Large Hump(t_B=0)

- $t_B(r)$=0
- $k\,(r)$: **decreasing function**

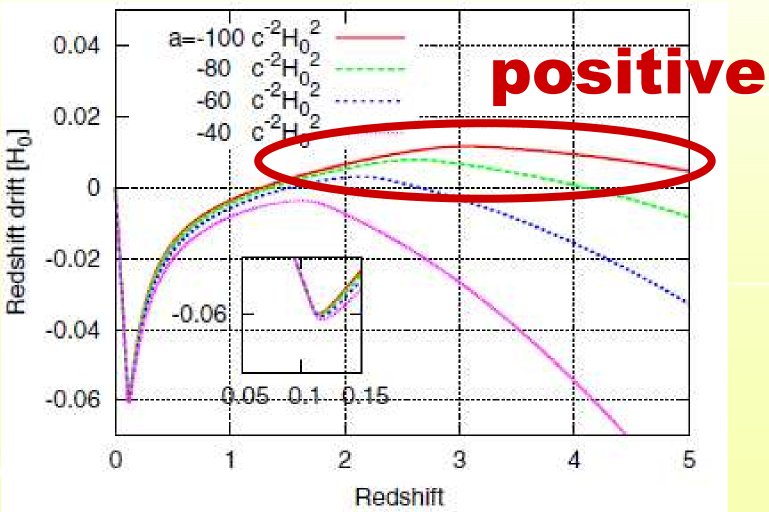


[c=G=H$_0$=1]

△ Density $\rho\,(t_0, r)$



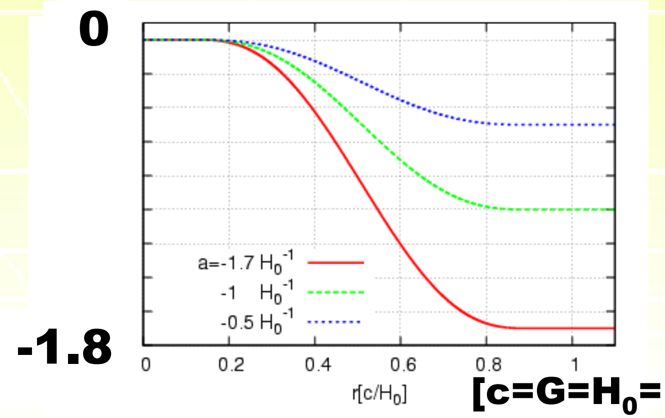
△ Redshift drift



It can be positive for very large hump models with t_B=0

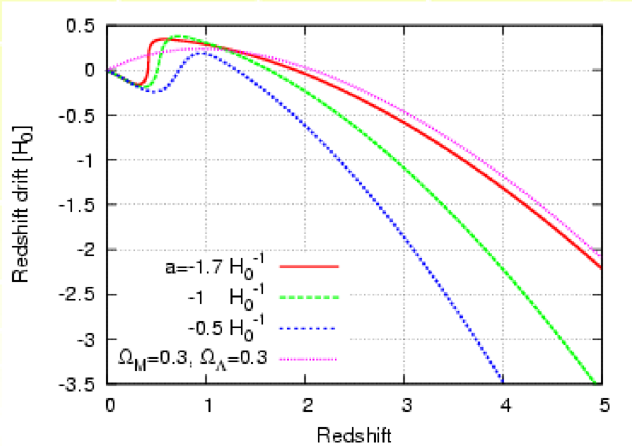
Distance-Redshift Relation

- $k(r)$=0
- $t_B(r)$: decreasing function

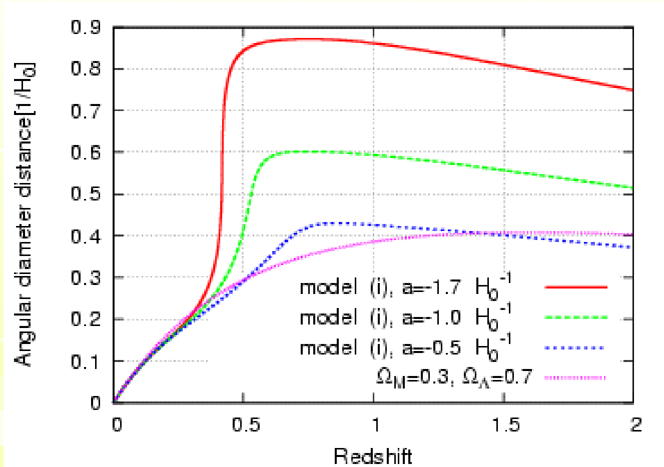


[c=G=H_0=1]

△ z-drift



△ Distance-Redshift



Even if the z-drift is positive, the distance-redshift relations are largely different from the observation in most cases.

The distance-redshift relations in the t_B=0 cases also are largely different from that of the LCDM.

z-drift & D-z relation is very good discriminator even for general models

Another Hump-type Model

- $t_B(r)$ and $k(r)$

[Celerier et al(arXiv:0906.0905)
Kolb, Lamb(arXiv:0911.3852)]

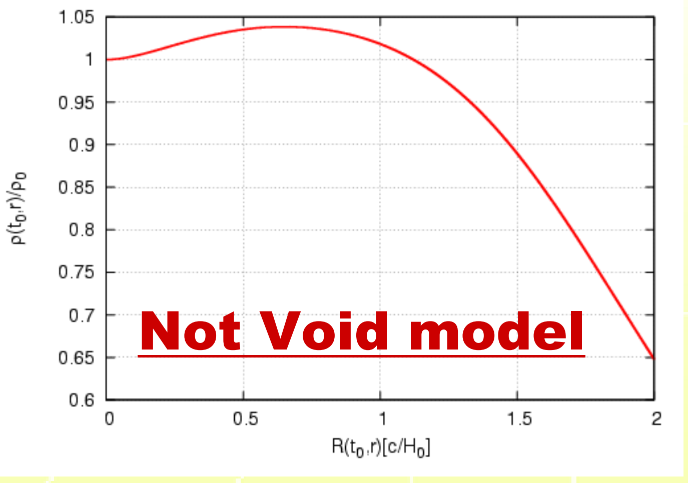
$$k(r) = -\frac{7}{10} + 0.728893r - 0.634917r^2 + 0.303959r^3 - 0.073768r^4 - \frac{0.00878216r}{r+0.303959}$$

$$t_B(r) = -0.21319r - 0.013605r^2 + 0.000925931r^3 + \frac{0.298252r^2}{r+1.5439}$$

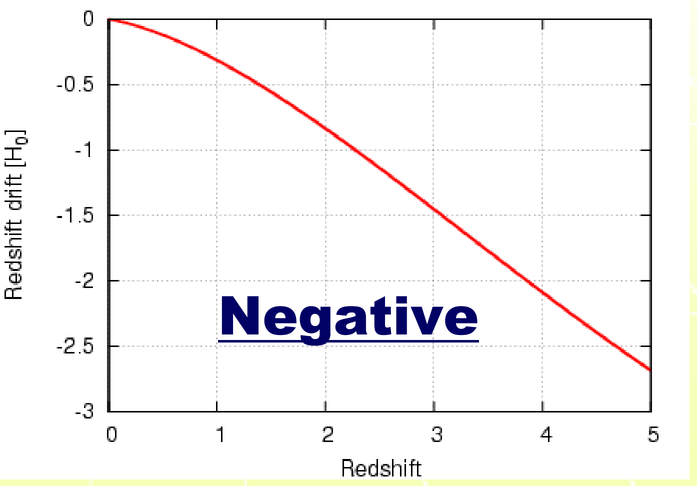
[CY(arXiv:1010.0530)]

The Distance-redshift relation and the redshift-space mass density coincide with those in LCDM (within 1% accuracy)

△ Density



△ z-drift



Summary

◎z-drift is negative for LTB void models.

◎z-drift can be partially positive if we introduce very large hump-type inhomogeneity.

◎Even if z-drift is positive, the distance-redshift relation is largely different from the observation in most cases.

If we observe a positive value of the redshift drift, we can rule out almost all LTB models (without dark energy)

Is it observable?

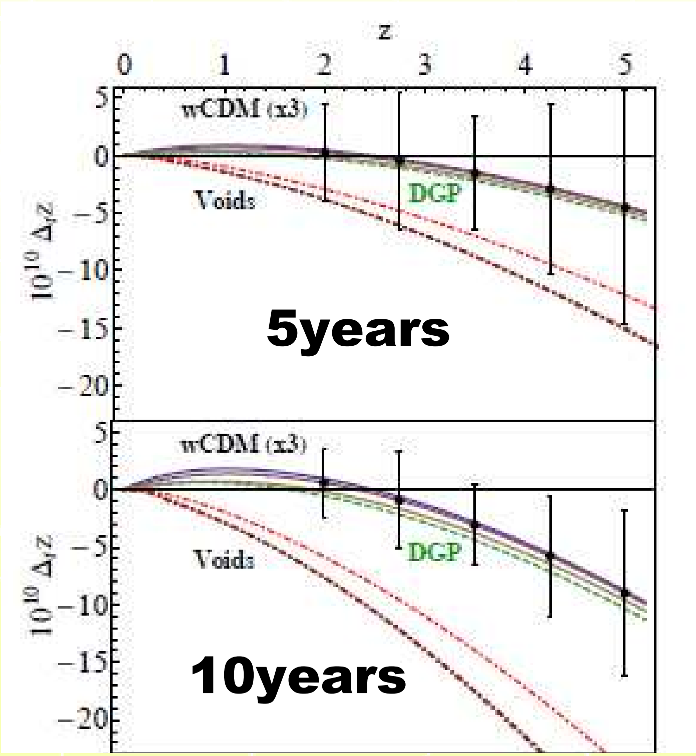
Observation

1. Precision Spectroscopy of QSOs

◎The Cosmic Dynamics Experiment (CODEX) at the European Extremely Large Telescope (E-ELT)

Using spectra of Ly α forest and metal lines

[Quartin, Amendola (arXiv:0909.4954)]



Assuming LCDM

Model	5 years	10 years	15 years
LTB model 1	1.1σ	6.2σ	12.5σ
	$\chi^2 = 6.5$	$\chi^2 = 52$	$\chi^2 = 176$
LTB model 2	$.5\sigma$	4.3σ	9.2σ
	$\chi^2 = 3.7$	$\chi^2 = 30$	$\chi^2 = 100$

Table I: Estimated achievable confidence levels by the CODEX mission in 5, 10 and 15 years.

Typical void models can be ruled out at the 4 σ level in a decade

E-ELT, CODEX

http://www.eso.org/public/teles-instr/e-elt.html

◎European Extremely Large Telescope (E-ELT)

△Aperture : 39.3m
△Location : Cerro Armazones, Chile
△Operations start : 2023

"The E-ELT will gather 100 000 000 times more light than the human eye, 8 000 000 times more than Galileo's telescope, and 26 times more than a single VLT Unit Telescope. In fact, the E-ELT will gather more light than all of the existing 8–10-metre class telescopes on the planet, combined."

◎COsmic Dyanamics and EXo-earth experiment

△Stable, high spectral resolution instrument

"very stable, high spectral resolution instrument proposed for the E-ELT. CODEX is currently undergoing the Phase A feasibility study "

Precision Spectroscopy of QSOs

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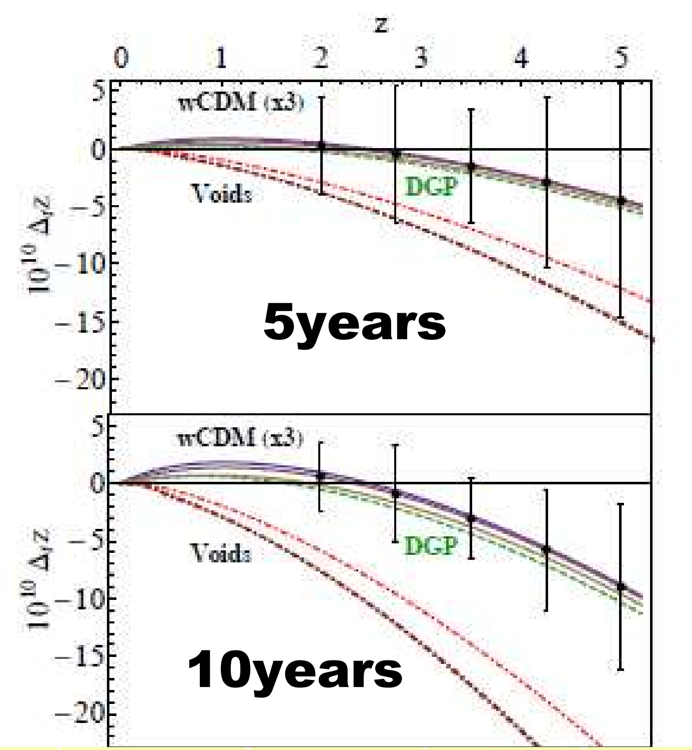

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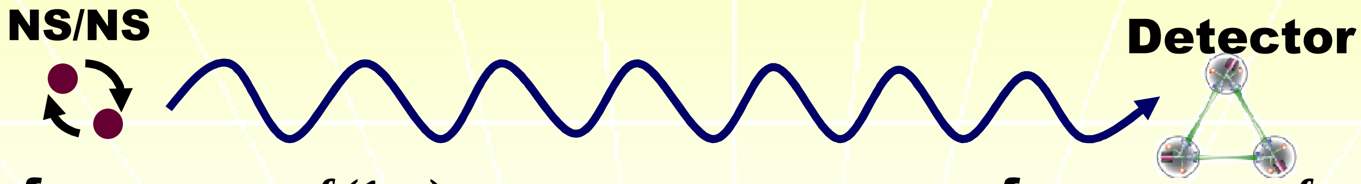
2. Gravitational Wave

◎Effect of acceleration in GW waveform

[Seto et.al (astro-ph/0108011)]



frequency: $f\,(1+z)$ **frequency:** f

time duration: Δt_e **time duration:** Δt_o

We have

$$\frac{dt_e}{dt_o} = \frac{1}{1+z}\,,\quad \frac{d^2 t_e}{dt_o^2} = -\frac{1}{(1+z)^2}\frac{dz}{dt_o}$$

z-drift

Using these, we obtain

$$(1+z)\Delta t_e \simeq \Delta t_o - \frac{1}{2(1+z)}\frac{dz}{dt_o}\Delta t_o^2 + \mathcal{O}(\Delta t_o^3)$$

Total phase shift

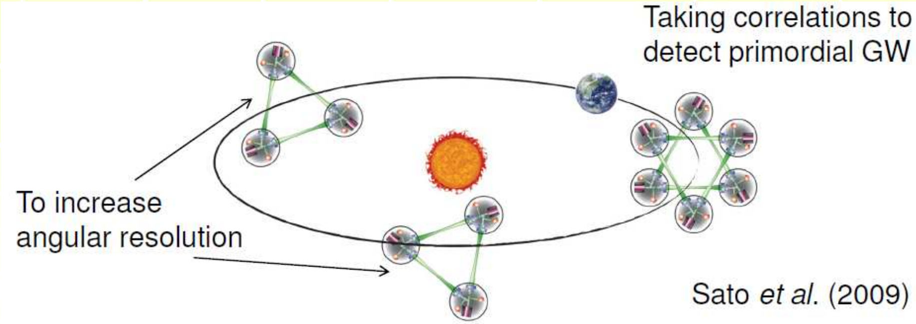
$$2\pi f(1+z)\Delta t_e \simeq 2\pi f \Delta t_o - \frac{\pi f}{(1+z)}\frac{dz}{dt_o}\Delta t_o^2$$

correction associated with z-drift

DECIGO

◎Space-based interferometers DECIGO

- •Japan
- •Fabry-Perrot cavity
- •Arm-length:1000km
- •Laser power: 10W
- •Launching year: 2027



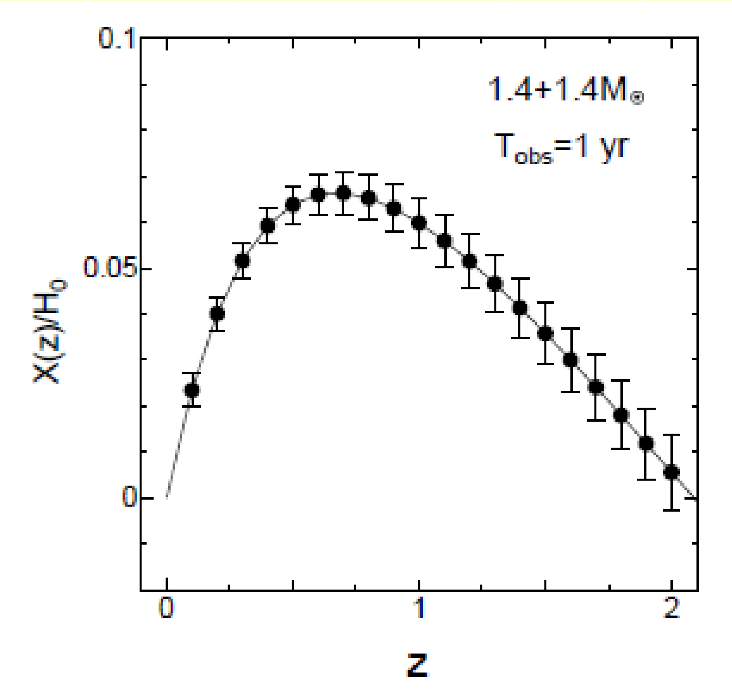
◎Advantages of using DECIGO

- •Longer observation time for a binary
- •Larger number of GW cycles: 0.1Hz × 5yr
- •Larger NS/NS binary detection rate: 10^6 /yr

z-drift observation with DECIGO

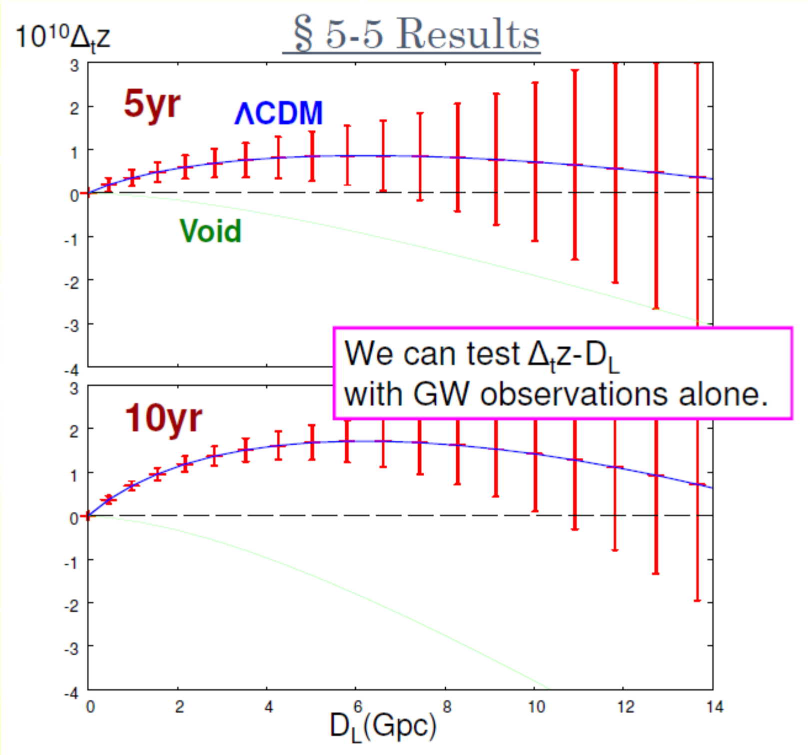
[Takahashi and Nakamura(2005)]

$$X(z) = \frac{1}{2(1+z)}\frac{dz}{dt_o}$$



for ultimate DECIGO

[Yagi, Nishizawa, CY in prep.]



for DECIGO(more realistic)

We can observe the sign of z-drift in z<2 region!

Observational Tests with z-drift

◎Precision Spectroscopy of QSOs

- E-ELT, CODEX (from 2023) ⟶ **2033**

<u>Typical void models can be ruled out at the 4σ level in a decade.</u>

◎Gravitational waves from NS/NS binary

- DECIGO (from 2027) ⟶ **2032**

<u>We can observe the sign of z-drift in z<2 region (5yr).</u>
<u>If z-drift is positive, we can rule out all LTB void models or even general ones.</u>

<u>Since the redshift drift is independent of the early stage of our universe, we do not need any assumption in the early universe.</u>

I wish you good health and long life

Thank you very much