What is dust?:
Physical foundations of the averaging problem

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DLW:

Class. Quantum Grav. 28 (2011) 164006

New J. Phys. 9 (2007) 377

Phys. Rev. Lett. 99 (2007) 251101

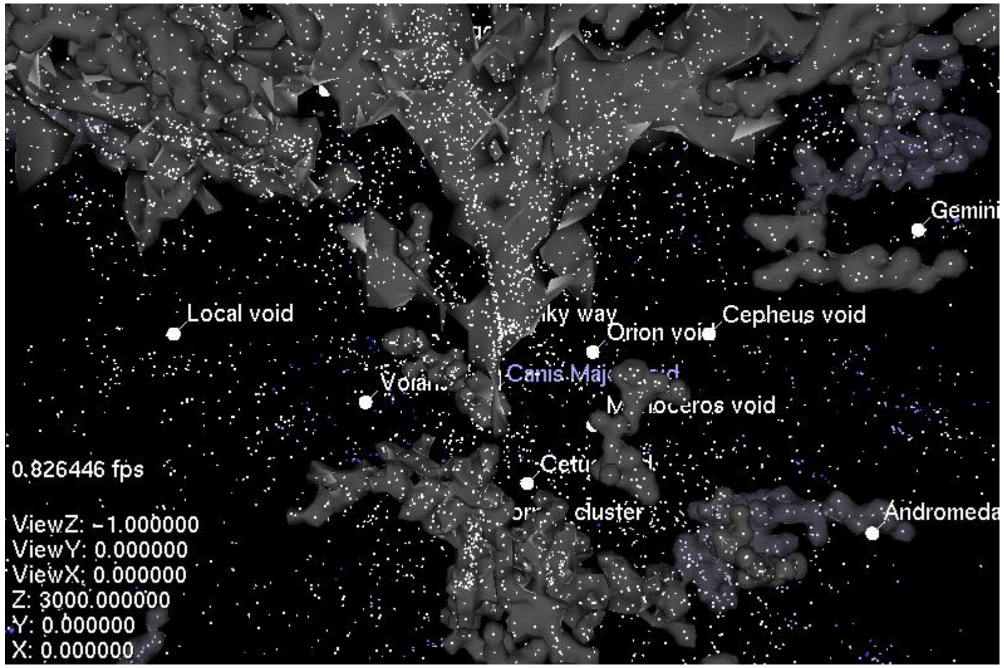
Phys. Rev. D78 (2008) 084032

Phys. Rev. D80 (2009) 123512

B.M. Leith, S.C.C. Ng & DLW:

ApJ 672 (2008) L91

P.R. Smale & DLW, **MNRAS 413 (2011) 367**



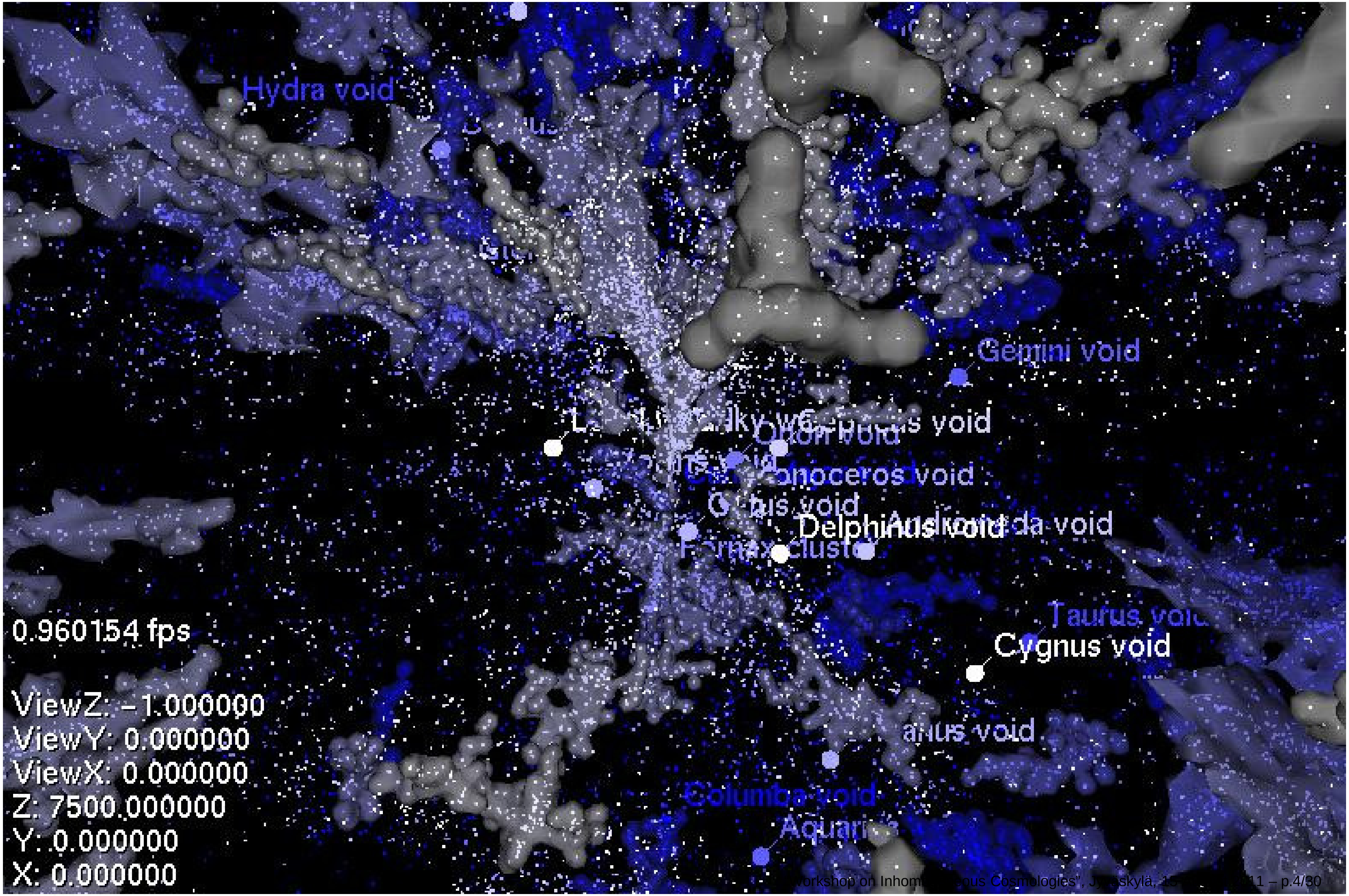
Overview

- Main physical questions:
    coarse-graining,
    average homogeneity,
    cosmic averages versus cosmic variance,
    identification of physical observables
- Implementing solutions to these questions in the timescape scenario
- Overview of timescape scenario (Hypothesis: must understand nonlinear evolution with backreaction, AND gravitational energy gradients within the inhomogeneous geometry)
- Brief overview of observational tests

From smooth to lumpy

- Universe was very smooth at time of last scattering; fluctuations in the fluid were tiny ($\delta\rho/\rho \sim 10^{-5}$ in photons and baryons; $\sim 10^{-4}$,10^{-3} in non–baryonic dark matter).
- FLRW approximation very good early on.
- Universe inhomogeneous today on scales $\lesssim 100h^{-1}$Mpc
- Recent surveys estimate that 40–50% of the volume of the universe is contained in voids of diameter $30h^{-1}$ Mpc. [Hubble constant $H_0 = 100h$ km/s/Mpc] (Hoyle & Vogeley, ApJ 566 (2002) 641; 607 (2004) 751)
- Add some larger voids, and many smaller minivoids, and the universe is *void–dominated* at present epoch.
- Clusters of galaxies are strung in filaments and bubbles around these voids.

6df: voids & bubble walls (A. Fairall, UCT)



Einstein's equations: valid on what scale?

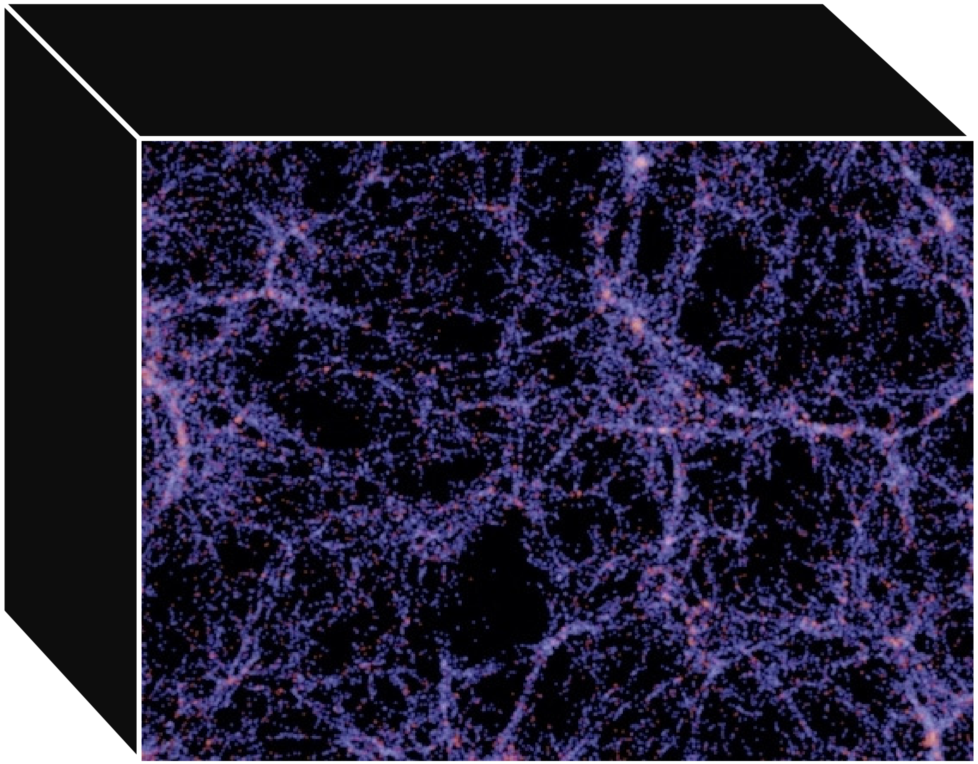
$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- Nonlinear averaging of full inhomogeneous Einstein equations leads to modified evolution equations
- Strong backreaction (many approaches), generally $\bar{\mathbf{G}}_{\mu\nu}(\mathbf{g}_{\alpha\beta}) \neq \mathbf{G}_{\mu\nu}(\bar{\mathbf{g}}_{\alpha\beta})$; and $\rho_{\rm cr} \neq 3H_{\rm av}^2/(8\pi G)$
- Dust not invariantly defined from last scattering today (except for atoms and ions in voids)
- Must coarse grain at least over galaxies
- LHS = geometry, RHS = matter no longer possible when coarse-graining over gravitational d.o.f.
- Viewpoint: foundational questions regarding statistical nature of gravity, especially gravitational energy and entropy, are an inextricable part of problem

Problem: a lack of principles

- Einstein: "in a consistent theory of relativity there can be no inertia relatively to 'space', but only an inertia of masses relatively to one another"
- Need to address Mach's principle: *"Local inertial frames are determined through the distributions of energy and momentum in the universe by some weighted average of the apparent motions"*
- *Fitting Problem*: How does coarse-graining affect relative calibration of clocks and rulers, from local to global, to account for average effects of gravity?
- Viewpoint: must coarse-grain over scales on which there is on average no flow of matter of one dust particle to another; i.e., on scales $\sim 100h^{-1}$Mpc on which space is expanding. Dust particle more like a fluid element.

Within a statistically average cell



- Need to consider relative position of observers over scales of tens of Mpc over which $\delta\rho/\rho \sim -1$.
- GR is a local theory: gradients in spatial curvature and gravitational energy can lead to calibration differences between our rulers and clocks and volume average ones

Cosmic averages versus variance

- Retain an average Copernican Principle - we are at an average position *for observers in a galaxy*
- Observers in bound systems are not at a volume average position in freely expanding space
- By Copernican principle other average observers should see an isotropic CMB
- BUT *nothing in theory, principle nor observation demands that such observers measure the same mean CMB temperature nor the same angular scales in the CMB anisotropies*
- Average mass environment (galaxy) can differ significantly from volume–average environment (void)

Average homogeneity

FLRW model has at least 3 notions of average homogeneity

1. The notion of average spatial homogeneity is described by a class of ideal comoving observers with synchronized clocks.
2. The notion of average spatial homogeneity is described by average surfaces of constant spatial curvature (orthogonal to the geodesics of the ideal comoving observers).
3. The expansion rate at which the ideal comoving observers separate within the hypersurfaces of average spatial homogeneity is uniform.

- Even at perturbative level (Bardeen 1980) can retain one notion as more fundamental than the others

Cosmological Equivalence Principle

- Viewpoint: in nonperturbative averaging need only one of these notions – the last one
- Key concept: regional symmetry. Averaging problem involves more than global diffeomorphisms and point symmetries (local Lorentz transformations)
- CEP: *It is always possible to choose a suitably defined average spacetime region, the cosmological inertial frame, in which average motions (timelike and null) can be described by geodesics of regional geometry*

$$\mathrm{d}s^2_{\mathrm{CIF}} = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right],$$

- CIF scale comparable to *matter horizon*, 1–10 Mpc, correlations on larger scales limited by causality and initial conditions at last scattering

Cosmic “rest frame”

- Patch together CIFs for observers who see an isotropic CMB by taking surfaces of uniform volume expansion

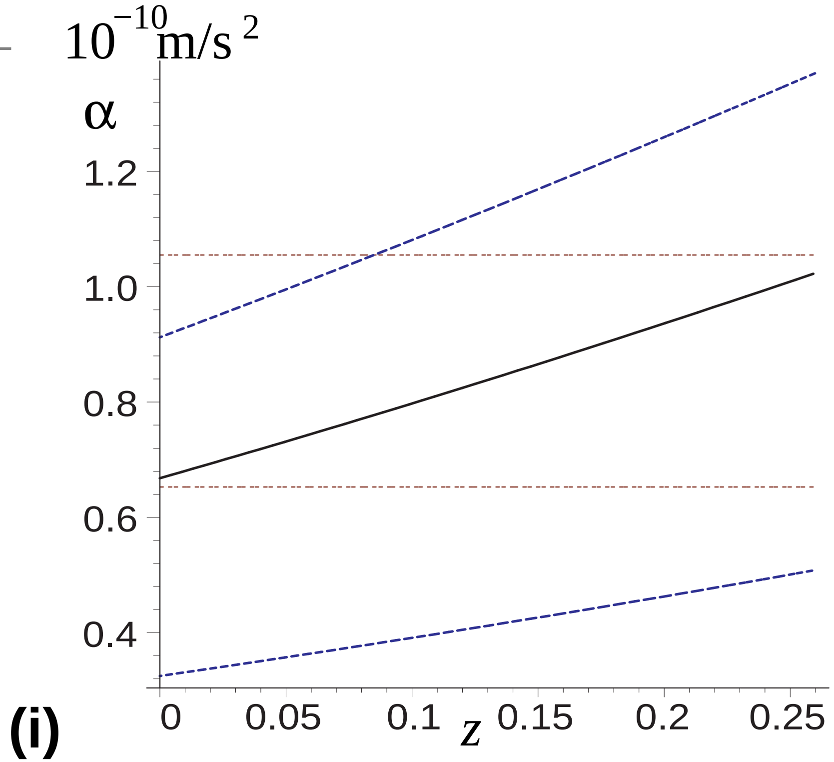
$$\left\langle \frac{1}{\ell_r(\tau)} \frac{\mathrm{d}\ell_r(\tau)}{\mathrm{d}\tau} \right\rangle = \frac{1}{3}\langle\theta\rangle_1 = \frac{1}{3}\langle\theta\rangle_2 = \cdots = \bar{H}(\tau)$$

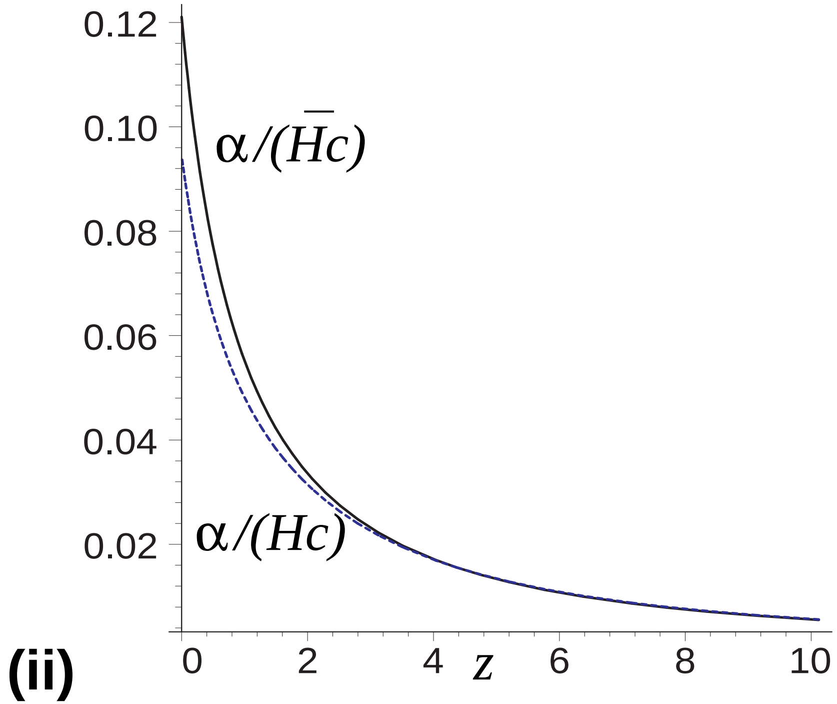
- Average over regions in which (i) scalar curvature is zero or negative; (ii) space is expanding at the boundaries, at least marginally.
- Solves the Sandage–de Vaucouleurs paradox implicitly.
- Voids appear to expand faster; but their local clocks tick faster, locally measured expansion can still be uniform.
- Global average H_{av} on large scales with respect to *any one set of clocks* will differ from $\bar{H}$

Cosmic “rest frame” is statistical

- For averages on scales larger than matter horizon there are different equivalent descriptions of average cosmic evolution
- Statistical description of geometries, with varying Ricci curvature required; no unique spacetime metric
- Interpret Buchert volume average as defined for observer whose local scalar spatial curvature matches global lightcone average; whereas galactic observers are by virtue of their existence in denser regions
- Relative volume deceleration and its integrated effect relevant to clock calibration of ideal observers
- *Weak field effect*: tiny relative deceleration leads to large cumulative difference in relative clock rates of galactic and volume average descriptions

Relative deceleration scale





Instantaneous relative volume deceleration of walls relative to volume average background $\alpha = H_0 c\bar{\gamma}_{\rm w}\dot{\bar{\gamma}}_{\rm w}/(\sqrt{\bar{\gamma}_{\rm w}^2 - 1})$ computed for timescape model which best fits supernovae luminosity distances: **(i)** as absolute scale nearby; **(ii)** divided by Hubble parameter to large z.

- With $\alpha_0 \sim 7 \times 10^{-11}$m s^{-2} and typically $\alpha \sim 10^{-10}$m s^{-2} for most of life of Universe, get 37% difference in calibration of volume average clocks relative to our own

Two/three scale “timescape” model

$$\bar{a}^3 = f_{\rm wi} a_{\rm w}{}^3 + f_{\rm vi} a_{\rm v}{}^3 \equiv \bar{a}^3 (f_{\rm w} + f_{\rm v})$$

- Split statistical volume average into *spatially flat* walls (containing galaxies), $f_{\rm w}$, and negatively curved voids with fraction, $f_{\rm v} = 1 - f_{\rm w}$.
- Solve Buchert equations, and construct a spherically symmetric average geometry (LTB metric NOT LTB solution) by radial null cone average
- Volume–average Buchert cosmological parameters, $\bar{\Omega}_M + \bar{\Omega}_k + \bar{\Omega}_{\mathcal{Q}} = 1$, defined w.r.t. coarse–grained 100/h Mpc cell with average (negative) spatial curvature
- Match to local spatially flat geometry within expanding wall regions by conformally matching radial null geodesics with uniform bare Hubble flow condition

Dressed cosmological parameters

- Conventional parameters for “wall observers” in galaxies: defined by assumption (no longer true) that others in entire observable universe have synchronous clocks and same local spatial curvature

$$\begin{aligned} \mathrm{d}s^2_{fi} &= -\mathrm{d}\tau_{\mathrm{w}}^2 + a_{\mathrm{w}}{}^2(\tau_{\mathrm{w}})\left[\mathrm{d}\eta_{\mathrm{w}}^2 + \eta_{\mathrm{w}}^2 \mathrm{d}\Omega^2\right] \\ \rightarrow \mathrm{d}s^2 &= -\mathrm{d}\tau_{\mathrm{w}}^2 + a^2\left[\mathrm{d}\bar{\eta}^2 + r_{\mathrm{w}}^2(\bar{\eta},\tau_{\mathrm{w}})\,\mathrm{d}\Omega^2\right] \end{aligned}$$

where $a \equiv \bar{a}/\bar{\gamma}_{\mathrm{w}}$, $r_{\mathrm{w}} \equiv \bar{\gamma}_{\mathrm{w}}\,(1 - f_{\mathrm{v}})^{1/3}\, f_{\mathrm{wi}}{}^{-1/3}\eta_{\mathrm{w}}(\bar{\eta},\tau_{\mathrm{w}})$, and volume–average conformal time $\mathrm{d}\bar{\eta} = \mathrm{d}t/\bar{a} = \bar{\gamma}_{\mathrm{w}}\,\mathrm{d}\tau_{\mathrm{w}}/\bar{a}$.

- This leads to dressed parameters *which do not sum to 1*, e.g.,

$$\Omega_M = \bar{\gamma}_{\mathrm{w}}^3 \bar{\Omega}_M\,.$$

Tracker solution PRL 99, 251101

- General exact solution possesses a "tracker limit"

$$\bar{a} = \frac{\bar{a}_0 (3\bar{H}_0 t)^{2/3}}{2 + f_{v0}} \left[3 f_{v0} \bar{H}_0 t + (1 - f_{v0})(2 + f_{v0}) \right]^{1/3}$$

$$f_v = \frac{3 f_{v0} \bar{H}_0 t}{3 f_{v0} \bar{H}_0 t + (1 - f_{v0})(2 + f_{v0})} ,$$

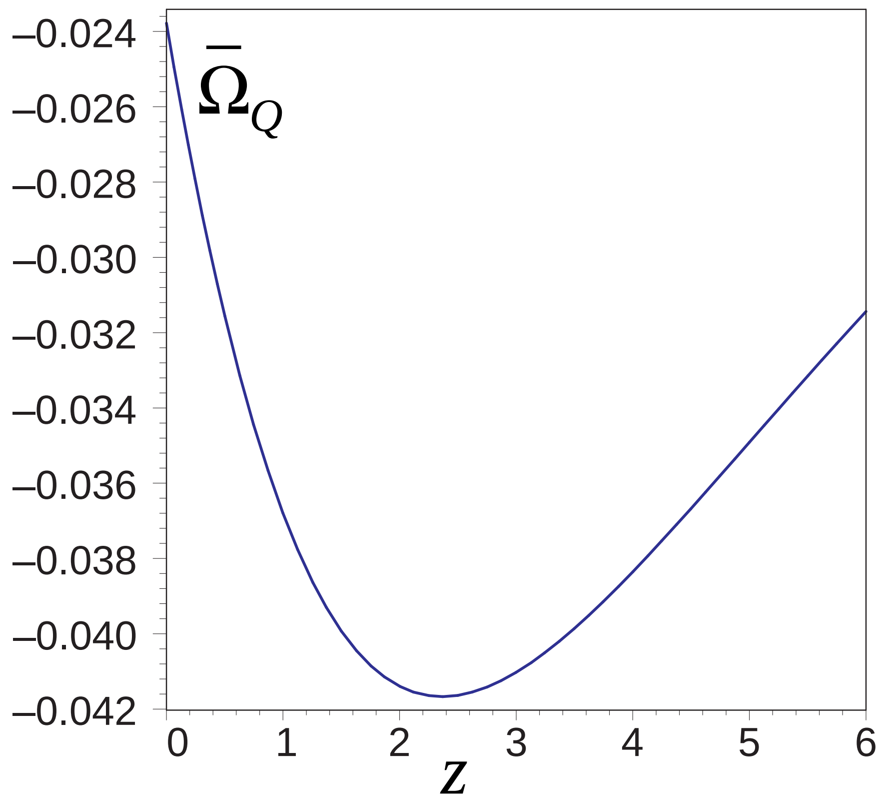
- Void fraction $f_v(t)$ determines many parameters:

$$\bar{\gamma}_w = 1 + \tfrac{1}{2} f_v = \tfrac{3}{2} \bar{H} t$$

$$\tau_w = \tfrac{2}{3} t + \frac{2(1 - f_{v0})(2 + f_{v0})}{27 f_{v0} \bar{H}_0} \ln \left(1 + \frac{9 f_{v0} \bar{H}_0 t}{2(1 - f_{v0})(2 + f_{v0})} \right)$$

$$H \equiv \frac{1}{a} \frac{\mathrm{d}a}{\mathrm{d}\tau_w} = \bar{\gamma}_w \bar{H} - \dot{\bar{\gamma}}_w = \bar{\gamma}_w \bar{H} - \bar{\gamma}_w^{-1} \tfrac{\mathrm{d}}{\mathrm{d}\tau_w} \bar{\gamma}_w$$

Magnitude of backreaction



- Magnitude of $\bar{\Omega}_{\mathcal{Q}}$ determines departure from FLRW evolution and variance of density of "statistically average" coarse–grained cells
- With realistic initial conditions $\bar{\Omega}_{\mathcal{Q}} \sim 4\%$ at most

Apparent cosmic acceleration

- Volume average observer sees no apparent cosmic acceleration

$$\bar{q} = \frac{2\,(1 - f_{\rm v})^2}{(2 + f_{\rm v})^2}\,.$$

As $t \to \infty$, $f_{\rm v} \to 1$ and $\bar{q} \to 0^+$.

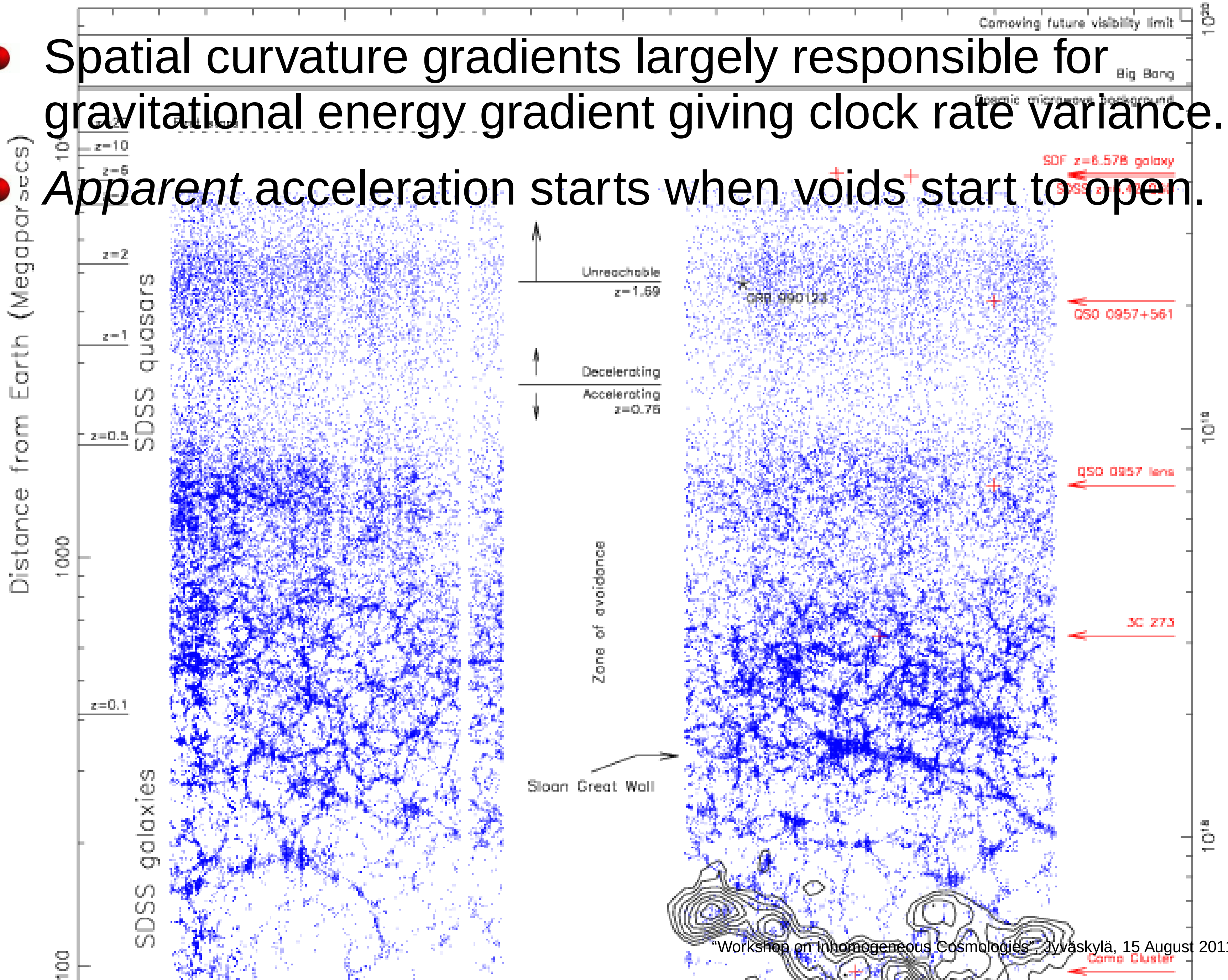
- A wall observer registers apparent cosmic acceleration

$$q = \frac{-\,(1 - f_{\rm v})\,(8 f_{\rm v}{}^3 + 39 f_{\rm v}{}^2 - 12 f_{\rm v} - 8)}{\left(4 + f_{\rm v} + 4 f_{\rm v}{}^2\right)^2}\,,$$

Effective deceleration parameter starts at $q \sim \frac{1}{2}$, for small $f_{\rm v}$; changes sign when $f_{\rm v} = 0.58670773\ldots$, and approaches $q \to 0^-$ at late times.

Cosmic coincidence problem solved

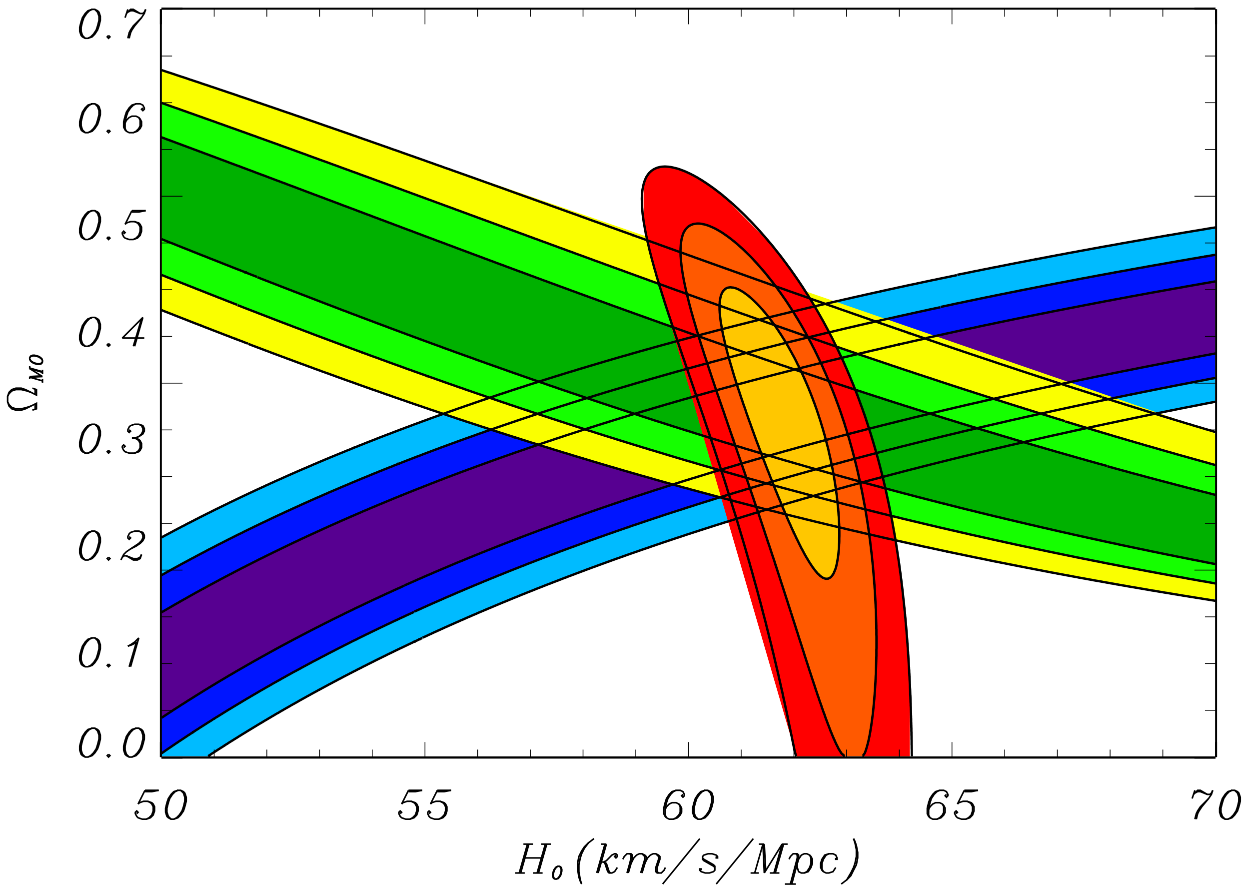
- Spatial curvature gradients largely responsible for gravitational energy gradient giving clock rate variance.
- *Apparent* acceleration starts when voids start to open.



Best fit parameters

- Hubble constant $H_0 + \Delta H_0 = 61.7^{+1.2}_{-1.1}$ km/s/Mpc
- present void volume fraction $f_{v0} = 0.76^{+0.12}_{-0.09}$
- bare density parameter $\bar{\Omega}_{M0} = 0.125^{+0.060}_{-0.069}$
- dressed density parameter $\Omega_{M0} = 0.33^{+0.11}_{-0.16}$
- non–baryonic dark matter / baryonic matter mass ratio $(\bar{\Omega}_{M0} - \bar{\Omega}_{B0})/\bar{\Omega}_{B0} = 3.1^{+2.5}_{-2.4}$
- bare Hubble constant $\bar{H}_0 = 48.2^{+2.0}_{-2.4}$ km/s/Mpc
- mean phenomenological lapse function $\bar{\gamma}_0 = 1.381^{+0.061}_{-0.046}$
- deceleration parameter $q_0 = -0.0428^{+0.0120}_{-0.0002}$
- wall age universe $\tau_0 = 14.7^{+0.7}_{-0.5}$ Gyr

Key observational tests



Best–fit parameters: $H_0 = 61.7^{+1.2}_{-1.1}$ km/s/Mpc, $\Omega_m = 0.33^{+0.11}_{-0.16}$ (1σ errors for SneIa only) [Leith, Ng & Wiltshire, ApJ 672 (2008) L91]

The value of H_0

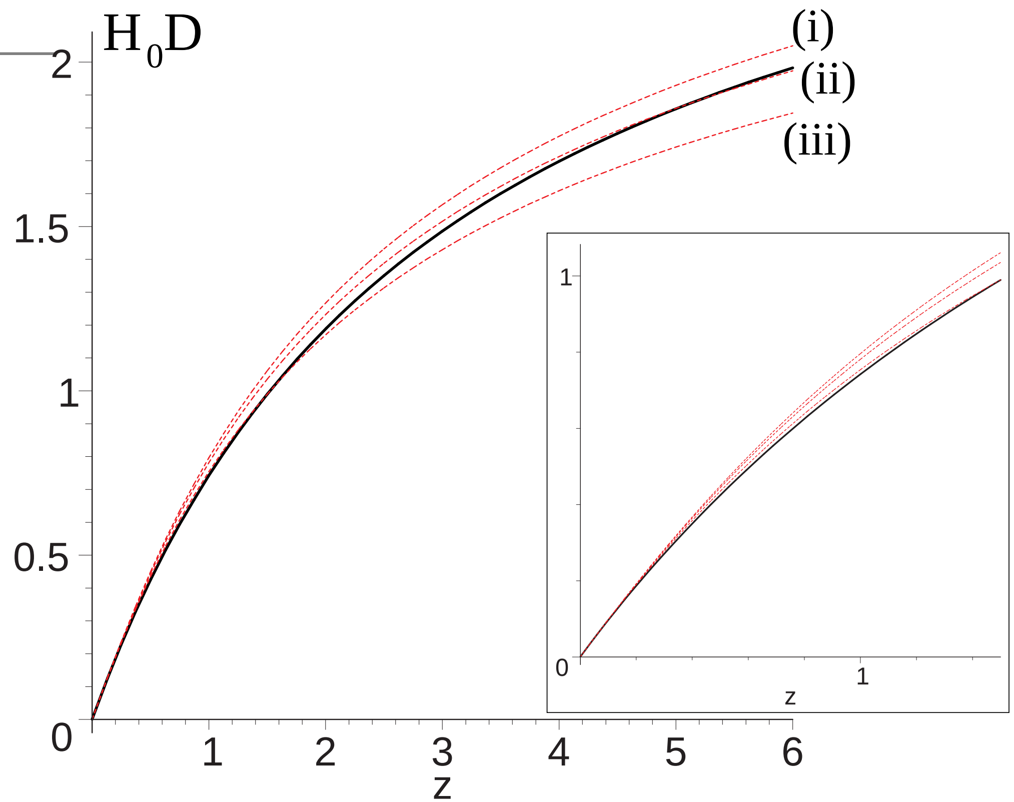
Value of $H_0 = 74.2 \pm 3.6$ km/s/Mpc of SH_0ES survey (Riess *et al*., 2009) calibrated by NGC4258 maser distance at 7.5 Mpc is a challenge for the timescape model. BUT

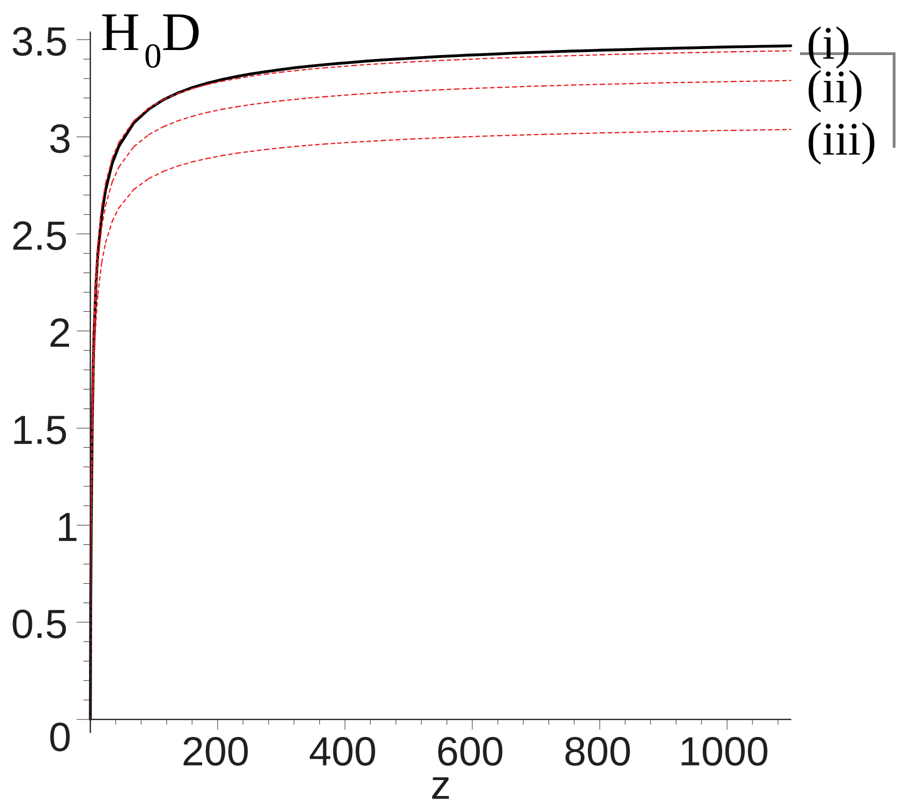
- Expect variance in Hubble flow below scale of homogeneity with typical higher value $H_{vw0} = 72.3$ km/s/Mpc at $30h^{-1}$ Mpc scale

H_0 determinations independent of local distance ladder:

- WiggleZ FLRW BAO value (Beutler et al, arXiv:1106.3366): $H_0 = 67 \pm 3.2$ km/s/Mpc
- Quasar strong lensing time delays; e.g., (Courbin et al, 1009.1473): $H_0 = 62^{+6}_{-4}$ km/s/Mpc
- Megamaser distance of UGC3789 $H_0 = 66.6 \pm 11.4$ km/s/Mpc, (69 ± 11 km/s/Mpc with“flow modeling”).

Dressed "comoving distance" $D(z)$

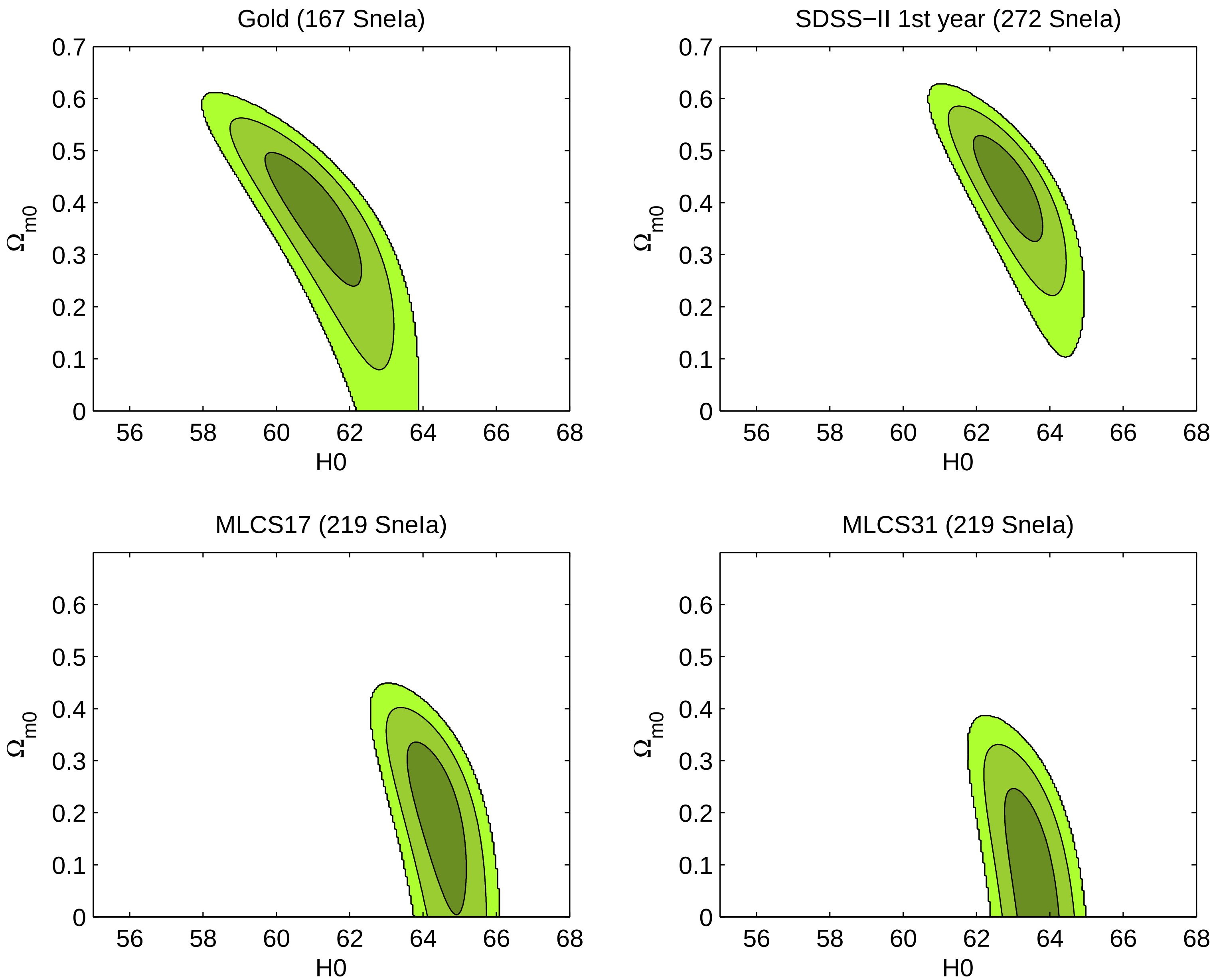




Best-fit timescape model **(red line)** compared to 3 spatially flat ΛCDM models: **(i)** best–fit to WMAP5 only ($\Omega_\Lambda = 0.75$); **(ii)** joint WMAP5 + BAO + SneIa fit ($\Omega_\Lambda = 0.72$); **(iii)** best flat fit to (Riess07) SneIa only ($\Omega_\Lambda = 0.66$).

- Three different tests with hints of tension with ΛCDM agree well with TS model.

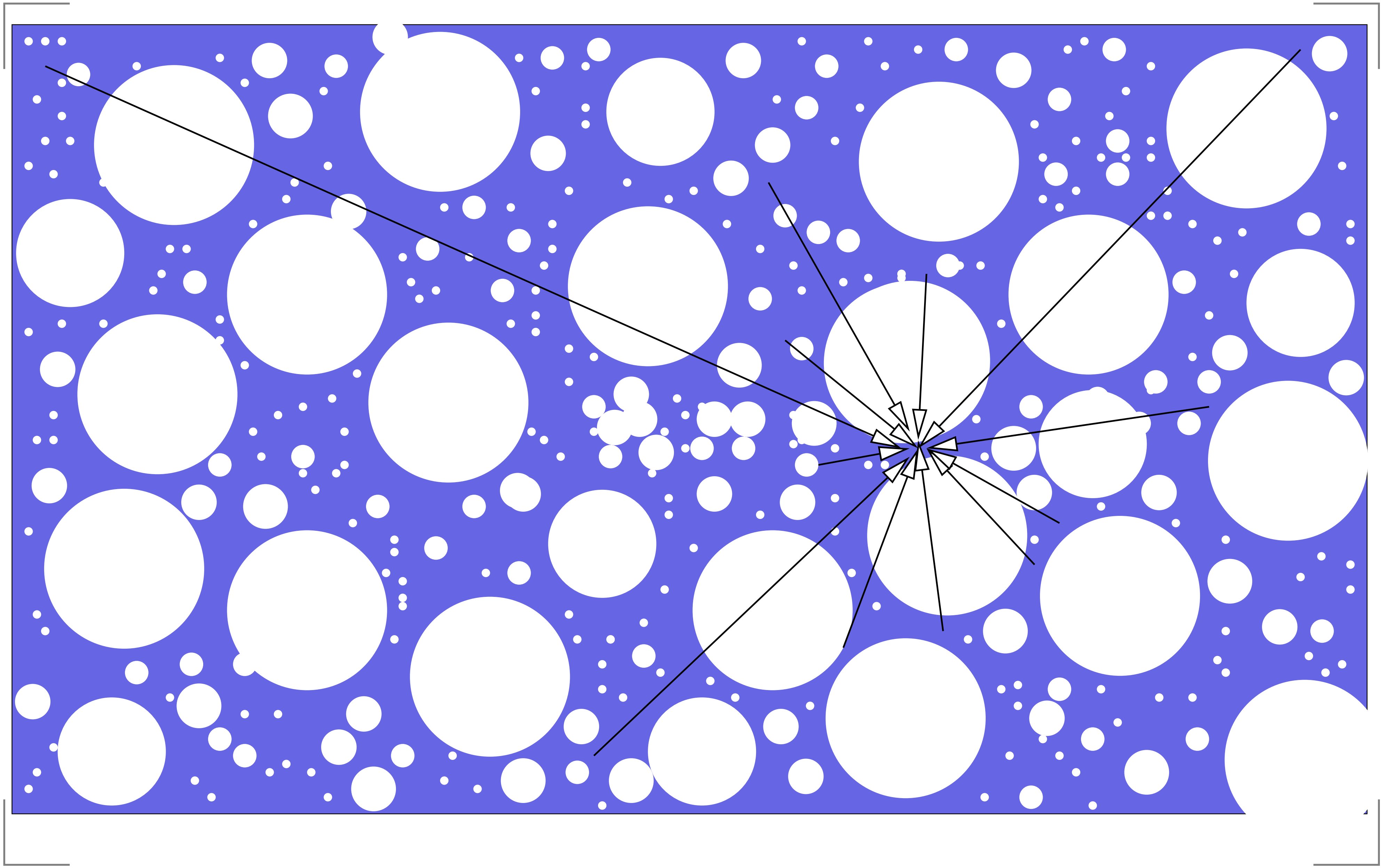
Supernovae systematics



Recent Sne Ia results; arXiv:1009.5855

- SALT/SALTII fits (Constitution,SALT2,Union2) favour ΛCDM over TS: $\ln B_{\mathrm{TS:\Lambda CDM}} = -1.06, -1.55, -3.46$
- MLCS2k2 (fits MLCS17,MLCS31,SDSS-II) favour TS over ΛCDM: $\ln B_{\mathrm{TS:\Lambda CDM}} = 1.37, 1.55, 0.53$
- Different MLCS fitters give different best-fit parameters; e.g. with cut at statistical homogeneity scale, for
  MLCS31 (Hicken et al 2009) $\Omega_{M0} = 0.12^{+0.12}_{-0.11}$;
  MLCS17 (Hicken et al 2009) $\Omega_{M0} = 0.19^{+0.14}_{-0.18}$;
  SDSS-II (Kessler et al 2009) $\Omega_{M0} = 0.42^{+0.10}_{-0.10}$
- Supernovae systematics (reddening/extinction, intrinsic colour variations) must be understood
- TS model most obviously consistent if dust in other galaxies not significantly different from Milky Way

Apparent Hubble flow variance

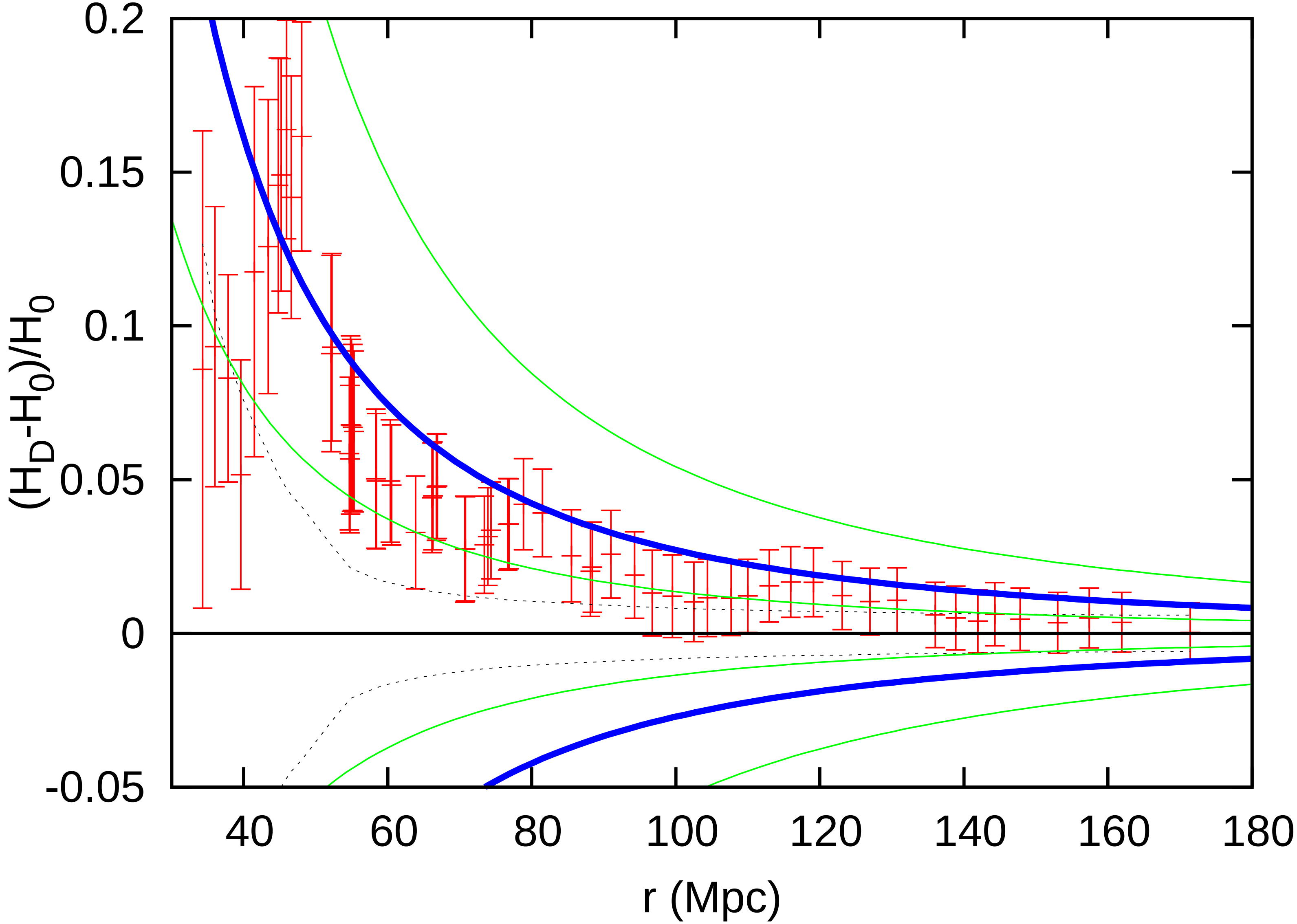


Apparent Hubble flow variance

- As voids occupy largest volume of space expect to measure higher average Hubble constant locally until the global average relative volumes of walls and voids are sampled at scale of homogeneity; thus expect maximum H_0 value for isotropic average on scale of dominant void diameter, $30h^{-1}$Mpc, then decreasing til levelling out by $100h^{-1}$Mpc.
- Consistent with a Hubble bubble feature (Jha, Riess, Kirshner ApJ 659, 122 (2007)); or "large scale flows" with certain characteristics (cf Watkins et al).
- Expected maximum "bulk dipole velocity"

$$v_{pec} = (\tfrac{3}{2}\bar{H}_0 - H_0)\frac{30}{h}\,\text{Mpc} = 510^{+210}_{-260}\,\text{km/s}$$

N. Li & D. Schwarz, arxiv:0710.5073v1–2



But is this GR?

- Apparent cosmic acceleration can be understood purely within general relativity; by (i) treating geometry of universe more realistically; (ii) understanding fundamental aspects of general relativity of statistical description of general relativity which have not been fully explored – *quasi–local gravitational energy*, of *gradients* in spatial curvature etc.
- Extra ingredients – regional averages etc – go beyond conventional general relativity
- Description of spacetime as a causal relational structure – retains principles consistent with GR
- Many details – averaging scheme etc – may change, but fundamental questions remain in any approach

Outlook

Other work

- Several observational tests (Alcock-Paczynski test, Clarkson, Basset and Lu test, redshift time drift etc) discussed in PRD 80 (2009) 123512

Work in progress

- Adapting Korzyński's "covariant coarse-graining" approach to more rigorously define regional averages (with James Duley)
- Analysis of variance of Hubble flow in style of Li and Schwarz on large datasets (with Peter Smale and Rick Watkins)
- Full analysis of CMB anisotropy spectrum in timescape model (with Ahsan Nazer)