The dependence of CMB anisotropies on void profile parameters in the local void model

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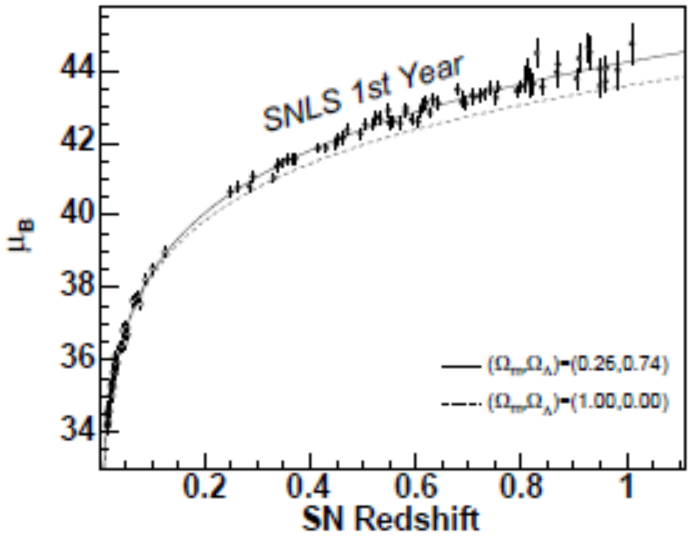
Reference: H. Kodama, K. Saito and A. Ishibashi,

Prog. Theor. Phys. **124**, 1 (2010), arXiv:1004.3089.

Introduction



P. Astier et al (2005)

Type Ia SN

→apparent accelerated expansion

⇩

- dark energy
- modified gravity
- local void model

Lemaitre-Tolman-Bondi (LTB) model

(spherically symmetric, dust dominant model)

G. Lemaitre (1933), R.C. Tolman (1934), H. Bondi (1947)

- metric $\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{\{R'(t,r)\}^2}{1-k(r)r^2}\mathrm{d}r^2 + R^2(t,r)\mathrm{d}\Omega^2$

$$\begin{cases} \dot{} \equiv \partial_t \\ ' \equiv \partial_r \end{cases}$$

- Einstein eq. $\left(\frac{\dot{R}}{R}\right)^2 = \frac{2GM(r)}{R^3} - \frac{k(r)r^2}{R^2} \qquad 4\pi\rho(t,r) = \frac{M'}{R^2R'}$

$$\begin{cases} k(r) > 0: \quad R(t,r) = \dfrac{M(r)}{k(r)r^2}(1-\cos\eta), \quad t - t_s(r) = \dfrac{M(r)}{\{k(r)r^2\}^{\frac{3}{2}}}(\eta - \sin\eta) \\ k(r) = 0: \quad R(t,r) = \left(\dfrac{9}{2}\right)^{\frac{1}{3}} M^{\frac{1}{3}}(r)\{t - t_s(r)\}^{\frac{2}{3}} \\ k(r) < 0: \quad R(t,r) = \dfrac{M(r)}{-k(r)r^2}(\cosh\eta - 1), \quad t - t_s(r) = \dfrac{M(r)}{\{-k(r)r^2\}^{\frac{3}{2}}}(\sinh\eta - \eta) \end{cases}$$

⇨ The solutions admit two arbitrary functions

$t_s(r),\ M(r)\ (\text{or } k(r))$

Big-bang time

Constraints from observations

- BAO → the size of void ～ Gpc
  （Baryon Acoustic Oscillation） S. Alexander et al. (2007)

- kSZ effect → the size of void ＜ 1.5Gpc
  （kinematic Sunyaev-Zeldovich） J. Garcia-Bellido, T. Haugbolle (2008)

- CMB dipole → **the distance from the observer to the center ＜ 15Mpc**
  ($a_{10} \sim 10^{-3}$)
  H. Alnes, M. Amarzguioui (2006)

we derive the analytic formulae for the CMB multipole.

・We have to test the local void model in addition to the m-z relation of SNIa.

・We need general methods which can be applied to any local void model.

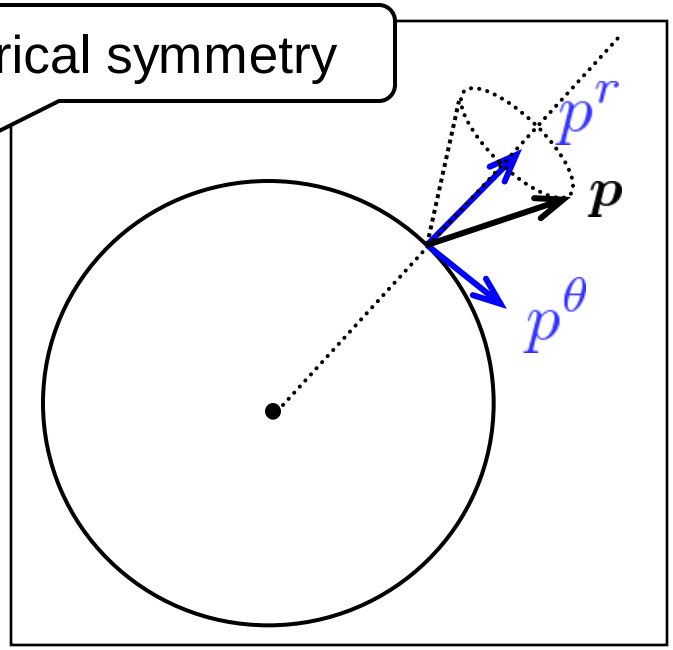
⇒ Here I derive analytic formulae for the CMB dipole and quadrupole by calculating **Taylor series for the distribution function of CMB photon at r=0** in the general local void model.

We test the dependence of CMB anisotropies on void profile parameters in local void models.

CMB temperature anisotropies

(1) CMB dipole

spherical symmetry



Distribution function of CMB photon

$$F(x, p, p^i) = F_0(t, r, \omega, \mu)$$

massless

$$\omega = p$$

$$\mu \equiv \frac{R'}{\sqrt{1-kr^2}} \frac{p^r}{p}$$

Taylor series at r=0 $\rightarrow$ $\delta F = \delta x^i (\partial_i F_0)_{t=t_0, r=0}$

the universe is locally in thermal equilibrium at LSS $\rightarrow$ $\delta F = -\frac{\delta T}{T} \omega \partial_\omega F_0$

present time

CMB dipole $\Longrightarrow$ $\frac{\delta T}{T} = -\frac{\delta x^i (\partial_i F_0)_{t=t_0, r=0}}{\omega \partial_\omega F_0}$

$$\partial_i F_0 = (\partial_i r)\partial_r F_0 + (\partial_i \omega)\partial_\omega F_0 + (\partial_i \mu)\partial_\mu F_0$$

We have to determine $\partial_\alpha F_0 \ (\alpha = r, \omega, \mu)$

Boltzmann equation

$$\frac{d}{dt}F_0 = \partial_t F_0 + \dot{r}\partial_r F_0 + \dot{\omega}\partial_\omega F_0 + \dot{\mu}\partial_\mu F_0 = 0$$

By differentiating this equation by α $\quad (\alpha = r, \omega, \mu)$

$$\partial_\alpha \left(\frac{d}{dt}F_0\right) = \left(\partial_\alpha \frac{d}{dt}\right) F_0 + \frac{d}{dt}(\partial_\alpha F_0) = 0$$

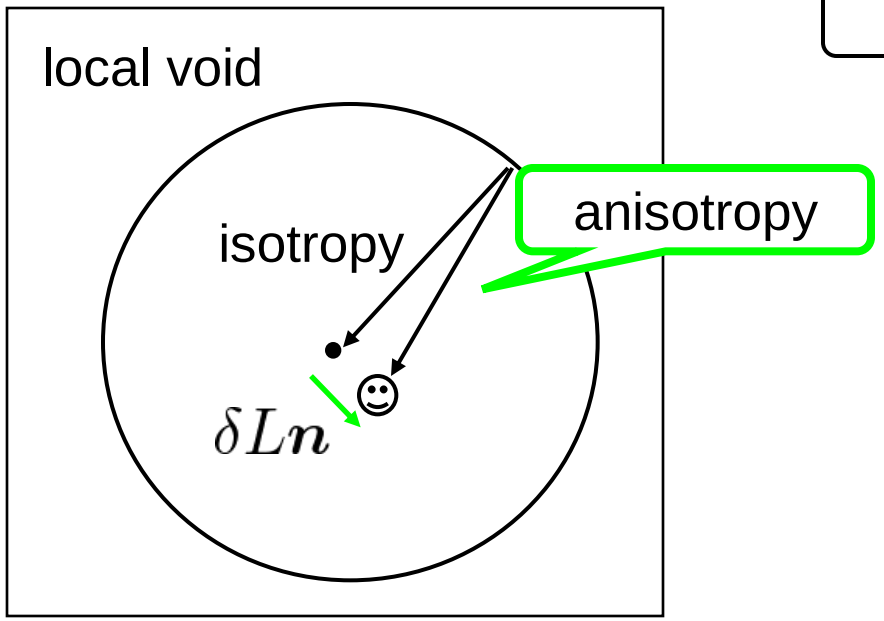
$$\frac{d}{dt}\left[\begin{pmatrix} \partial_\omega F_0 \\ \partial_r F_0 \\ \partial_\mu F_0 \end{pmatrix}\right] = -\begin{pmatrix} \partial_\omega \dot{\omega} & 0 & 0 \\ \partial_r \dot{\omega} & \partial_r \dot{r} & 0 \\ \partial_\mu \dot{\omega} & \partial_\mu \dot{r} & \partial_\mu \dot{\mu} \end{pmatrix} \begin{pmatrix} \partial_\omega F_0 \\ \partial_r F_0 \\ \partial_\mu F_0 \end{pmatrix}$$

We can easily solve this set of ODEs!!

CMB dipole formula

$$\left(\frac{\delta T}{T}\right)^{(1)} = \delta L \boldsymbol{n}\cdot\boldsymbol{\Omega}\left\{\frac{\xi_i}{a_0}e^{-\tilde{P}(t_0,t_i)}\left(\frac{\partial_r F_0}{\omega\partial_\omega F_0}\right)_i + \int_0^{r_i} dr H'_{//}\exp\left[\int_{t_0}^{t} dt_1 H_{//}(t_1)\right]\right\}$$

Initial condition



$$\Omega^i = \frac{x^i}{r} \qquad a_0 = (R')_0$$

$$\xi = \sqrt{1-kr^2} \qquad H_{//} \equiv \frac{\dot{R}'}{R'}$$

$$\tilde{P}(t_0,t) = \int_0^r dr_1 \left(\frac{R''}{R'}\right)_{r_1}$$

(2) CMB quadrupole formula

$$a_{20} = \frac{2}{3}\sqrt{\frac{\pi}{5}}(\delta L)^2 \left[\frac{2(f_2)_0}{\omega\partial_\omega F_0} - \int_{t_i}^{t_0} dt \frac{1}{(1+z)^2 S^2}\left\{H_\parallel'' + (1+z)\frac{S}{S''}\int_r^{r_i} dr \frac{H_\parallel}{1+z}\right\} + \frac{a_\perp''}{a_\perp} - \frac{S''}{a_0}\right]$$

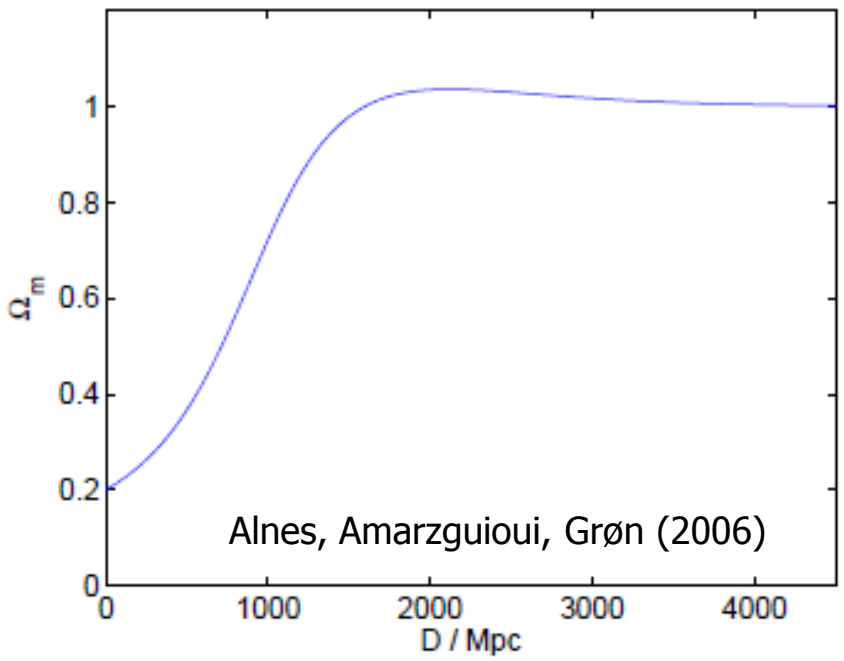
$$(f_2)_0 = \frac{a_0^2}{2} e^{-\tilde{Q}(t_0,t_i)}\left(\frac{\partial_\mu F_0}{R^2}\right)_i - \frac{a_0^2}{a_{\perp i}^2}\frac{e^{-\tilde{Q}(t_0,t_i)}}{2r_i}\left(-\partial_r F_0 + \frac{R'}{\xi}B\omega\partial_\omega F_0\right)_i$$
$$- \frac{a_0^2}{2}\int_{t_i}^{t_0}\frac{dt}{r}\left[-\left\{2\left(H_{//} - H_\perp + \frac{\xi}{R'}\frac{a_\perp'}{a_\perp}\right) + \left(\frac{\xi}{R'}\right)'\right\}\partial_r F_0 + \left\{-H_{//}' + 2H_{//}\frac{R'}{\xi}B + a_\perp^2\frac{d}{dt}\left(\frac{R'B}{a_\perp^2\xi}\right)\right\}(\omega\partial_\omega F_0)_i\right]\frac{e^{-\tilde{Q}(t_0,t)}}{a_\perp^2}$$

$$\partial_r F_0 = \frac{\xi_i}{\xi}e^{-\tilde{P}(t,t_i)}(\partial_r F_0)_i + \frac{(\omega\partial_\omega F_0)_i}{\xi}\int_{t_i}^{t} dt_1 \xi(t_1) e^{-\tilde{P}(t,t_1)} H_{//}(t_1)$$

$$S = \frac{R'}{\sqrt{1-kr^2}}$$

$$H_\perp \equiv \frac{\dot{R}}{R} \qquad B = -2(H_{//} - H_\perp) \qquad a_\perp = \frac{R}{r}$$

Constraints on local void models



density distribution at present time

$$M(r) = \frac{1}{2}H_\perp^2(t_0, r_{\rm out})r^3\left[1 - \Delta\alpha\left(\frac{1}{2} - \frac{1}{2}\tanh\frac{r - r_0}{2\Delta r}\right)\right]$$

$$k(r) = H_\perp^2(t_0, r_{\rm out})\Delta\beta\left(\frac{1}{2} - \frac{1}{2}\tanh\frac{r - r_0}{2\Delta r}\right)$$

$$t_s(r) = 0$$

$$H_\perp(t_0, r_{\rm out}) = 51\ {\rm km/s/Mpc}, \quad \Delta\alpha = 0.90,$$
$$r_0 = 1.34\ {\rm Gpc}, \quad \Delta r/r_0 = 0.4, \quad \Delta\beta = -\Delta\alpha = -0.90$$

$a_{10} < 2.52 \times 10^{-3}$ (G. Hinshaw et al (2008))

The distance from observers to the center $\delta L \lesssim 25\,{\rm Mpc}$

⟹ We can check the consistency with the numerical studies previously made (H. Alnes, M. Amarzguioui (2006))

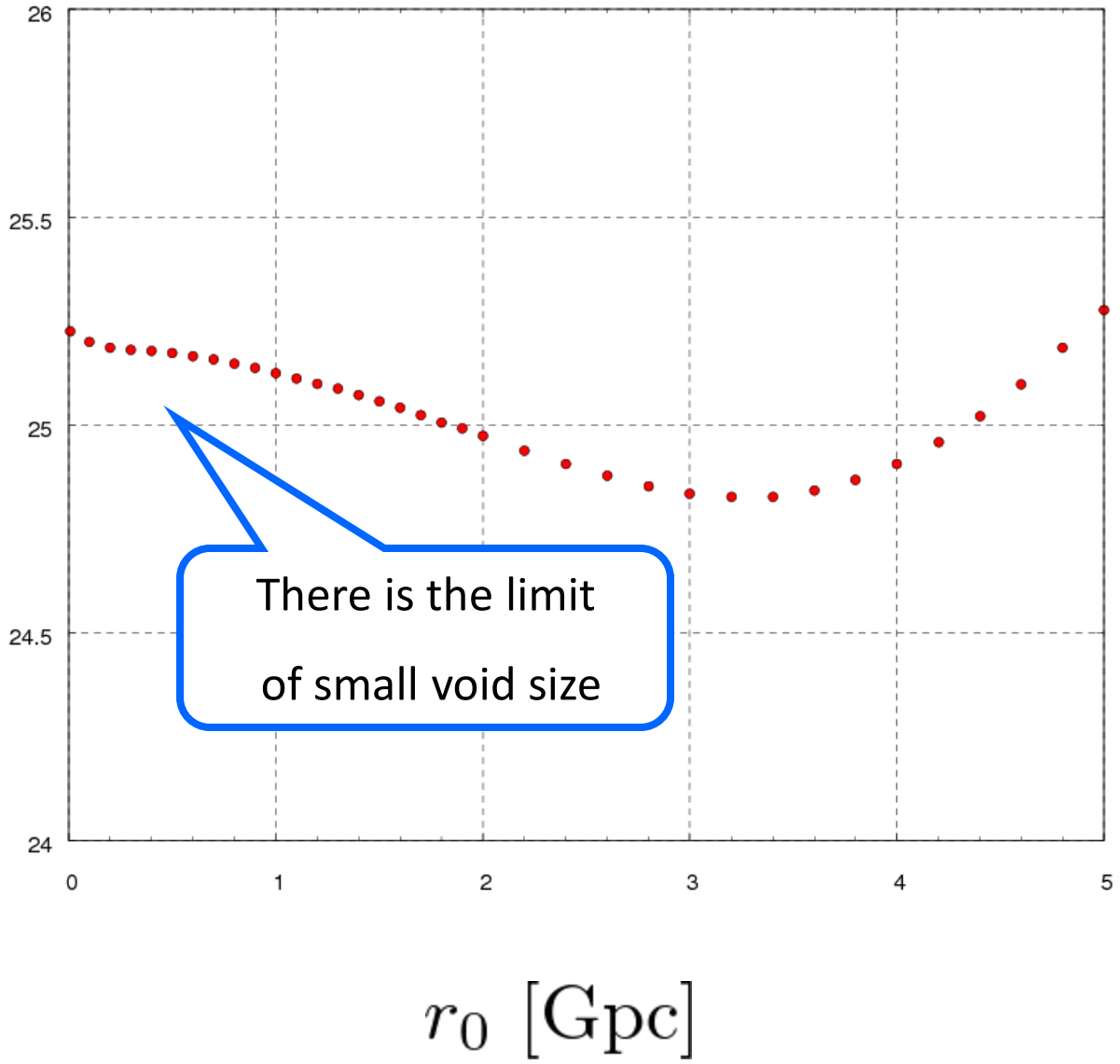
Dependence on void profile parameters

We test the dependence of CMB anisotropies on void profile parameters in local void models.

First, we consider the dependence on the void size r_0.

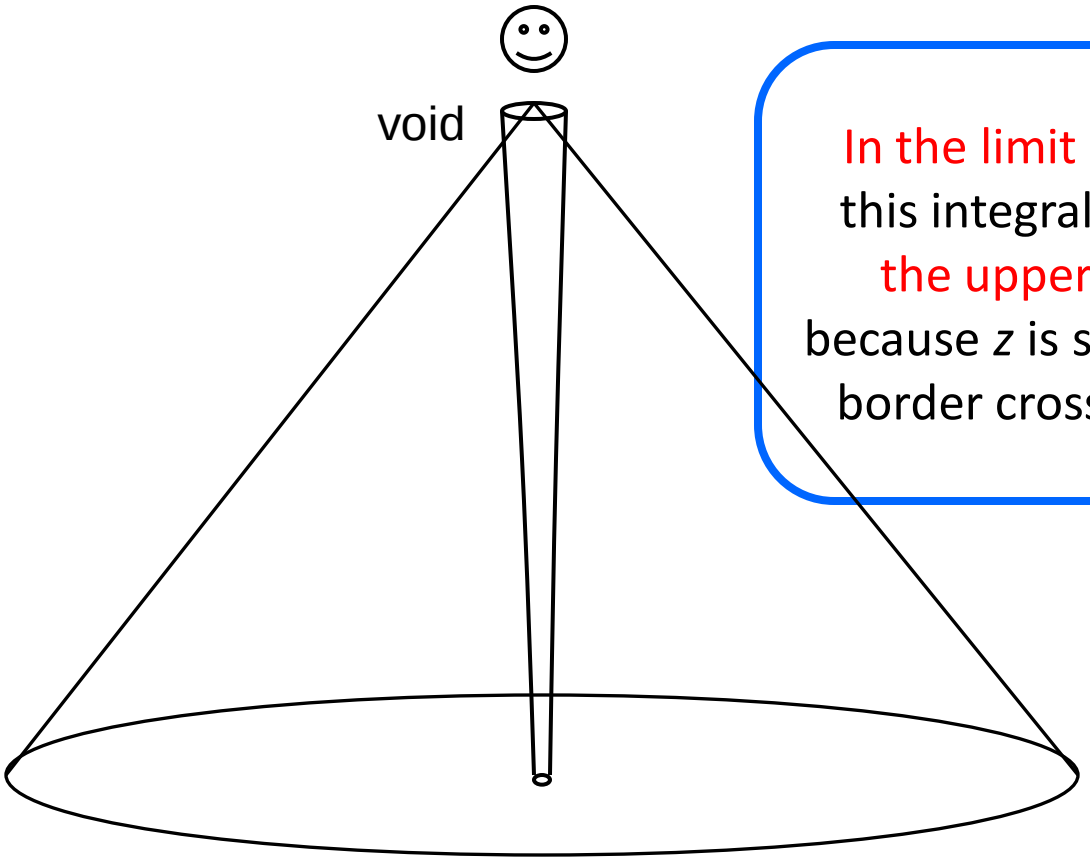
Modified AA model with varying void size r_0 and fixing $\Delta r / r_0$ (= 0.4)

($a_{10}=2.52\times10^{-3}$)



dipole : $a_{10} \sim \sqrt{\frac{4\pi}{3}}\delta L \int_0^{r_i} dr H'_{\parallel} \exp\left[\int_{t_0}^{t_i} dt H_{\parallel}\right]$

$= \sqrt{\frac{4\pi}{3}}\delta L \int_0^{z_i} dz \frac{1}{(1+z)^2} \frac{\sqrt{1-kr^2}}{\dot{R}'} H'_{\parallel}$

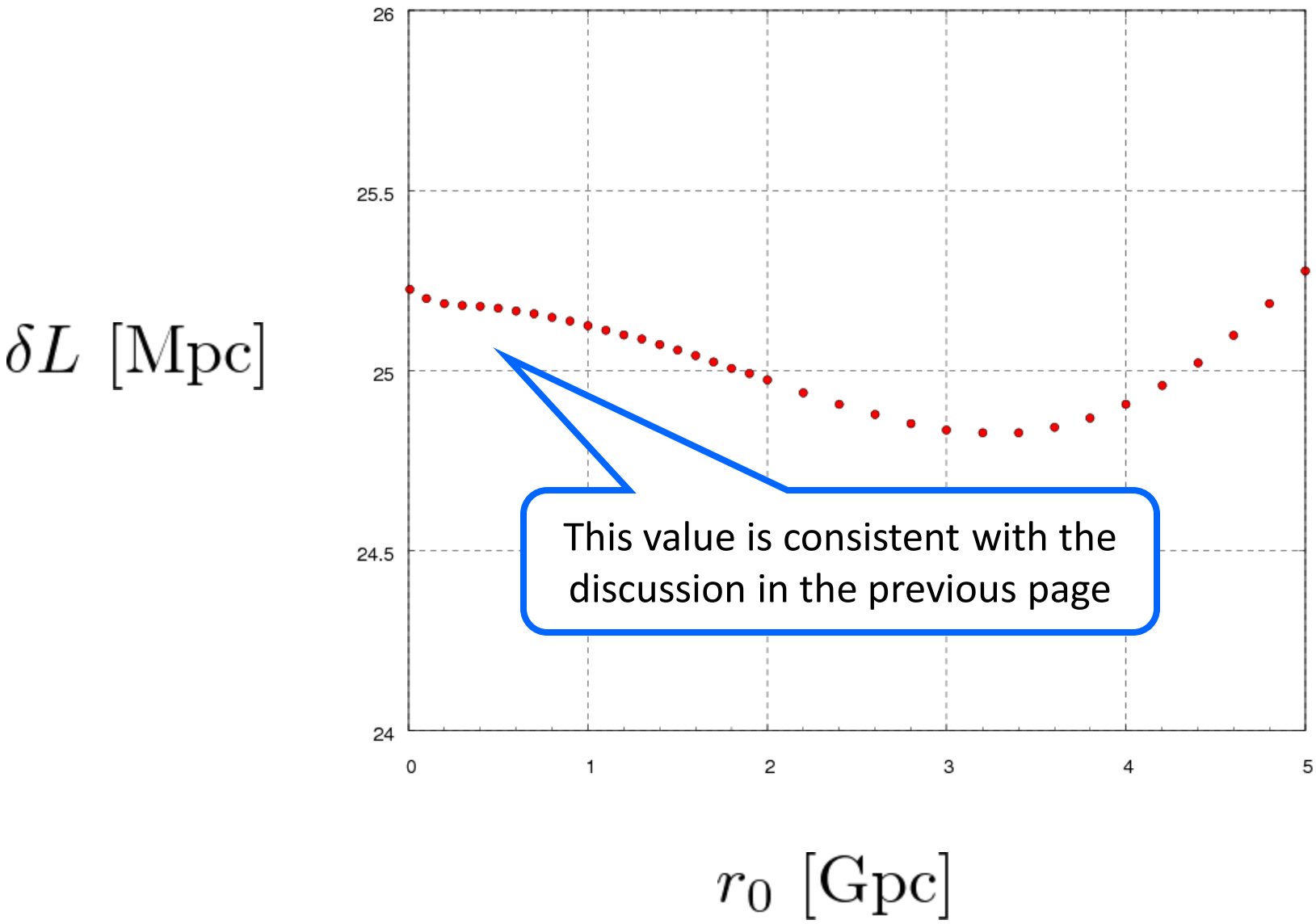


In the limit of small void size,
this integral is determined by
the upper and lower limit
because *z* is small when the void
border crosses the light cone.

$H_{\rm in} - H_{\rm out}$

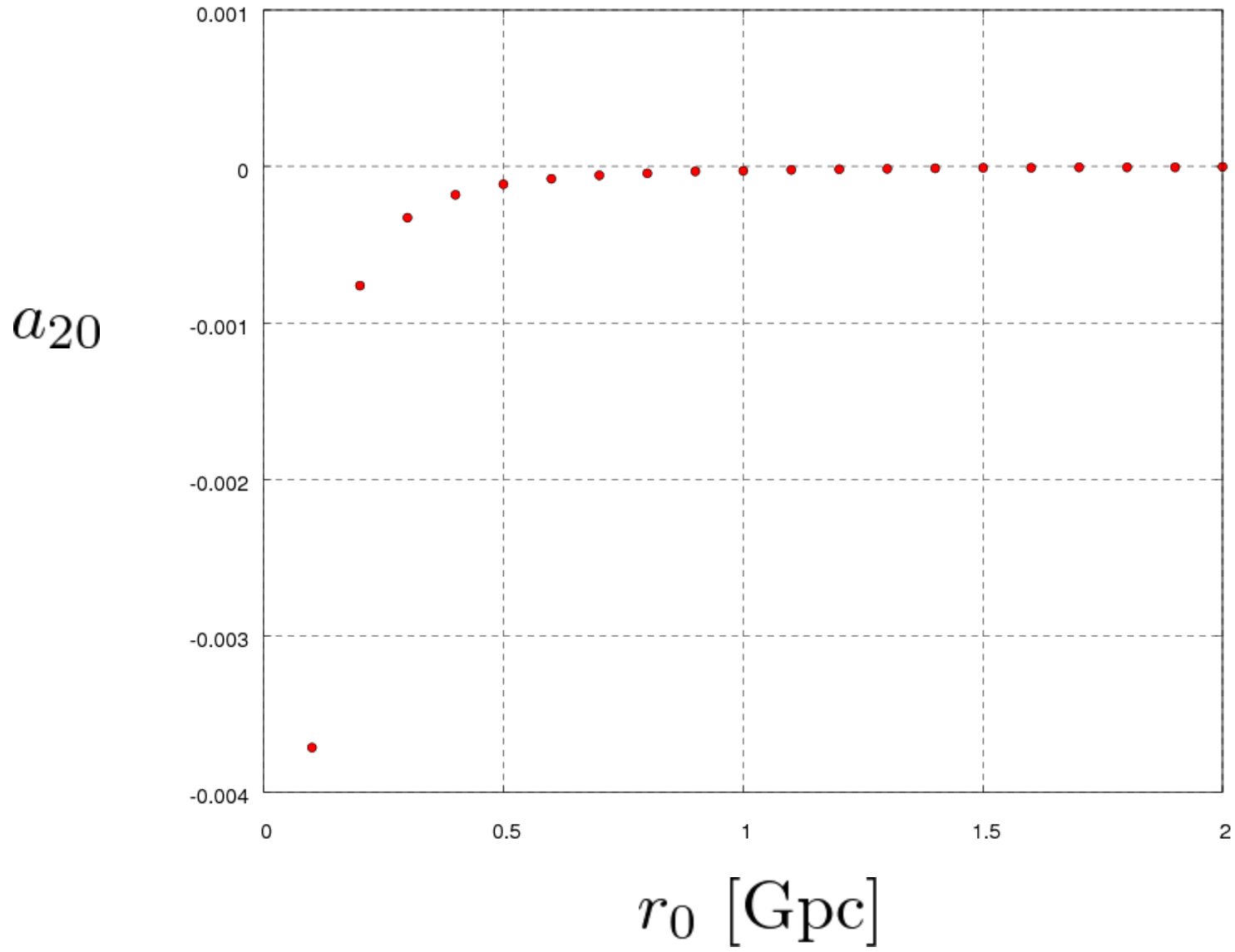
Modified AA model with varying void size r_0 and fixing $\Delta r / r_0$ (= 0.4)

($a_{10}=2.52\times10^{-3}$)



Modified AA model with varying void size r_0 and fixing $\Delta r / r_0$ (= 0.4)

($a_{10}=2.52\times10^{-3}$)



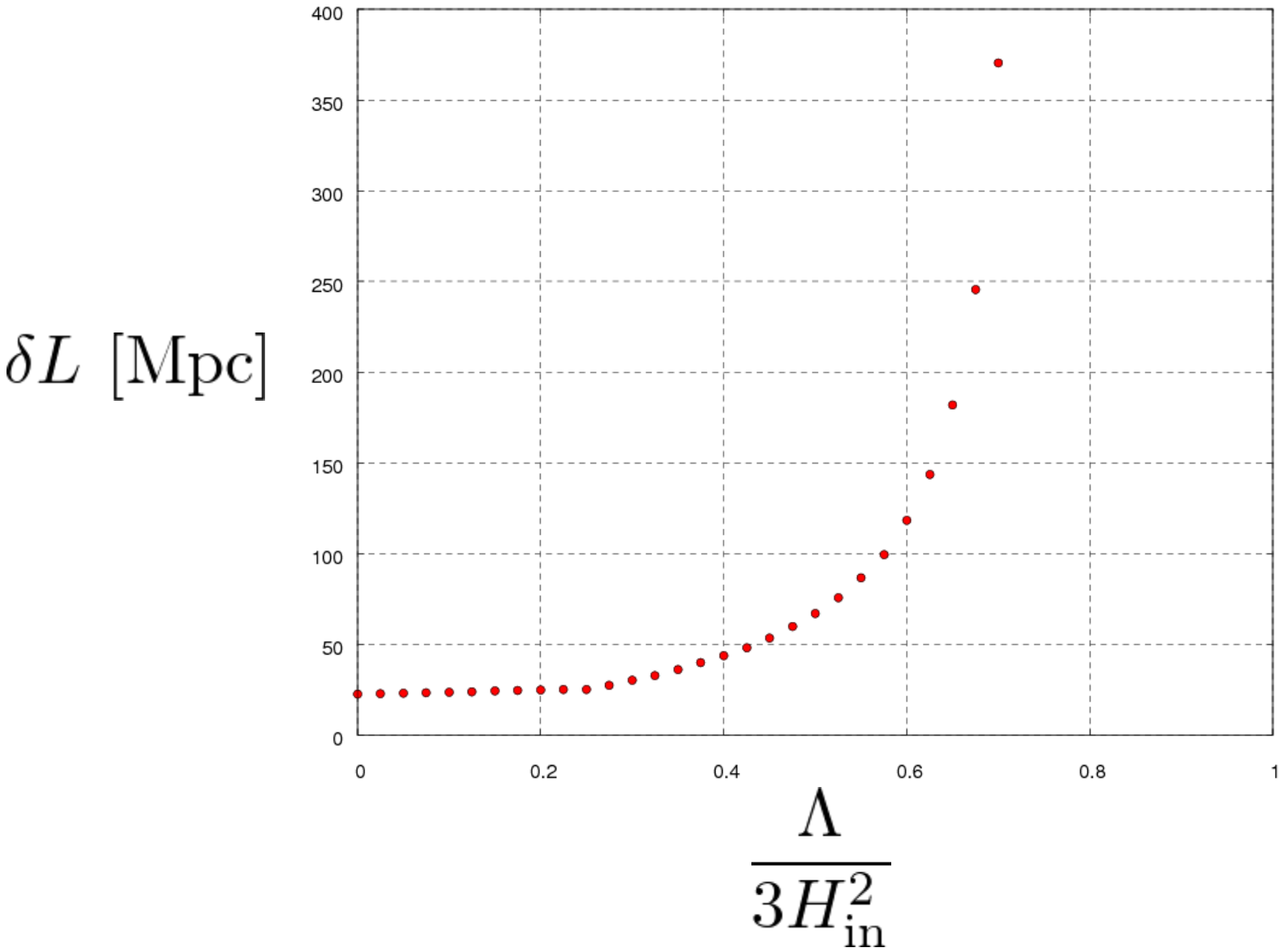
the local void model + Λ

We consider the ΛLTB model.

The cosmological constant Λ may or may not be exist, but there is the inhomogeneity in the universe.

Even if Λ is exist, we should consider the effect of inhomogeneity in order to estimate the real value of Λ.

Modified AA model with varying Λ and fitting density contrast Δα to 288 SNIa of SDSS II (Kessler et al. (2009)) ($a_{10}=2.52\times10^{-3}$)



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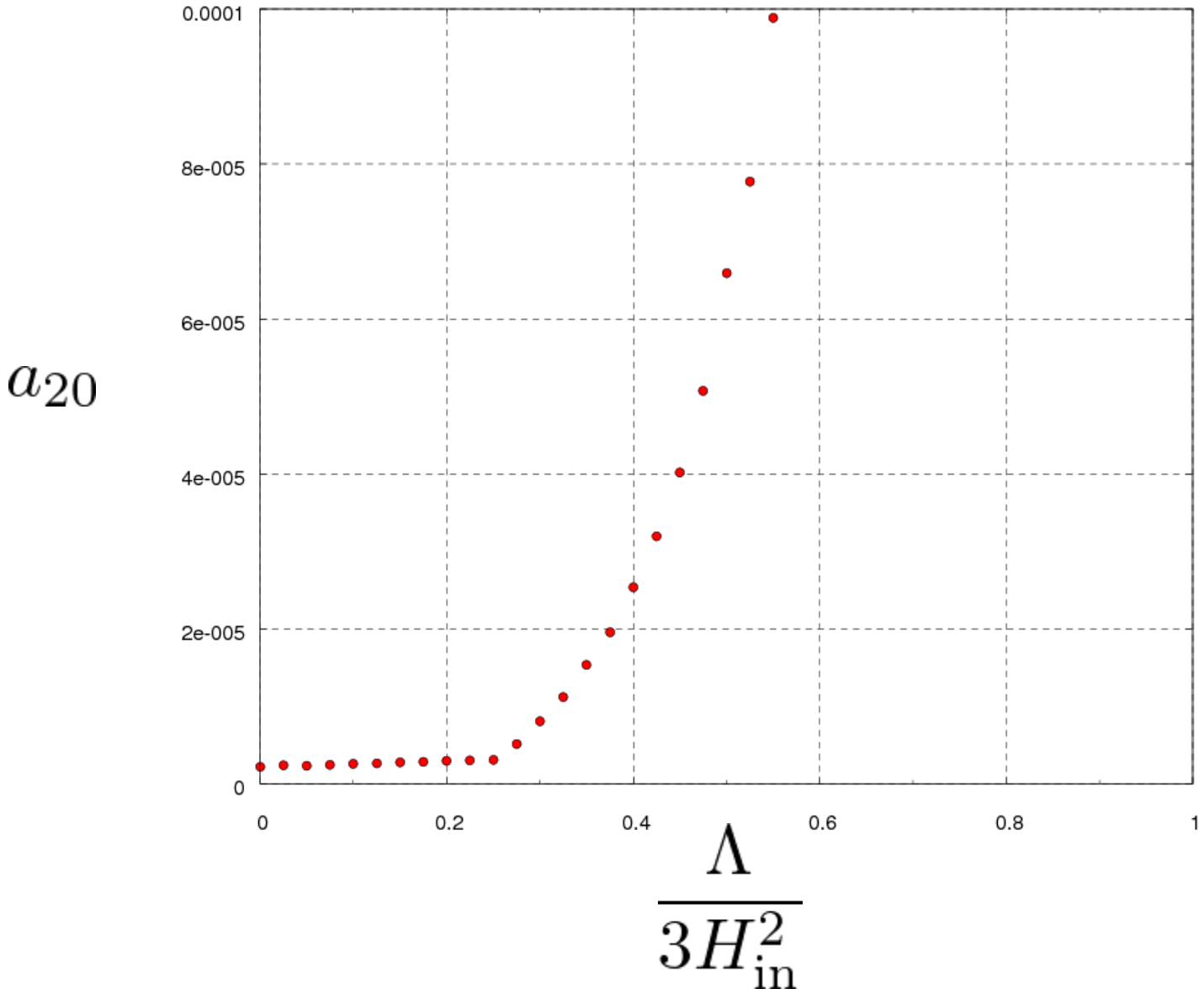

Summary

- We derived analytic formulae for the CMB dipole and quadrupole by calculating Taylor series for the distribution function of CMB photon at r=0 in the general local void model.

- We test the dependence on the void size r_0 and the cosmological constant Λ.