

# Generalizing $\Lambda$ CDM with Inhomogenous Dark Energy: Observational Constraints

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# Main Points

The consistency level of  $\Lambda$ CDM with geometrical data probes has been increasing with time during the last decade.

There are some puzzling conflicts between  $\Lambda$ CDM predictions and dynamical data probes

(bulk flows, alignment and magnitude of low CMB multipoles, alignment of quasar optical polarization vectors, cluster halo profiles)

Most of these puzzles are related to the existence of **preferred anisotropy axes** which appear to be surprisingly close to each other!

The simplest mechanism that can give rise to a **cosmological preferred axis** is based on an **off-center observer** in a spherical energy inhomogeneity (dark matter or dark energy)

# Geometric Probes: Recent SnIa Datasets

Dataset	Date Released	Redshift Range	# of SnIa	Filtered subsets included
SNLS1	2005	$0.015 \leq z \leq 1.01$	115	SNLS, LR
Gold06	2006	$0.024 \leq z \leq 1.76$	182	SNLS1, HST, SCP, HZSST
ESSENCE	2007	$0.016 \leq z \leq 1.76$	192	SNLS1, HST, ESSENCE,
Union	2008	$0.015 \leq z \leq 1.55$	307	Gold06, ESSENCE
Constitution	2009	$0.015 \leq z \leq 1.55$	397	Union, CfA3
SDSS	2009	$0.022 \leq z \leq 1.55$	288	Nearby, SDSS-II, ESSENCE, SNLS , HST
Union2	2010	$0.015 \leq z \leq 1.4$	557	Union, CfA3, SDSS-II

Q1: What is the Figure of Merit of each dataset?

Q2: What is the consistency of each dataset with  $\Lambda$ CDM?

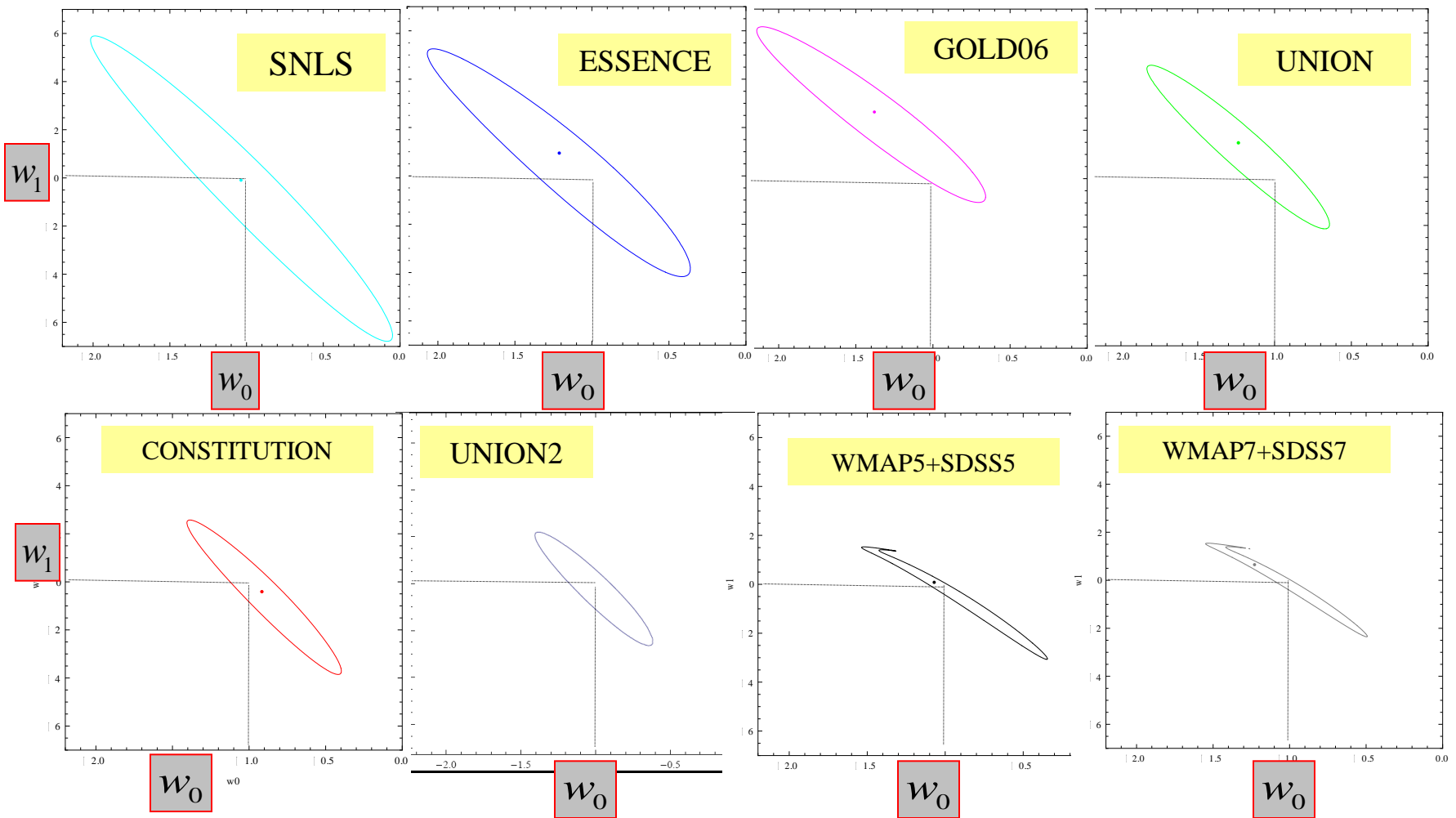
Q3: What is the consistency of each dataset with Standard Rulers?

$$\Omega_{0m} = 0.28$$

# Figures of Merit

$$w(z) = w_0 + w_1 \frac{z}{1+z}$$

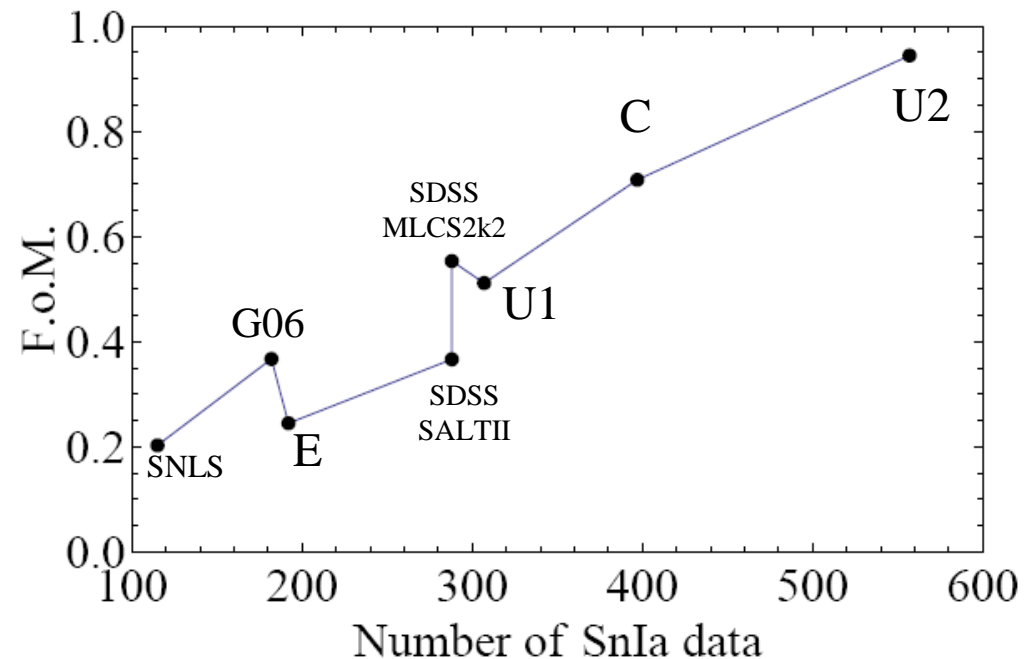
**The Figure of Merit:** Inverse area of the  $2\sigma$  CPL parameter contour.  
A measure of the effectiveness of the dataset in constraining the given parameters.



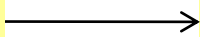
# Figures of Merit

**The Figure of Merit:** Inverse area of the  $2\sigma$  CPL parameter contour.  
A measure of the effectiveness of the dataset in constraining the given parameters.

Dataset	# of SnIa	Figure of Merit
SNLS1	115	0.208
Gold06	182	0.367
ESSENCE	192	0.245
SDSS-II (SALT2)	288	0.366
SDSS-II (MLCS2k2)	288	0.553
Union	307	0.512
Constitution (SALT2)	397	0.708
Union2	557	0.95
CMB+SDSS5	-	2.028
CMB+SDSS7	-	2.541



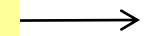
**SDSS5**



**Percival et. al.**

[arXiv:0705.3323](https://arxiv.org/abs/0705.3323)

**SDSS7**



**Percival et. al.**

[arXiv:0907.1660](https://arxiv.org/abs/0907.1660)

# Approaches in Cosmology Research

## A1:

- a. Focus on majority of data which are consistent with  $\Lambda$ CDM.
- b. Assume validity of  $\Lambda$ CDM and constrain standard model parameters with best possible accuracy

## A2:

- a. Focus on Theoretical Motivation and construct more general models.
- b. Use data to constrain the larger space of parameters.

## A3:

- a. Focus on minority of data that are inconsistent with  $\Lambda$ CDM at a level more than  $2-3\sigma$ .
- b. Identify common features of these data and construct theoretical models consistent with these features.
- c. Make non-trivial predictions using these models.

# Puzzles for $\Lambda$ CDM

From LP, 0811.4684,  
I. Antoniou, LP, **JCAP 1012:012,**  
**2010**, arxiv:1007.4347

## Large Scale Velocity Flows

R. Watkins et. al. , **Mon.Not.Roy.Astron.Soc.392:743-756,2009**, 0809.4041.  
A. Kashlinsky et. al. **Astrophys.J.686:L49-L52,2009** arXiv:0809.3734

- **Predicted:** On scale larger than  $50 h^{-1}\text{Mpc}$  Dipole Flows of  $110\text{km/sec}$  or less.
- **Observed:** Dipole Flows of more than  $400\text{km/sec}$  on scales  $50 h^{-1}\text{Mpc}$  or larger.
- **Probability of Consistency:** **1%**

## Alignment of Low CMB Spectrum Multipoles

M. Tegmark et. al., **PRD 68, 123523 (2003)**,  
Copi et. al. **Adv.Astron.2010:847541,2010**.

- **Predicted:** Orientations of coordinate systems that maximize planarity ( $|a_n|^2 + |a_{l-n}|^2$ ) of CMB maps should be independent of the multipole  $l$ .
- **Observed:** Orientations of  $l=2$  and  $l=3$  systems are unlikely close to each other.
- **Probability of Consistency:** **1%**

## Large Scale Alignment of QSO Optical Polarization Data

D. Hutsemekers et. al.. **AAS, 441,915**  
(2005), astro-ph/0507274

- **Predicted:** Optical Polarization of QSOs should be randomly oriented
- **Observed:** Optical polarization vectors are aligned over  $1\text{Gpc}$  scale along a preferred axis.
- **Probability of Consistency:** **1%**

## Cluster and Galaxy Halo Profiles:

Broadhurst et. al. ,ApJ 685, L5, 2008, 0805.2617,  
S. Basilakos, J.C. Bueno Sanchez, LP, 0908.1333, PRD, 80, 043530, 2009.

- **Predicted:** Shallow, low-concentration mass profiles ( $c_{\text{vir}} \sim 4-5$ )
- **Observed:** Highly concentrated, dense halos ( $c_{\text{vir}} \sim 10-15$ )
- **Probability of Consistency:** **3-5%**

# Proximity of Axes Directions

I. Antoniou, LP,  
JCAP 1012:012, 2010,  
arxiv: 1007.4347

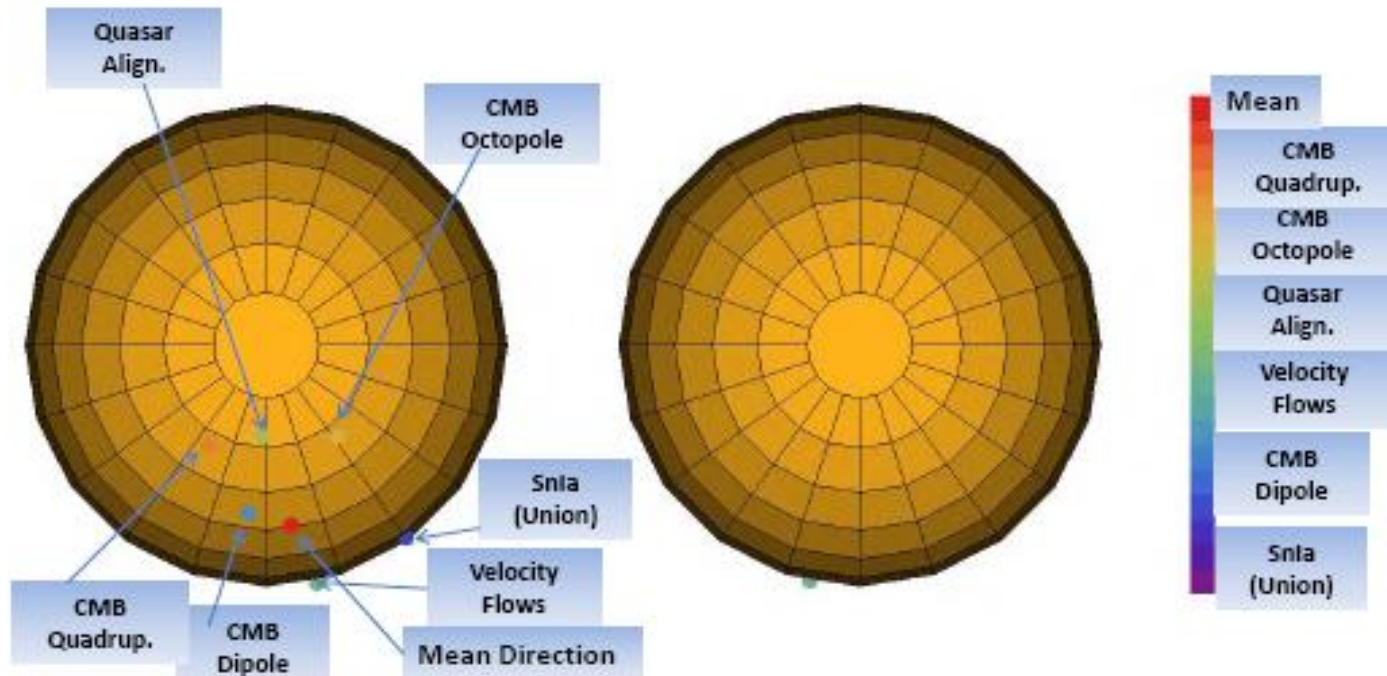
**Q:** What is the probability that these independent axes lie so close in the sky?

**Calculate:**

$$\langle |\cos\theta_{ij}| \rangle = \langle |\hat{r}_i \cdot \hat{r}_j| \rangle = \sum_{i,j=1, j \neq i}^N \frac{|\hat{r}_i \cdot \hat{r}_j|}{N(N-1)}$$

**Compare 6 real directions  
with 6 random directions**

Cosmological Obs.	$l$	$b$
SnIa Union2	306°	15°
CMB Dipole	264°	48°
Velocity Flows	282°	6°
Quasar Alignment	267°	69°
CMB Octopole	308°	63°
CMB Quadrupole	240°	63°
Mean	$277^\circ \pm 26^\circ$	$44^\circ \pm 27^\circ$





# Proximity of Axes Directions

**Q:** What is the probability that these independent axes lie so close in the sky?

**Calculate:**

$$\langle |\cos\theta_{ij}| \rangle = \langle |\hat{r}_i \cdot \hat{r}_j| \rangle = \sum_{i,j=1, j \neq i}^N \frac{|\hat{r}_i \cdot \hat{r}_j|}{N(N-1)}$$

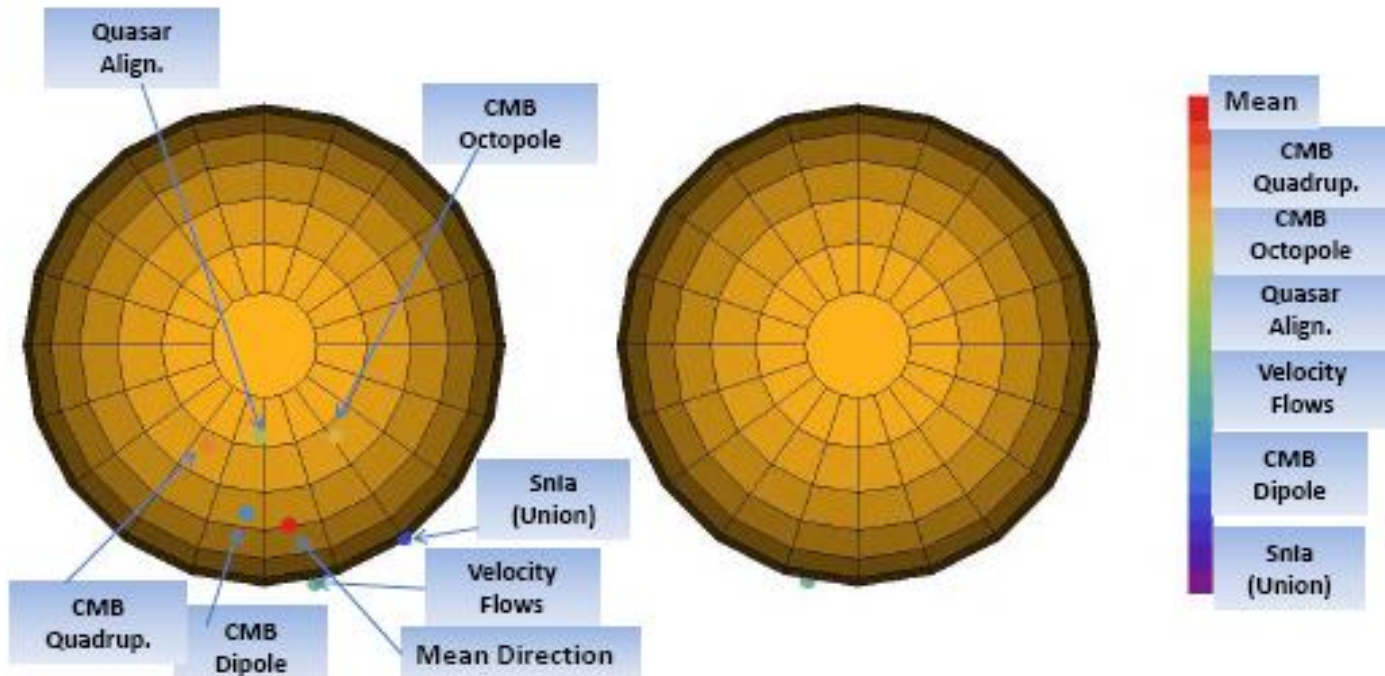
**Compare 6 real directions  
with 6 random directions**

**Simulated 6 Random Directions:**

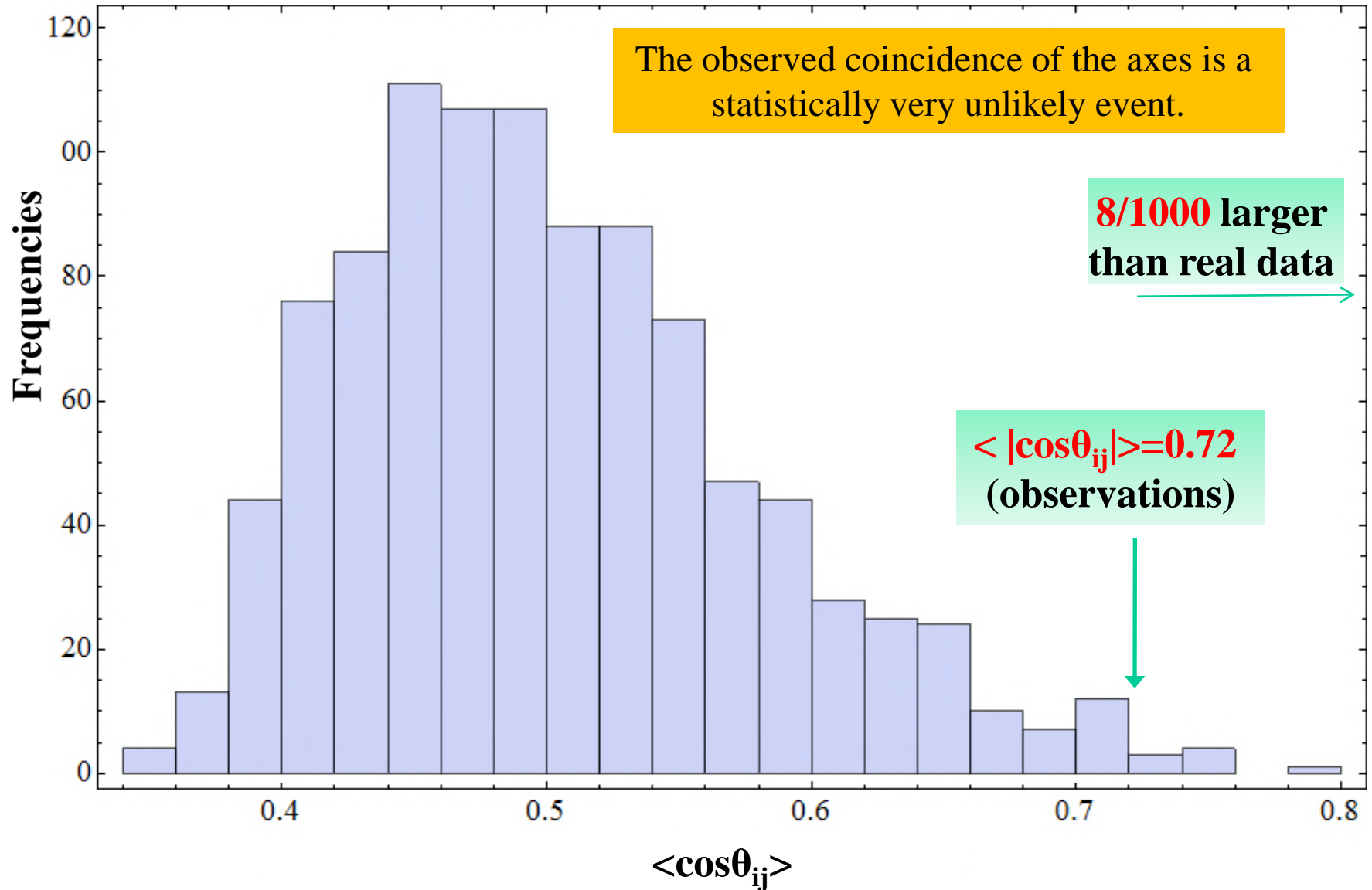
$$\langle |\cos\theta_{ij}| \rangle = 0.5 \pm 0.072$$

**6 Real Directions ( $3\sigma$  away from mean value):**

$$\langle |\cos\theta_{ij}| \rangle = 0.72$$



# Distribution of Mean Inner Product of Six Preferred Directions (CMB included)

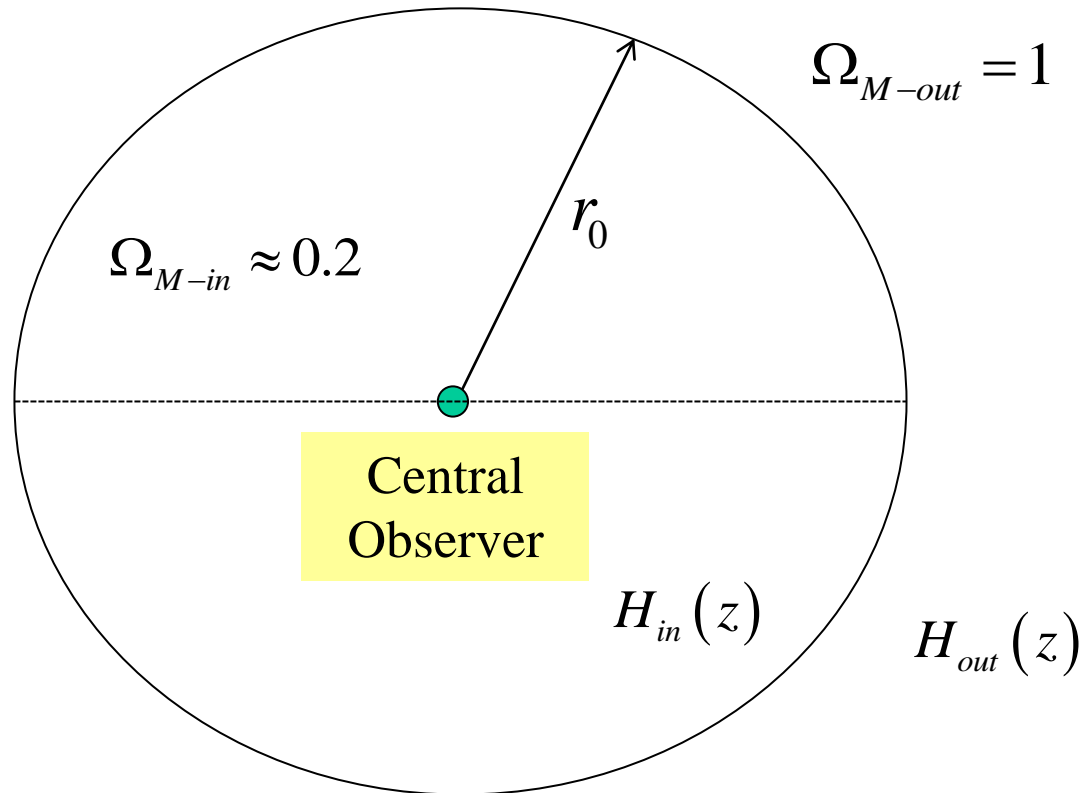


# Models Predicting a Preferred Axis

- Anisotropic dark energy equation of state (eg vector fields)  
(T. Koivisto and D. Mota (2006), R. Battye and A. Moss (2009))
- Fundamentally Modified Cosmic Topology or Geometry (rotating universe, horizon scale compact dimension, non-commutative geometry etc)  
(J. P. Luminet (2008), P. Bielewicz and A. Riazuelo (2008), E. Akofor, A. P. Balachandran, S. G. Jo, A. Joseph, B. A. Qureshi (2008), T. S. Koivisto, D. F. Mota, M. Quartin and T. G. Zlosnik (2010))
- Statistically Anisotropic Primordial Perturbations (eg vector field inflation)  
(A. R. Pullen and M. Kamionkowski (2007), L. Ackerman, S. M. Carroll and M. B. Wise (2007), K. Dimopoulos, M. Karciauskas, D. H. Lyth and Y. Rodriguez (2009))
- Horizon Scale Primordial Magnetic Field.  
(T. Kahniashvili, G. Lavrelashvili and B. Ratra (2008), L. Campanelli (2009), J. Kim and P. Naselsky (2009))
- **Horizon Scale Dark Matter or Dark Energy Perturbations (eg few Gpc void)**  
(J. Garcia-Bellido and T. Haugboelle (2008), P. Dunsby, N. Goheer, B. Osano and J. P. Uzan (2010), T. Biswas, A. Notari and W. Valkenburg (2010))

# Simplest Model: Lematre-Tolman-Bondi

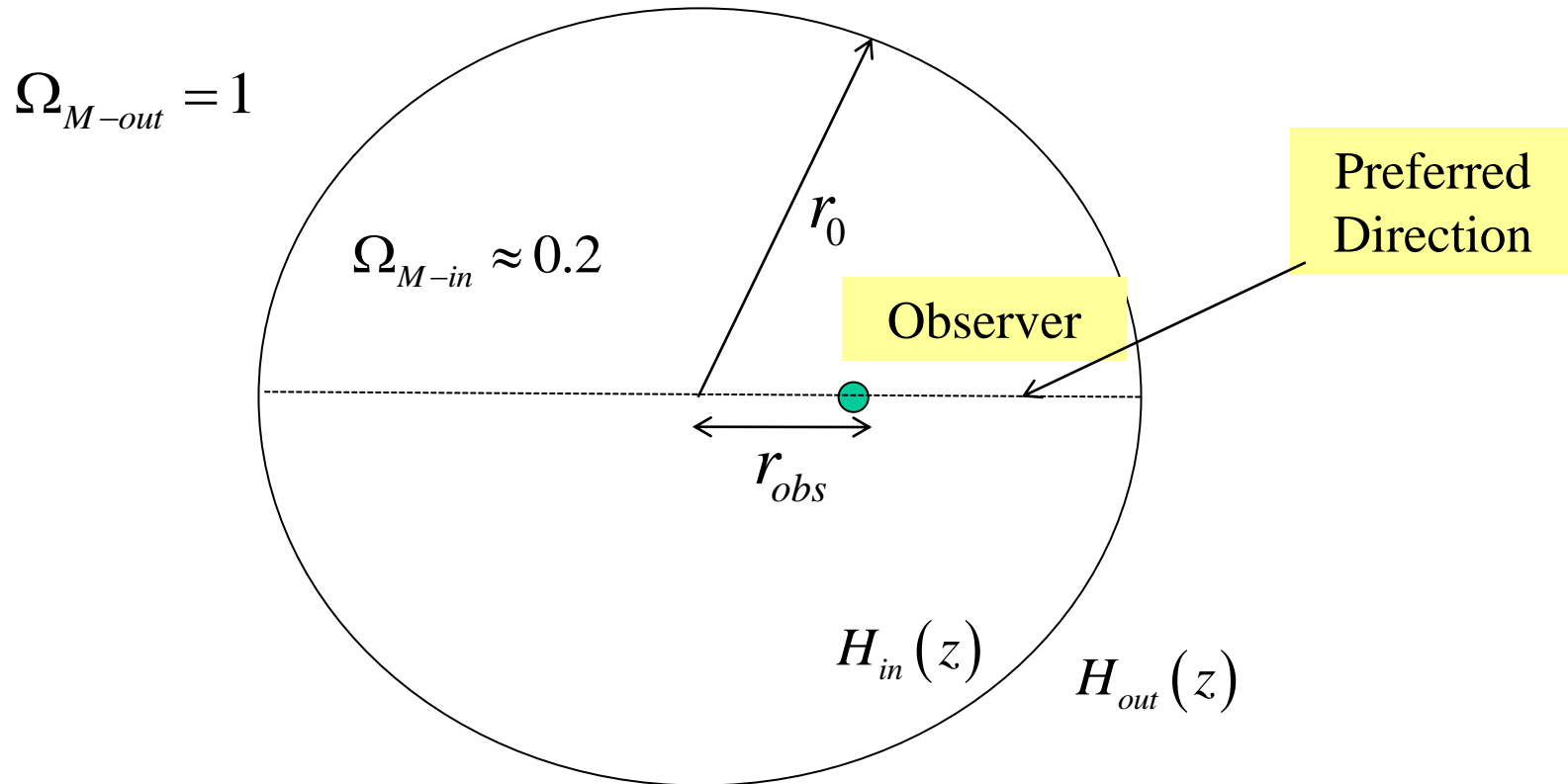
Local spherical underdensity of matter (Void), no dark energy



Faster expansion rate at low redshifts  
(local space equivalent to recent times)

# Shifted Observer: Preferred Direction

Local spherical underdensity of matter (Void)



Faster expansion rate at low redshifts  
(local space equivalent to recent times)

# Constraints: Union2 Data – Central Observer

Metric:

FRW limit:

$$A(r, t) = r a(t), \quad k(r) = k$$

$$ds^2 = -dt^2 + \frac{(A'(r, t))^2}{1 - k(r)} dr^2 + A^2(r, t) (d\theta^2 + \sin^2 \theta d\varphi^2)$$

Cosmological Equation:

$$H^2(r, t) = H_0^2(r) \left[ \Omega_M(r) \left( \frac{A_0}{A} \right)^3 + \Omega_X(r) + \Omega_c(r) \left( \frac{A_0}{A} \right)^2 \right]$$

$$1 - \Omega_M(r) = -\frac{k(r)}{H_0(r)^2 A(r, t_0)^2}$$

$$H(r, t) \equiv \frac{\dot{A}(r, t)}{A(r, t)}$$

$$\Omega_M(r) = \Omega_{M, \text{out}} + (\Omega_{M, \text{in}} - \Omega_{M, \text{out}}) \frac{1 - \tanh([r - r_0]/2\Delta r)}{1 + \tanh(r_0/2\Delta r)}$$

$$H_0(r) = \frac{1}{t_0} \int_0^1 \frac{dx}{\sqrt{\Omega_M(r)x^{-1} + \Omega_X(r)x^2 + \Omega_c(r)}}$$

Geodesics:

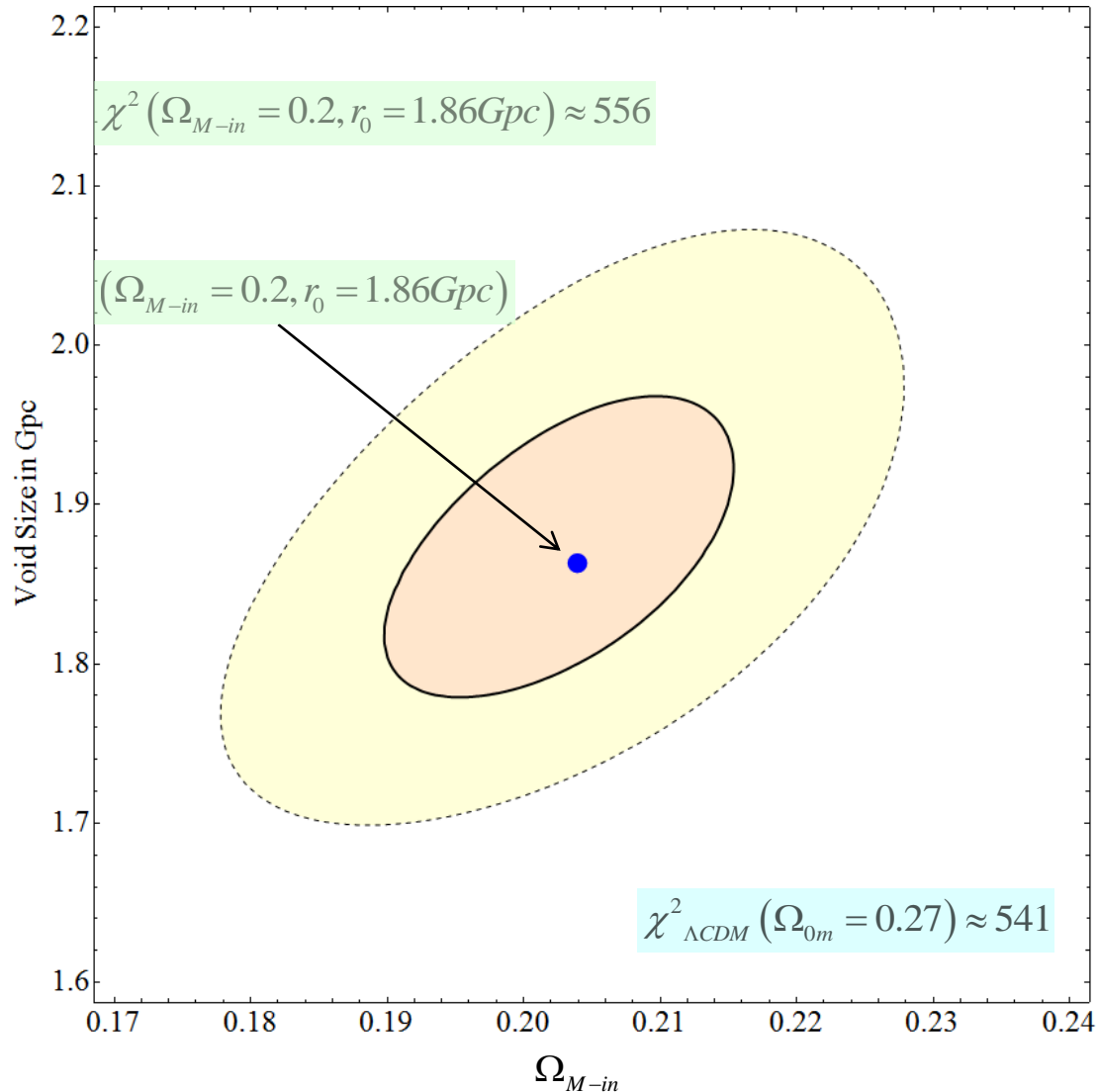
$$\frac{dt}{dz} = -\frac{A'(r, t)}{(1 + z)\dot{A}'(r, t)}$$

$$\frac{dr}{dz} = \frac{c\sqrt{1 - k(r)}}{(1 + z)\dot{A}'(r, t)}$$

Luminosity Distance:

$$d_L(z, r_0, \Omega_{M, \text{in}}) = (1 + z)^2 A(r(z), t(z))$$

# Constraints: Union2 Data – Central Observer



## Advantages:

1. No need for dark energy.
2. Natural Preferred Axis.
3. Coincidence Problem

## Problems:

1. No simple mechanism to create such large voids.
2. Off-Center Observer produces too large CMB Dipole.
3. Worse Fit than LCDM.
4. Ruled out (Zibin, Bull, Stebbins)

J. Grande, L.P., Phys. Rev. D **84**,  
023514 (2011).

# Inhomogeneous Dark Energy: Why Consider?

## Coincidence Problem:

Why Now?  $\longrightarrow$  Time Dependent Dark Energy

## Standard Model ( $\Lambda$ CDM):

1. Homogeneous - Isotropic Dark and Baryonic Matter.
2. Homogeneous-Isotropic-Constant Dark Energy (Cosmological Constant)
3. General Relativity

## Alternatively:

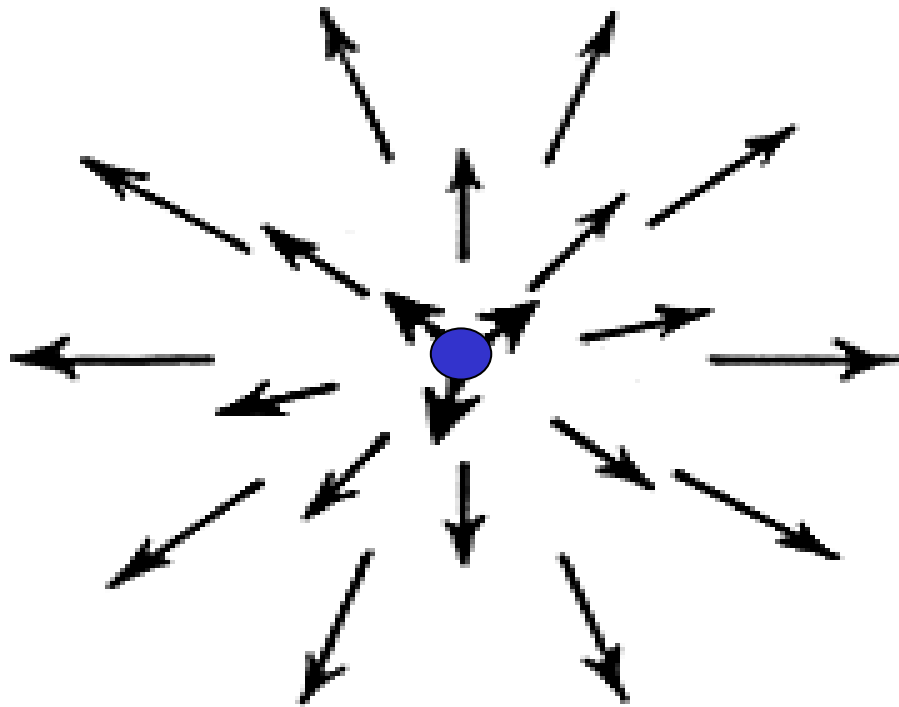
Why Here?  $\longrightarrow$  Inhomogeneous Dark Energy

## Consider Because:

1. New generic generalization of  $\Lambda$ CDM (breaks homogeneity of dark energy). Includes  $\Lambda$ CDM as special case.
2. Natural emergence of preferred axis (off – center observers)
3. Well defined physical mechanism (topological quintessence with Hubble scale global monopoles).

J. Grande, L.P., Phys. Rev. D **84**,  
023514 (2011).

J. B. Sanchez, LP, in preparation.



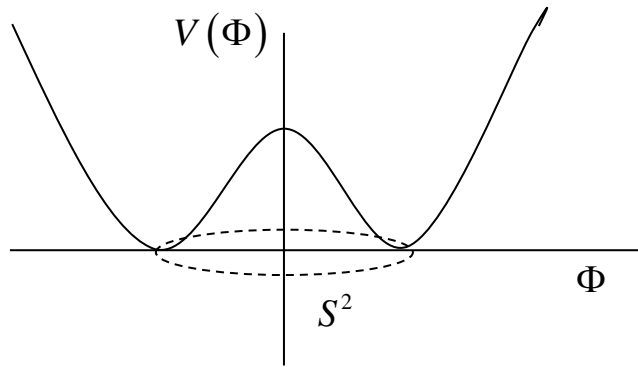


# Generalized LTB: Inhomogeneous Dark Energy

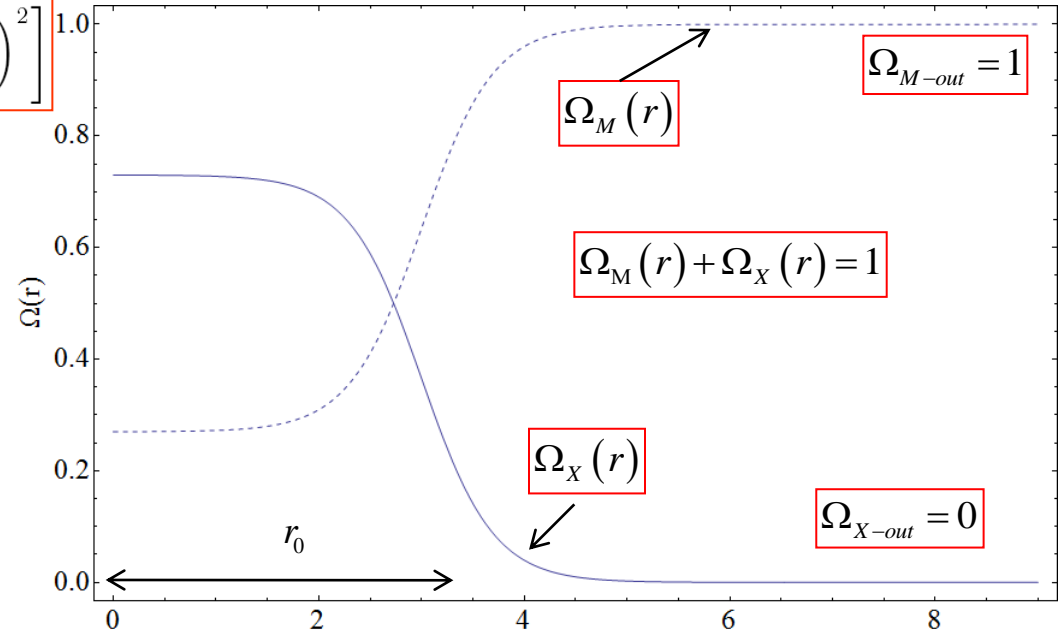
Cosmological Equation:

$$H^2(r, t) = H_0^2(r) \left[ \Omega_M(r) \left( \frac{A_0}{A} \right)^3 + \Omega_X(r) + \Omega_c(r) \left( \frac{A_0}{A} \right)^2 \right]$$

$\nearrow 0$



Isocurvature Profiles (Flat):



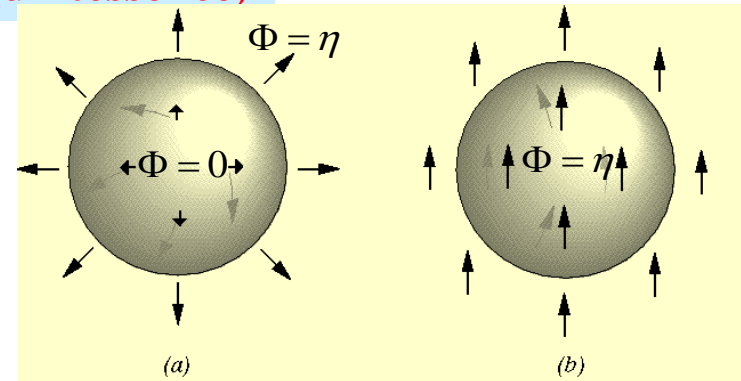
Physical Motivation:

Global Monopole with Hubble scale Core (**Topological Quintessence**)

$$S = \int d^4x \sqrt{-g} \left[ \frac{m_{Pl}^2}{16\pi} \mathcal{R} - \frac{1}{2} (\partial_\mu \Phi^a)^2 - V(\Phi) + \mathcal{L}_m \right]$$

$$V(\Phi) = \frac{1}{4} \lambda (\Phi^2 - \eta^2)^2, \quad \Phi \equiv \sqrt{\Phi^a \Phi^a}$$

$$\Phi^a = \Phi(t, r) \hat{r}^a \equiv \Phi(t, r) (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$$



# Constraints for On-Center Observer

Geodesics:

Luminosity Distance:

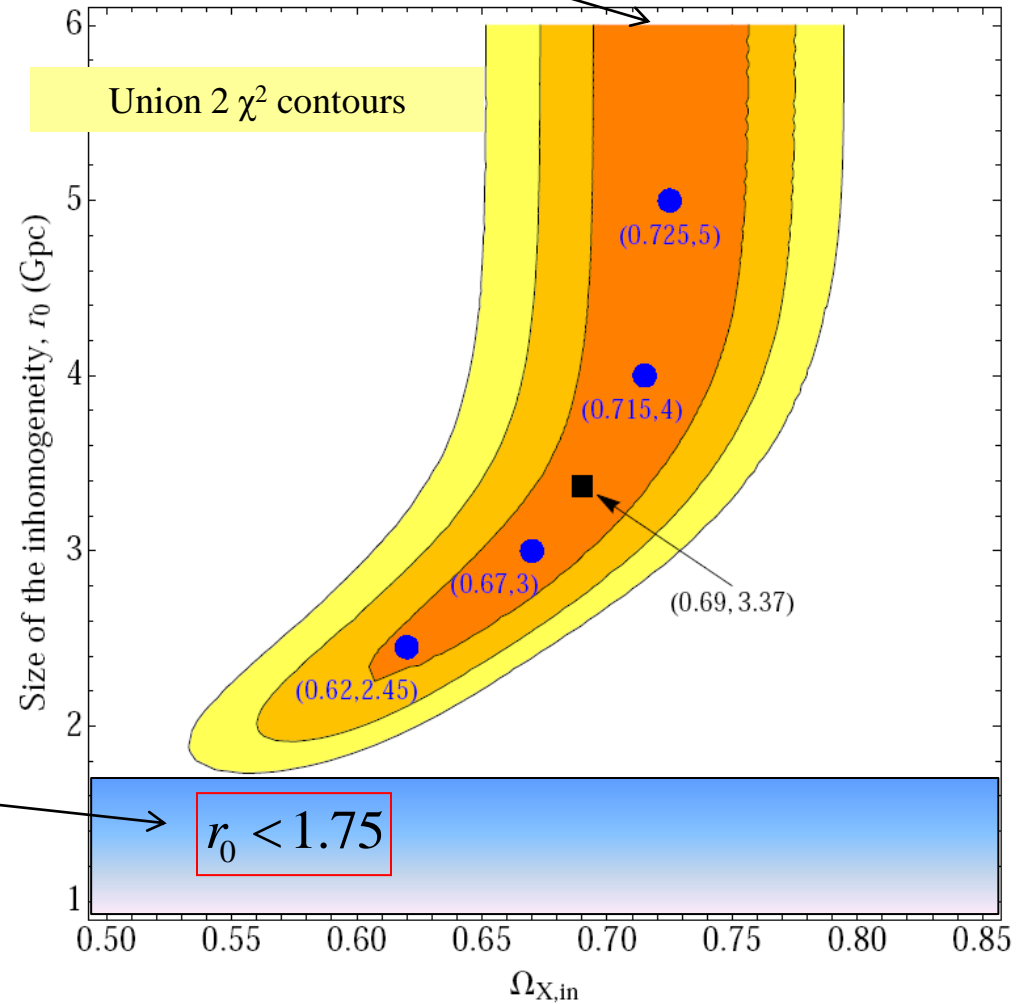
$$d_L(z, r_0, \Omega_{X,\text{in}}) = (1+z)^2 A(r(z), t(z))$$

$$\chi^2(r_0, \Omega_{X,\text{in}})$$

Ruled out region at  $3\sigma$

$\Lambda$ CDM limit

Union 2  $\chi^2$  contours



# Constraints for On-Center Observer

Geodesics:



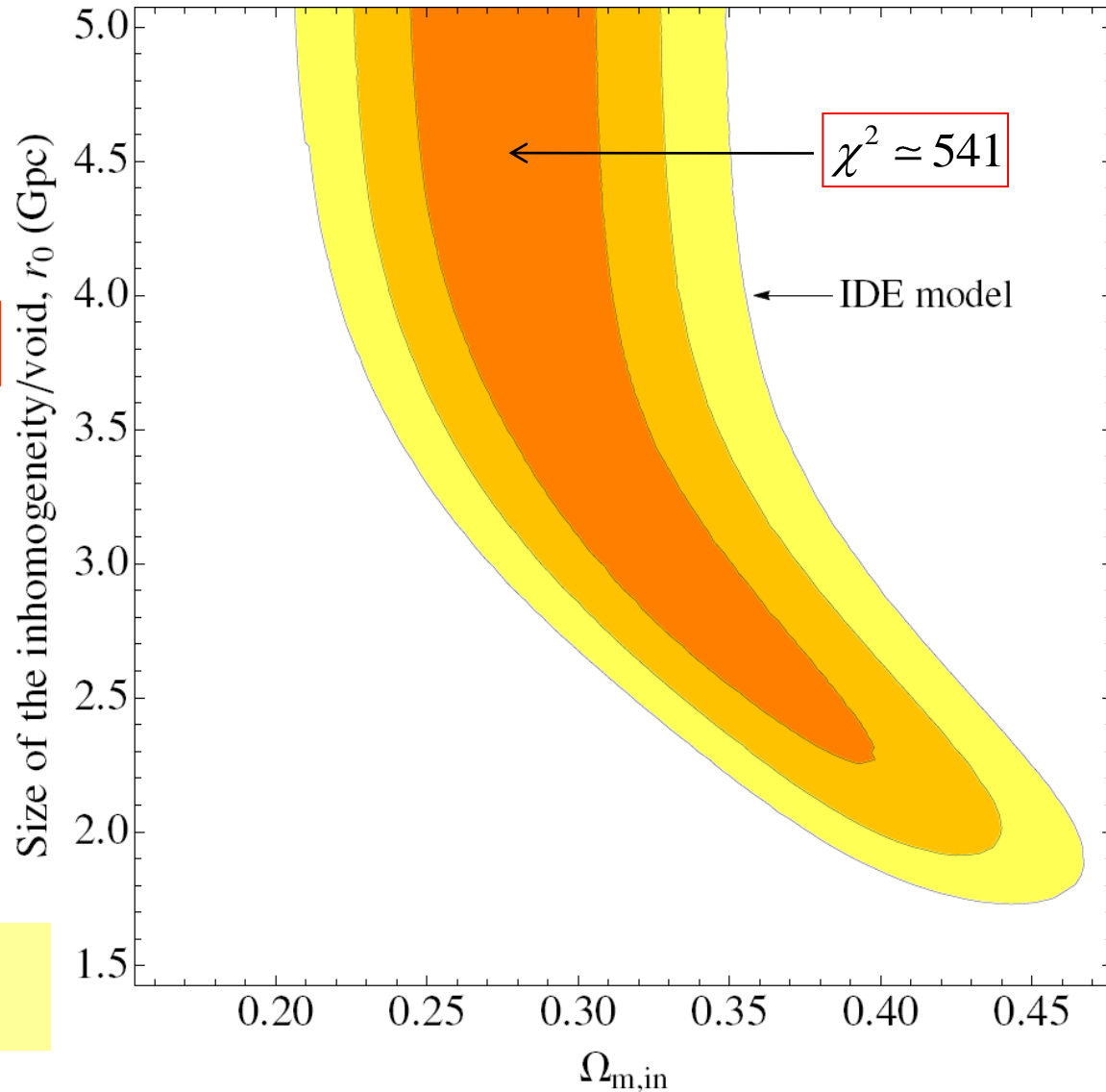
Luminosity Distance:

$$d_L(z, r_0, \Omega_{X,\text{in}}) = (1+z)^2 A(r(z), t(z))$$

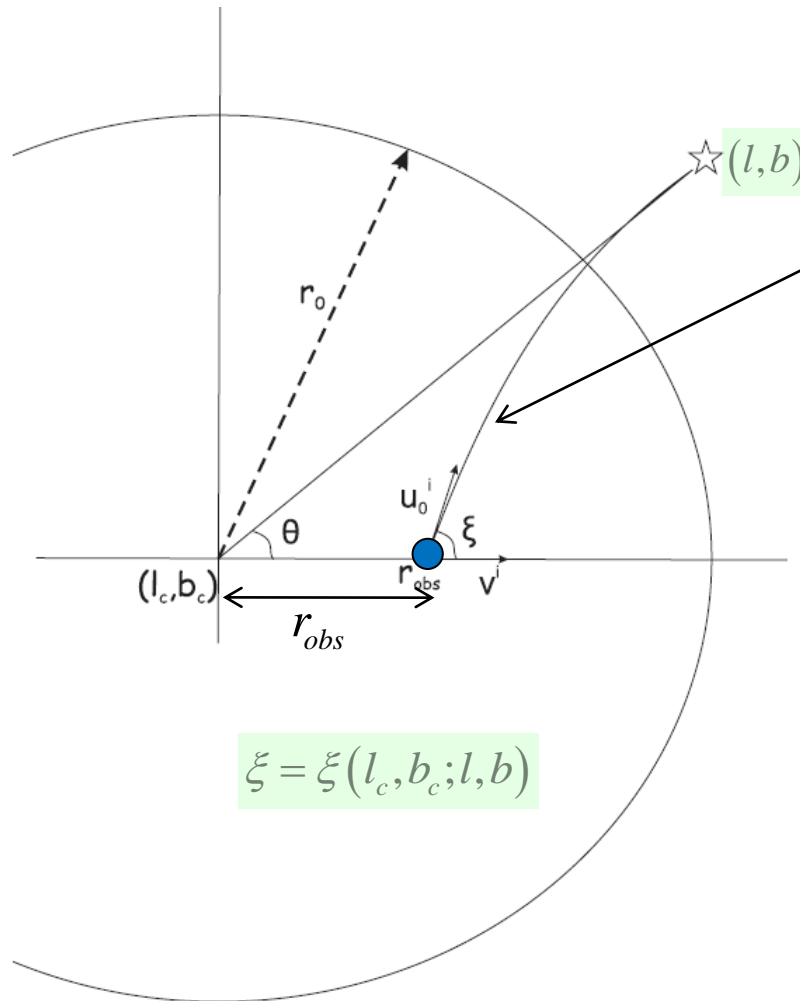


$$\chi^2(r_0, \Omega_{X-\text{in}})$$

J. Grande, L.P., Phys. Rev. D **84**,  
023514 (2011).



# Off-Center Observer



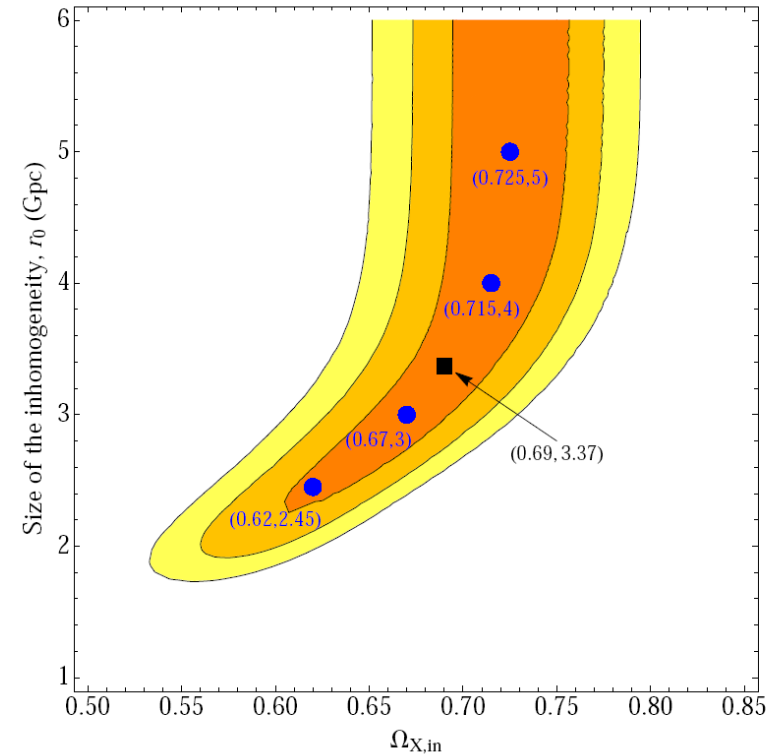
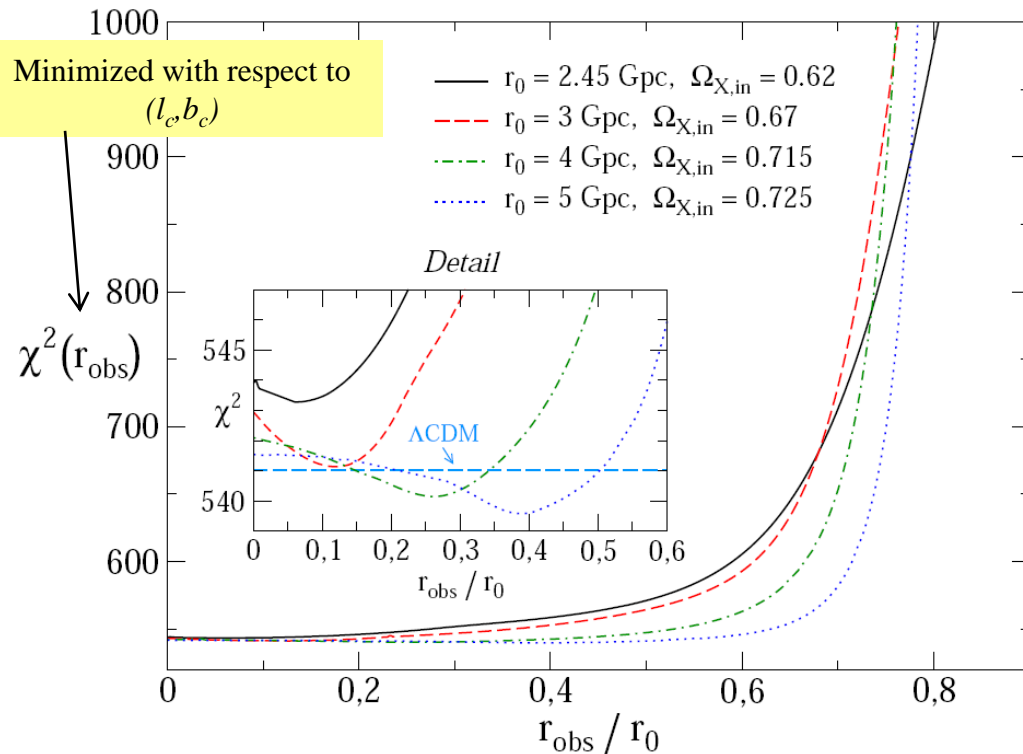
Geodesics

Luminosity Distance

$$d_L(z, r_0, \Omega_{X-in}; \xi, r_{obs})$$

$$\chi^2(r_0, \Omega_{X-in}; r_{obs}, l_c, b_c)$$

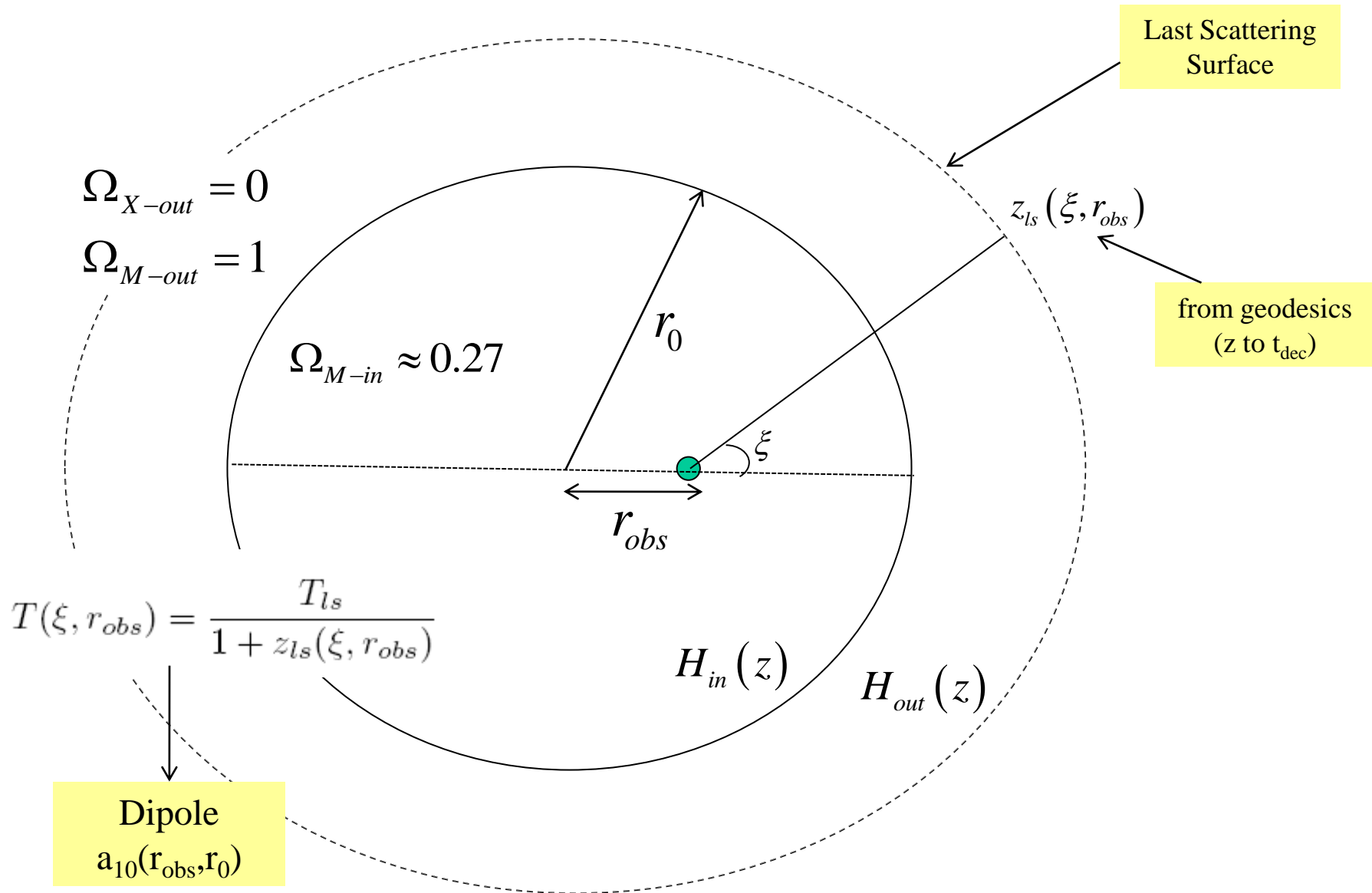
# Off-Center Observer: Union2 Constraints



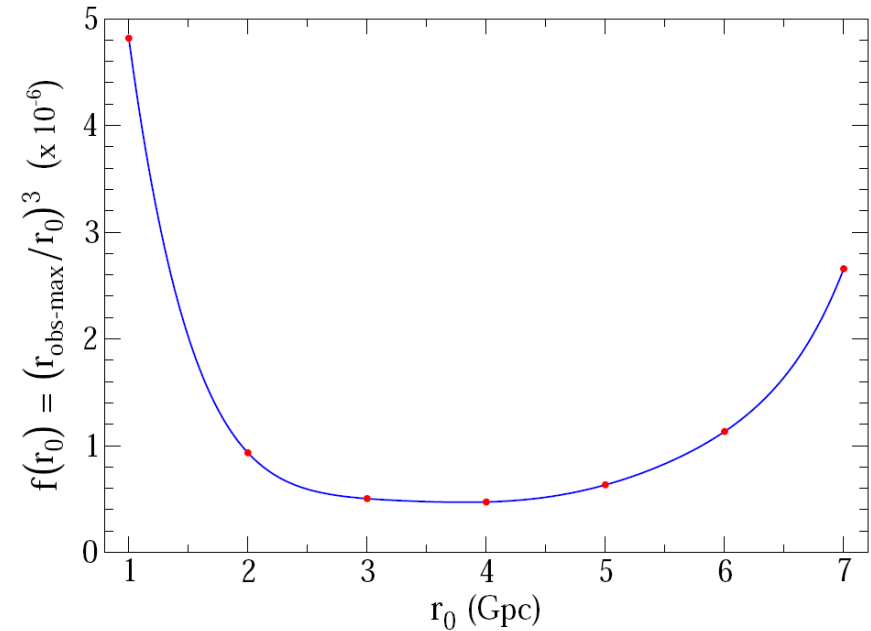
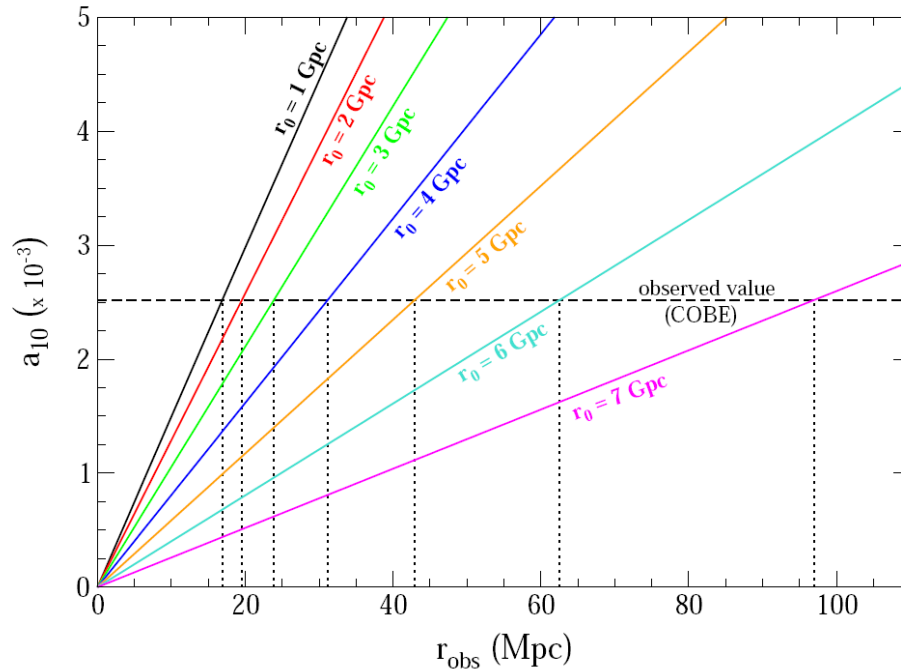
Minor improvement (if any) in quality of fit over  $\Lambda\text{CDM}$ , despite four additional parameters.

Copernican Principle Respected ( $r_{\text{obs}}/r_0$  as large as 0.7).

# Off-Center Observer: CMB dipole Constraints



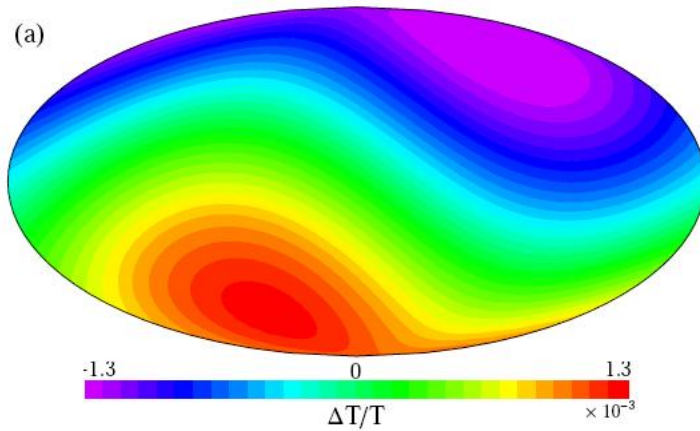
# Off-Center Observer: CMB dipole Constraints



Problem with Copernican Principle ( $(r_{\text{obs}}/r_0)^3 < 10^{-5}$ ).

Expected to be alleviated for  $r_0 \sim r_{\text{ls}}$

# Off-Center Observer: CMB multipoles alignment





# Summary

Early hints for **deviation from the cosmological principle** and **statistical isotropy** are being accumulated. This appears to be one of the most likely directions which may lead to **new fundamental physics** in the coming years.

The simplest mechanism that can give rise to a **cosmological preferred axis** is based on an **off-center observer** in a spherical energy inhomogeneity (dark matter or dark energy)

Such a mechanism can give rise to **aligned CMB multipoles** and **large scale velocity flows**. If the inhomogeneity is attributed to a Hubble scale **global monopole** with a Hubble scale core other interesting effects may occur (direction dependent variation of fine structure constant, quasar polarization alignment etc).

# Standard Cosmological Models

Static Universe with Matter

Einstein (1917)

Static Universe with Matter and Cosmological Constant

Hubble (1930)

Expanding Universe with Visible Matter

Zwicky 1933 Coma galaxy cluster (virial theorem)

Expanding Universe with Dark Baryonic Matter

late 70s to mid 80s (low CMB perts + experiment of 20eV neutrino, inflation)

Flat Expanding Universe with Hot Dark Matter (Neutrinos)

free streaming-top down structure formation (1984)

Flat Expanding Universe with Cold Dark Matter (Exotic)

$\Omega_{\text{dyn}} < 1$ , too much small scale structure, SNIa data, age problem (1998)

Flat Expanding Universe with Cold Dark Matter (Exotic) and Cosmological Constant ( $\Lambda$ CDM)

?

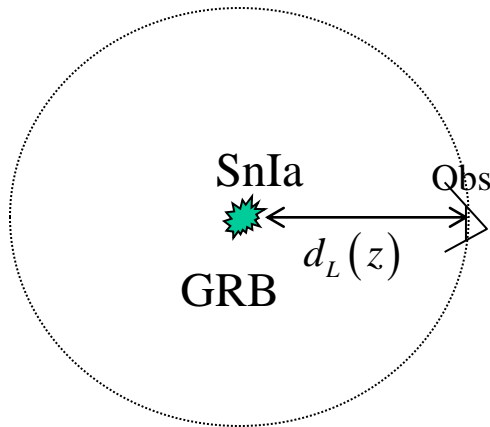
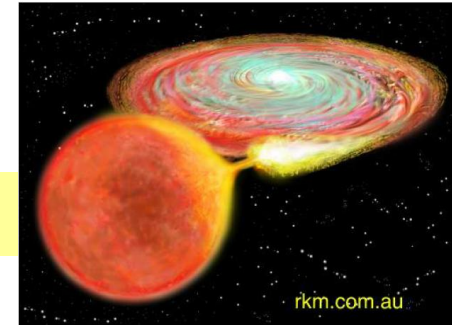
?

# Direct Probes of the Cosmic Metric:

## Geometric Observational Probes

Direct Probes of  $H(z)$ :

Luminosity Distance (standard candles: SnIa, GRB):



$$l = \frac{L}{4\pi d_L^2}$$

$$d_L(z)_{th} = c(1+z) \int_0^z \frac{dz'}{H(z')}$$

flat

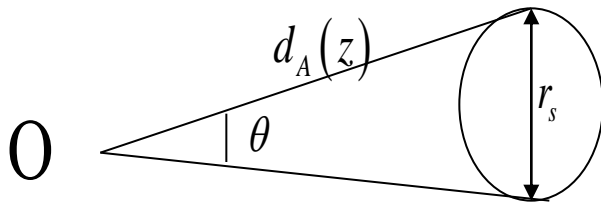
*SnIa* :  $z \in (0, 1.7]$

*GRB* :  $z \in [0.1, 6]$

Significantly less accurate probes  
S. Basilakos, LP, **MBRAS**, **391**, 411, 2008

arXiv:0805.0875

Angular Diameter Distance (standard rulers: CMB sound horizon, clusters):



$$d_A(z) = \frac{r_s}{\theta}$$

$$d_A(z)_{th} = \frac{c}{(1+z)} \int_0^z \frac{dz'}{H(z')}$$

*BAO* :  $z = 0.35, z = 0.2$

*CMB Spectrum* :  $z = 1089$

# Geometric Constraints

Parametrize  $H(z)$ :

$$w(z) = w_0 + w_1 \frac{z}{1+z}$$

Chevallier, Pollarski, Linder

$$H^2(z) = H_0^2 [\Omega_{0m}(1+z)^3 + (1 - \Omega_{0m})(1+z)^{3(1+w_0+w_1)} e^{\frac{-3w_1 z}{(1+z)}}]$$

$$\Lambda\text{CDM} \rightarrow (w_0, w_1) = (-1, 0)$$

Minimize:

$$\chi^2(\Omega_m, w_0, w_1) = \sum_{i=1}^N \frac{[5 \log_{10}(d_{L,A}(z_i)_{obs}) - 5 \log(d_{L,A}(z_i; w_0, w_1)_{th})]^2}{\sigma_i^2} = \min$$

SALT-II  
light curve fitter

$$\mu_i = m_{B_i}^* - M + \alpha \cdot x_{1,i} - \beta \cdot c_i$$

R. Amanullah, et. al. (Union2)  
Astrophys.J.716:712-738,2010,

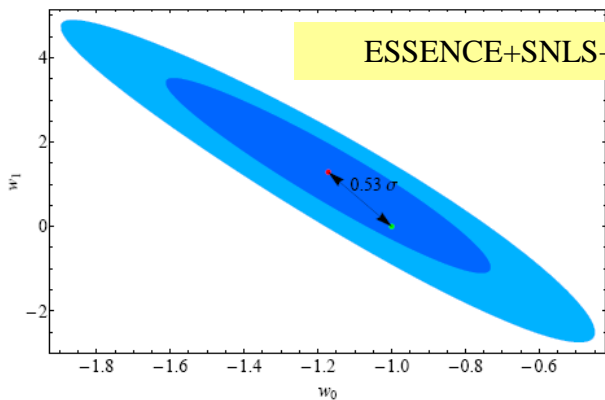
$$\chi_{\text{stat}}^2 = \sum_{\text{SNe}} \frac{[\mu_B(\alpha, \beta, M) - \mu(z; \Omega_M, \Omega_w, w)]^2}{\sigma_{\text{ext}}^2 + \sigma_{\text{sys}}^2 + \sigma_{\text{lc}}^2}$$

MLCS2k2  
light curve fitter

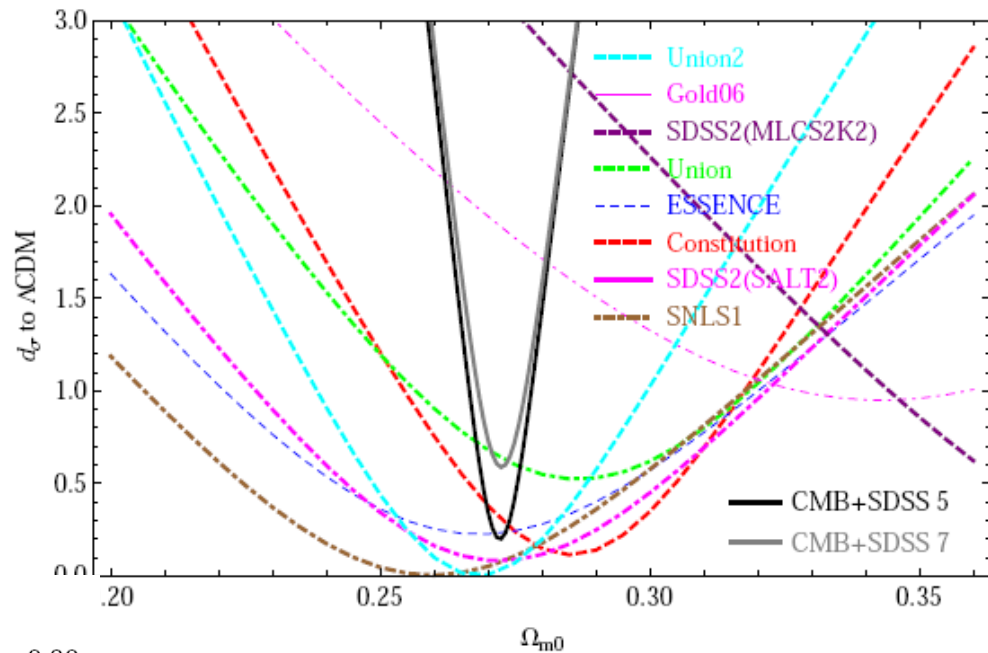
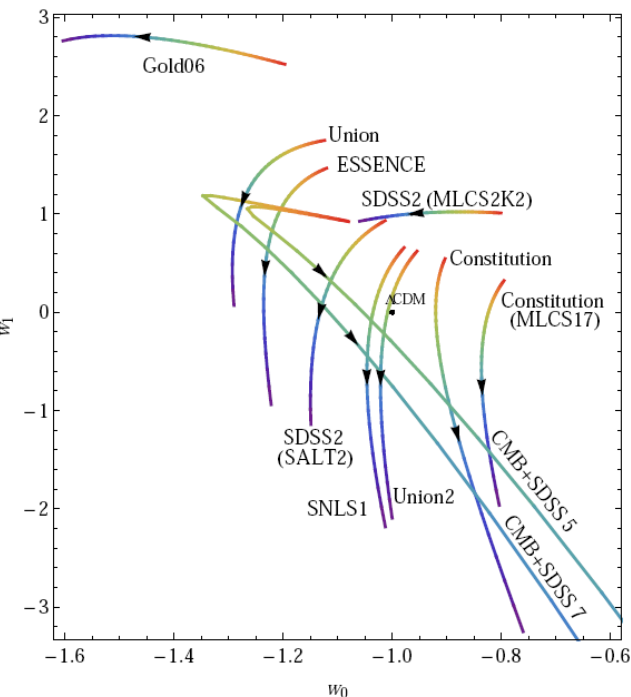
$$\chi_{\mu}^2 = \left\{ \sum_i \frac{[\mu_i - \mu(z_i; w, \Omega_M, \Omega_{DE}, H_0)]^2}{\sigma_{\mu}^2} \right\}$$

# The $\sigma$ -distance to $\Lambda$ CDM

ESSENCE+SNLS+HST data



Trajectories of Best Fit Parameter Point

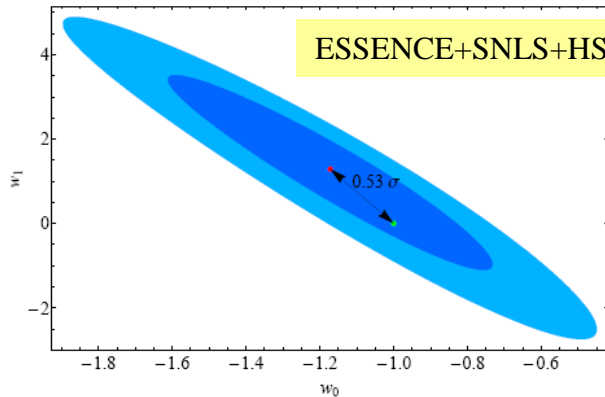


Consistency with  $\Lambda$ CDM Ranking:

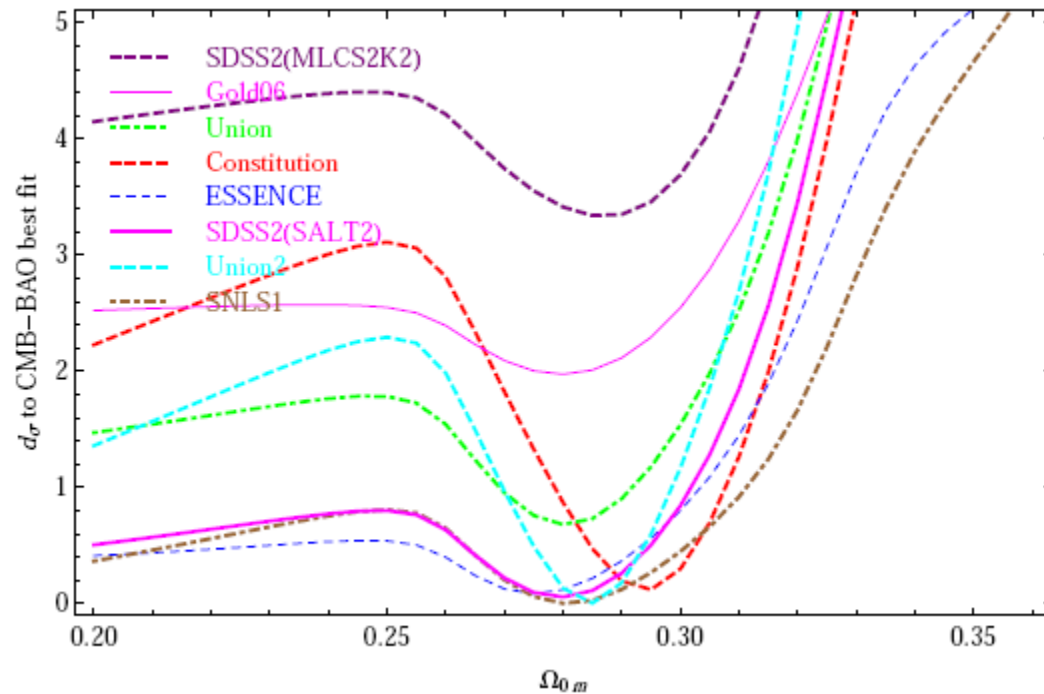
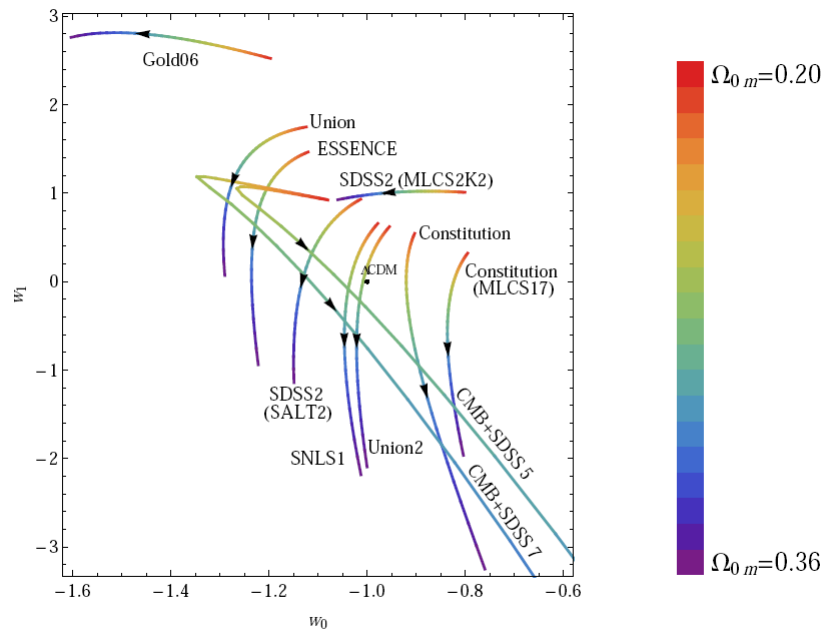
Dataset	$d_{\sigma}^{min}$	$\Omega_{0m}^{min}$	$w_0$	$w_1$
Union2	0.001	0.270	-1.01	0.01
SNLS1	0.004	0.260	-1.03	0.16
SDSS-II (SALT2)	0.084	0.270	-1.09	0.51
Constitution	0.114	0.285	-0.91	-0.54
ESSENCE	0.227	0.270	-1.20	1.04
Union	0.525	0.285	-1.25	1.40
SDSS-II (MLCS2K2)	0.623	0.360	-1.06	0.93
Gold06	0.950	0.345	-1.56	2.80
CMB+BAO (SDSS5)	0.200	0.272	-1.15	0.51
CMB+BAO (SDSS7)	0.588	0.272	-1.30	0.97

# The $\sigma$ -distance to Standard Rulers

ESSENCE+SNLS+HST



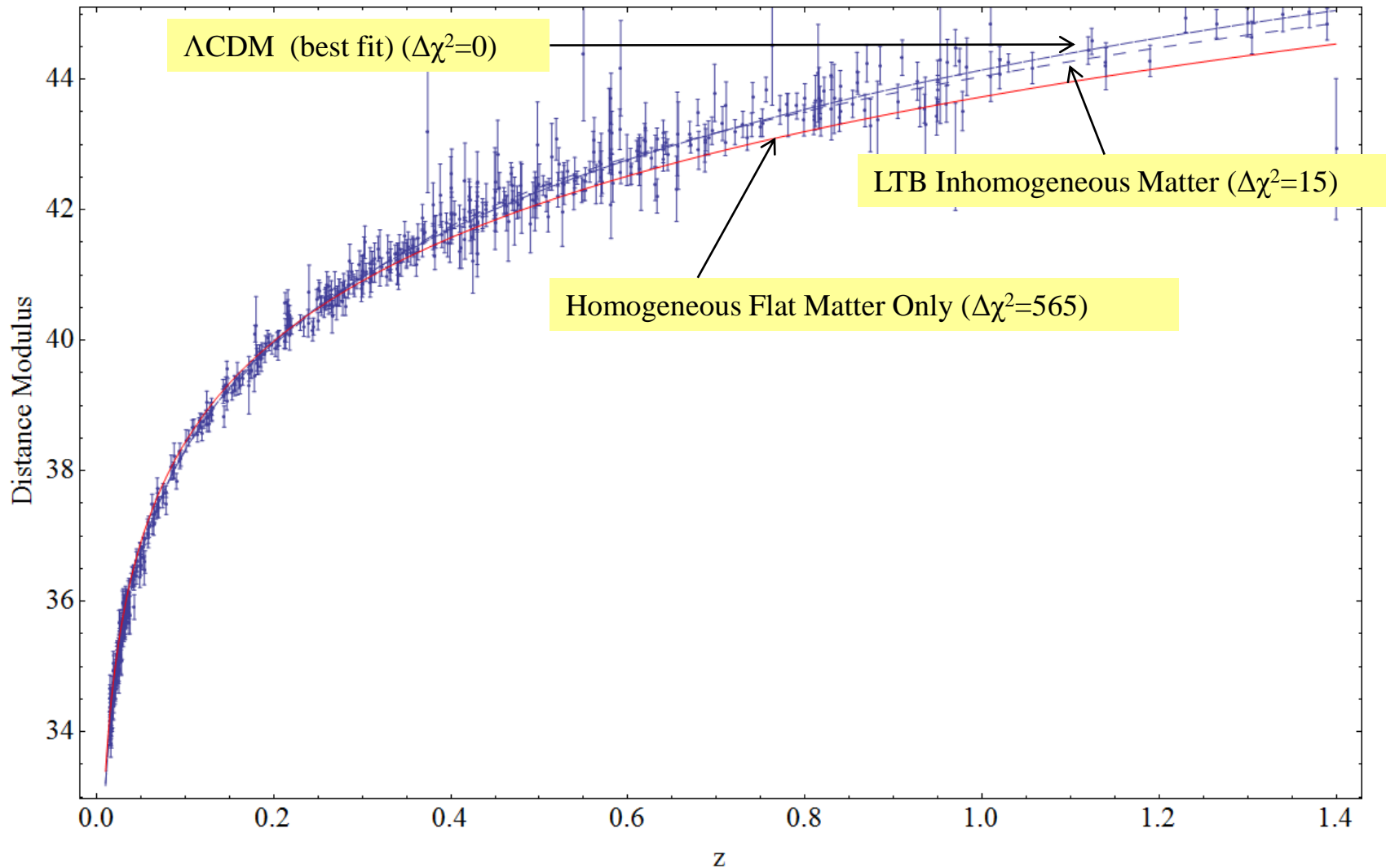
Trajectories of Best Fit Parameter Point



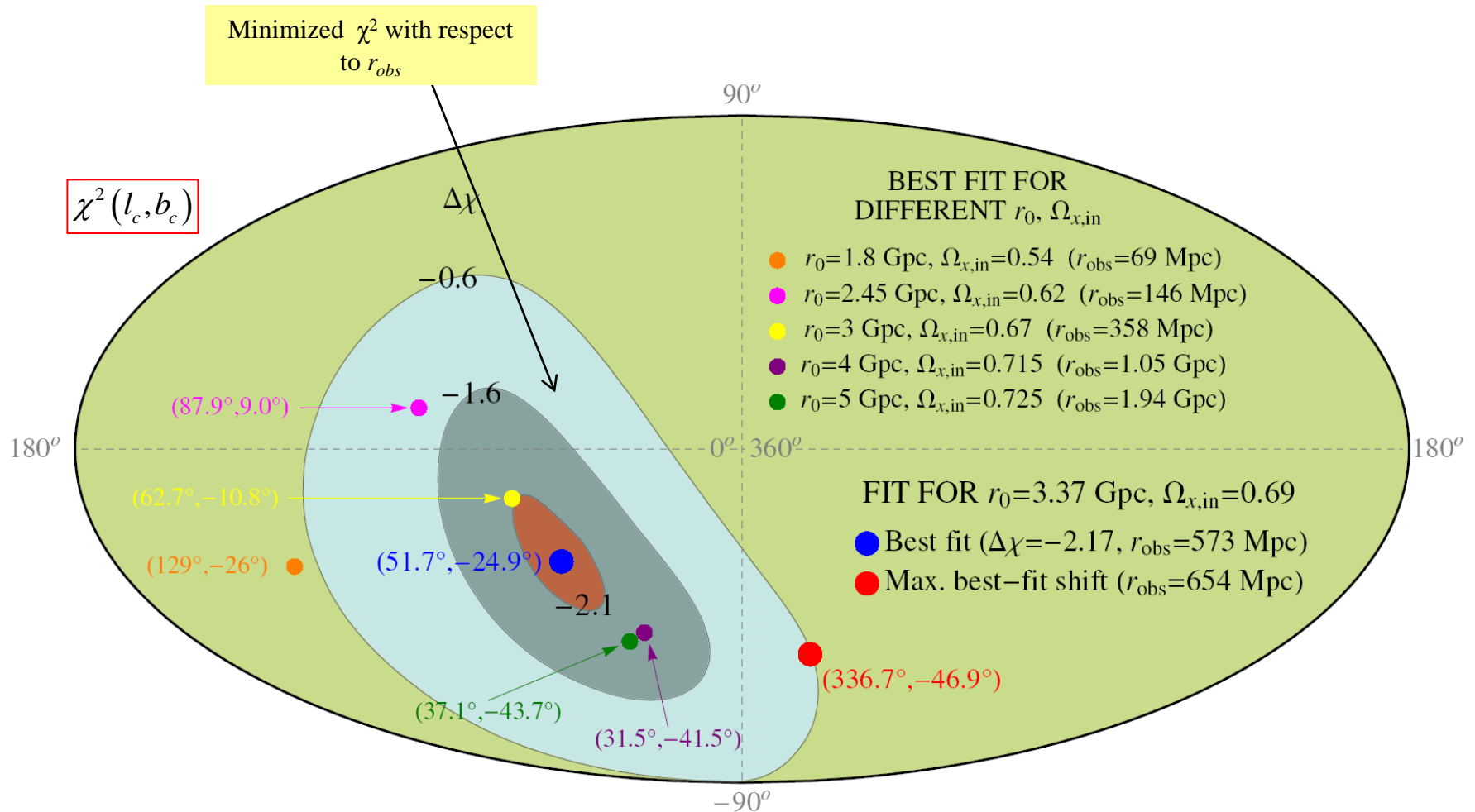
Consistency with Standard Rulers Ranking:

Dataset	$d_{\sigma}^{\min}$	$\Omega_{0m}^{\min}$	$w_0$	$w_1$	$w_0^{SR}$	$w_1^{SR}$
SNLS1	0.003	0.280	-1.04	-0.10	-1.07	0.08
Union2	0.004	0.280	-1.01	-0.12	-1.07	0.08
SDSS-II (SALT2)	0.058	0.280	-1.11	0.40	-1.07	0.08
ESSENCE	0.087	0.275	-1.21	0.99	-1.12	0.38
Constitution	0.121	0.295	-0.90	-0.76	-0.84	-1.28
Union	0.681	0.280	-1.24	1.44	-1.07	0.08
Gold06	1.976	0.280	-1.38	2.75	-1.07	0.08
SDSS-II(MLCS2K2)	3.342	0.285	-0.92	1.02	-1.00	-0.28

# Constraints: Union2 Data – Central Observer



# Off-Center Observer: Union2 Constraints II





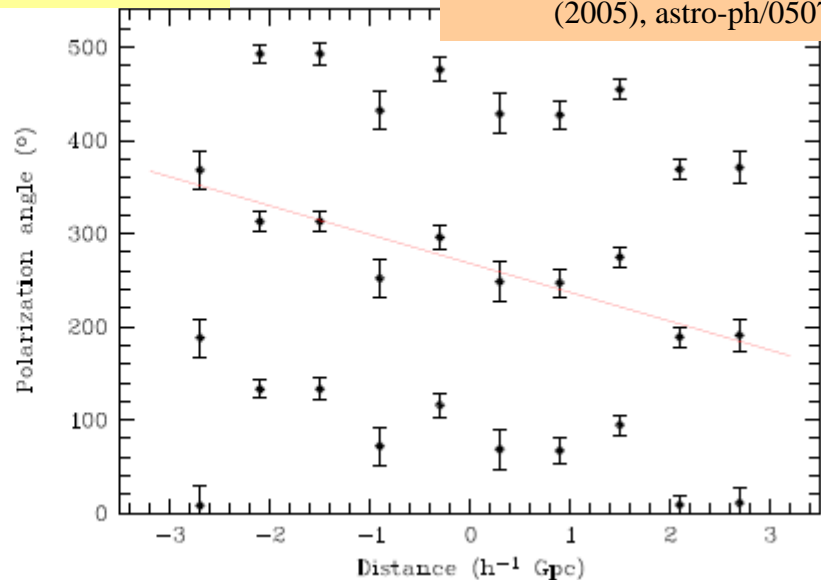
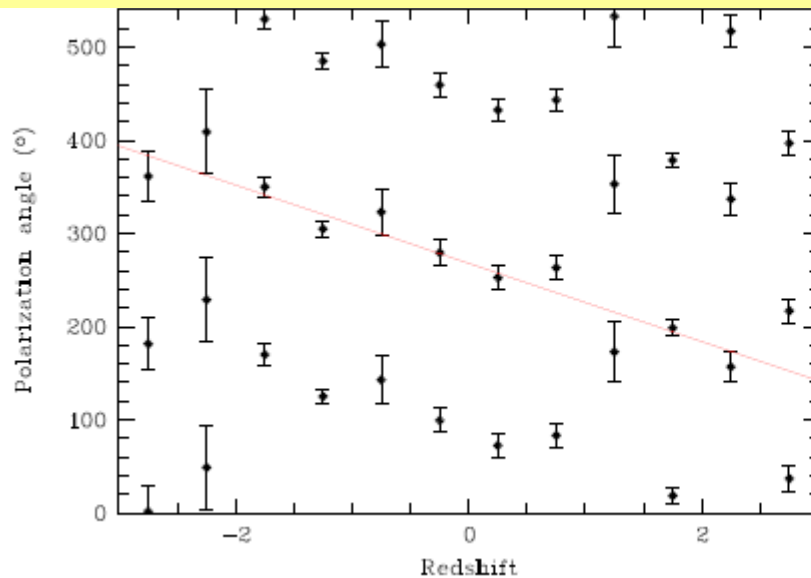
# Preferred Axes

Three of the four puzzles for  $\Lambda$ CDM are related to the existence of a preferred axis

Cosmological Obs.	$l$	$b$
CMB Dipole	$264^\circ$	$48^\circ$
Velocity Flows	$282^\circ$	$6^\circ$
Quasar Alignment	$267^\circ$	$69^\circ$
CMB Octopole	$308^\circ$	$63^\circ$
CMB Quadrupole	$240^\circ$	$63^\circ$

**QSO optical polarization angle** along the direction  $l=267^\circ$ ,  $b=69^\circ$

D. Hutsemekers et. al., AAS, 441,915  
(2005), astro-ph/0507274

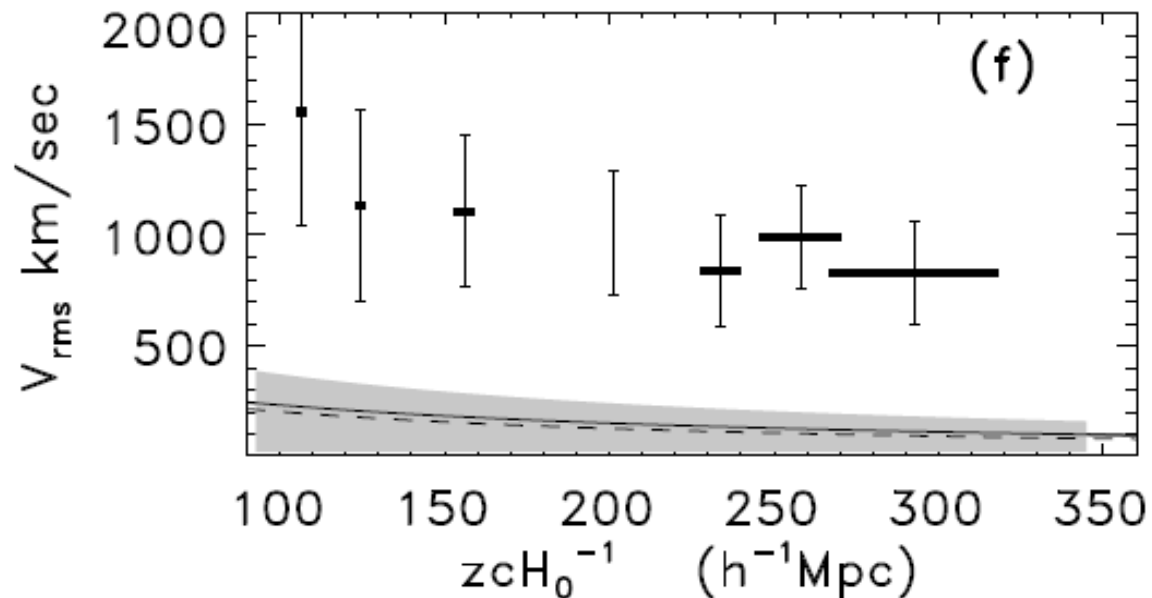


# Preferred Axes

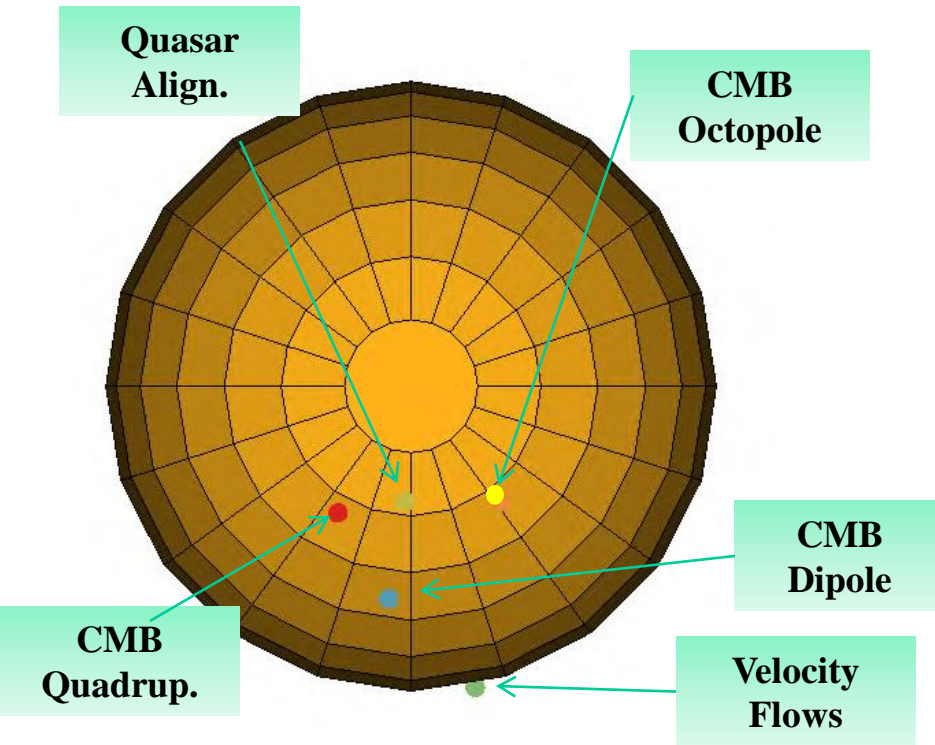
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CMB Quadrupole	$240^\circ$	$63^\circ$

A. Kashlinsky et. al.  
**Astrophys.J.686:L49-L52,2009**  
**arXiv:0809.3734**



# Preferred Axes

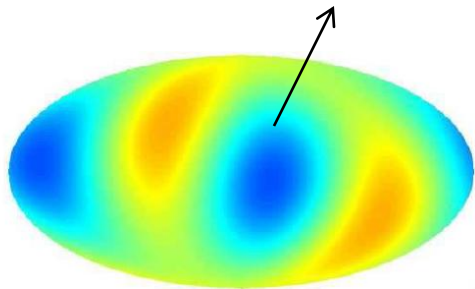


**Q1:** Are there other cosmological data with hints towards a preferred axis?

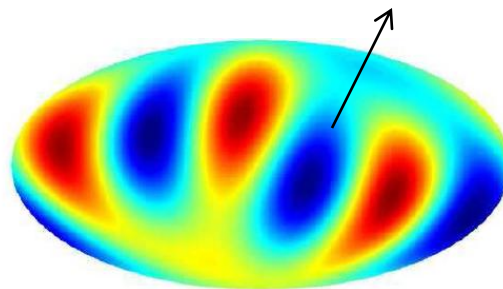
**Q2:** What is the probability that these independent axes lie so close in the sky?

I. Antoniou, LP,  
JCAP 1012:012, 2010,  
arxiv: 1007.4347

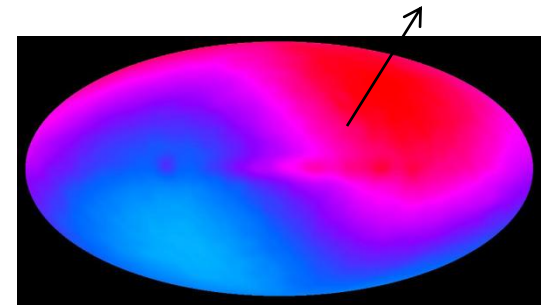
Quadrupole component of CMB map



Octopole component of CMB map



Dipole component of CMB map



M. Tegmark et. al., **PRD 68**, 123523 (2003),  
Copi et. al. **Adv.Astron.**2010:847541,2010.

# Union2 Data: Hemisphere Comparison Method

Q.: How isotropic is the accelerating expansion of the universe?

2. Evaluate Best Fit  $\Omega_m$  in each Hemisphere

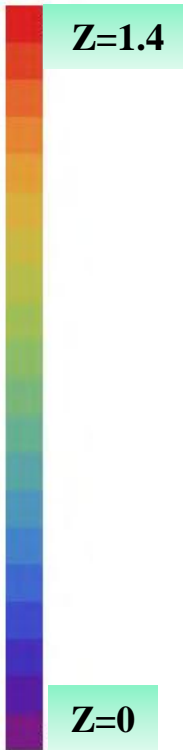
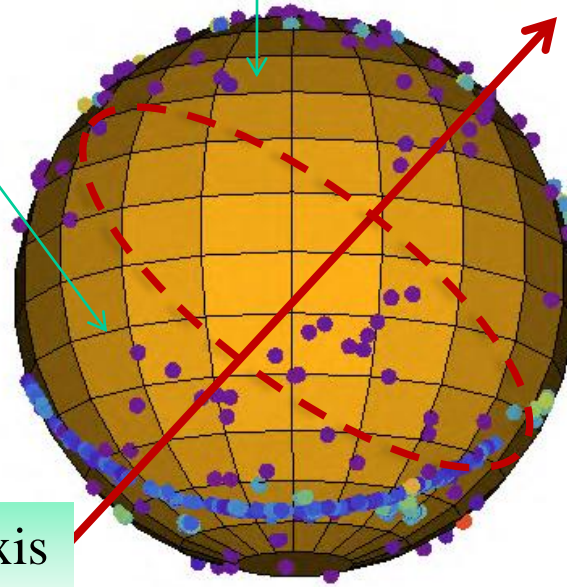
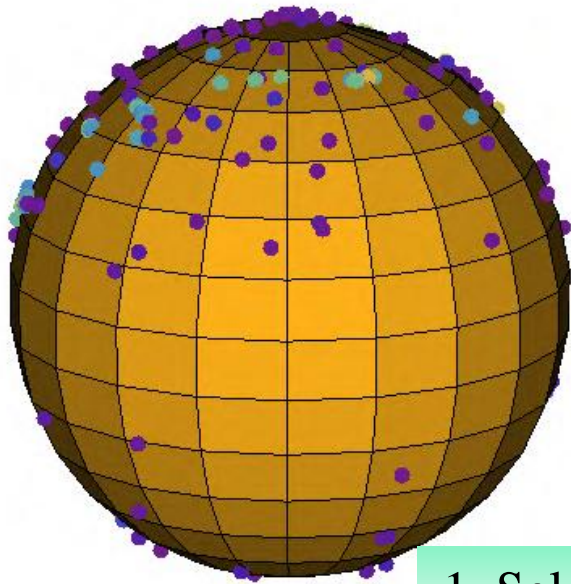
1. Select Random Axis

3. Evaluate  $\frac{\Delta\Omega_m}{\bar{\Omega}}$

4. Repeat with several random axes  
and find  $\frac{\Delta\Omega_{\max}}{\bar{\Omega}}$

Union2 Data  
Galactic Coordinates

(view of sphere from opposite directions)

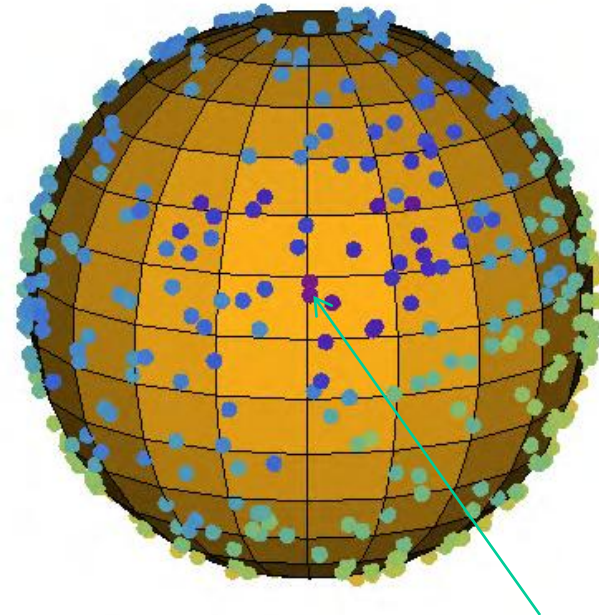


# Anisotropies for Random Axes (Union2 Data)

View from above Maximum Asymmetry Axis  
Galactic Coordinates

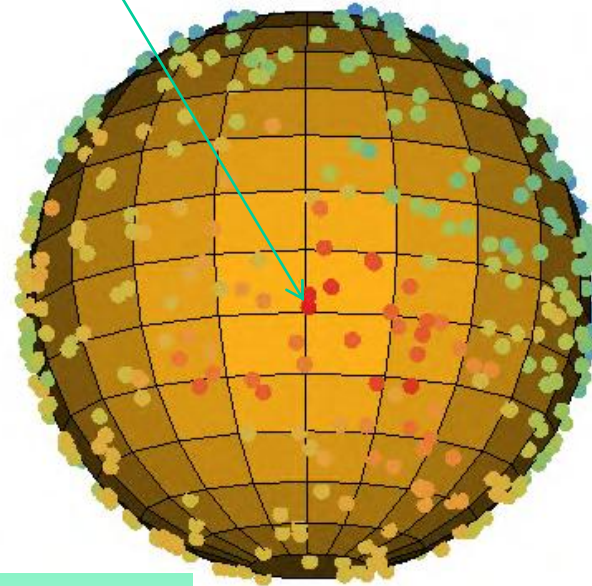
Minimum Acceleration:  
 $(l,b)=(126^{\circ},-15^{\circ})$

$$\frac{\Delta\Omega_{\max}}{\bar{\Omega}} = 0.42$$



$$\frac{\Delta\Omega_{\max}}{\bar{\Omega}} = -0.42$$

Maximum Acceleration Direction:  
 $(l,b)=(306^{\circ},15^{\circ})$



$\Delta\Omega/\bar{\Omega}=$   
0.43

$\Delta\Omega/\bar{\Omega}=$   
-0.43

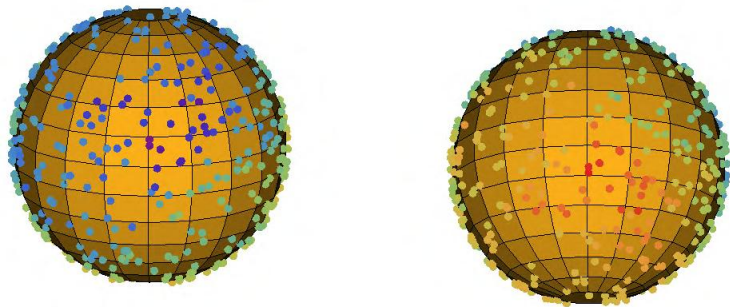


# Statistical Isotropy Test: Results

There is a direction of maximum anisotropy in the Union2 data  $(l,b)=(306^\circ,15^\circ)$ .

The level  $\left( \frac{\Delta\Omega_{0m-\max}}{\Omega_{0m}} \right)^{U/2}$  of this anisotropy is larger than the corresponding level of about 70% of isotropic simulated datasets but it is consistent with statistical isotropy.

Real Data Test Axes

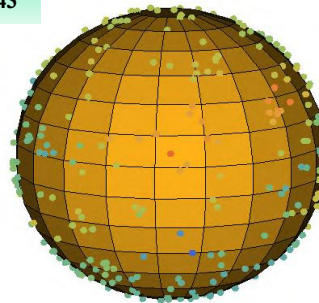


$\Delta\Omega/\Omega=0.43$



$\Delta\Omega/\Omega=-0.43$

Simulated Data Test Axes



$\Delta\Omega/\Omega=0.43$



$\Delta\Omega/\Omega=-0.43$