Department of Physics, University of Jyväskylä
Helsinki Institute of Physics, University of Helsinki

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Inhomogeneous Cosmologies

Gravitational lensing with the sGL method

Valerio Marra

Part I

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Weak-lensing magnification

(on a standard FLRW model)

Weak-lensing basics

$$\kappa \ll 1$$

optical weight: $G(r, r_s) = \frac{4\pi G}{c^2} \frac{f_K(r) f_K(r_s - r)}{f_K(r_s)} \frac{1}{a}$

$$\Delta m(z) \simeq -\frac{5}{\ln 10}\, \kappa(z)$$

$$\kappa(z) = \rho_{MC} \int_0^{r_s(z)} dr\, G(r, r_s(z))\, \delta_M(r, t(r))$$

shift in distance modulus

effective lens convergence

unperturbed light path

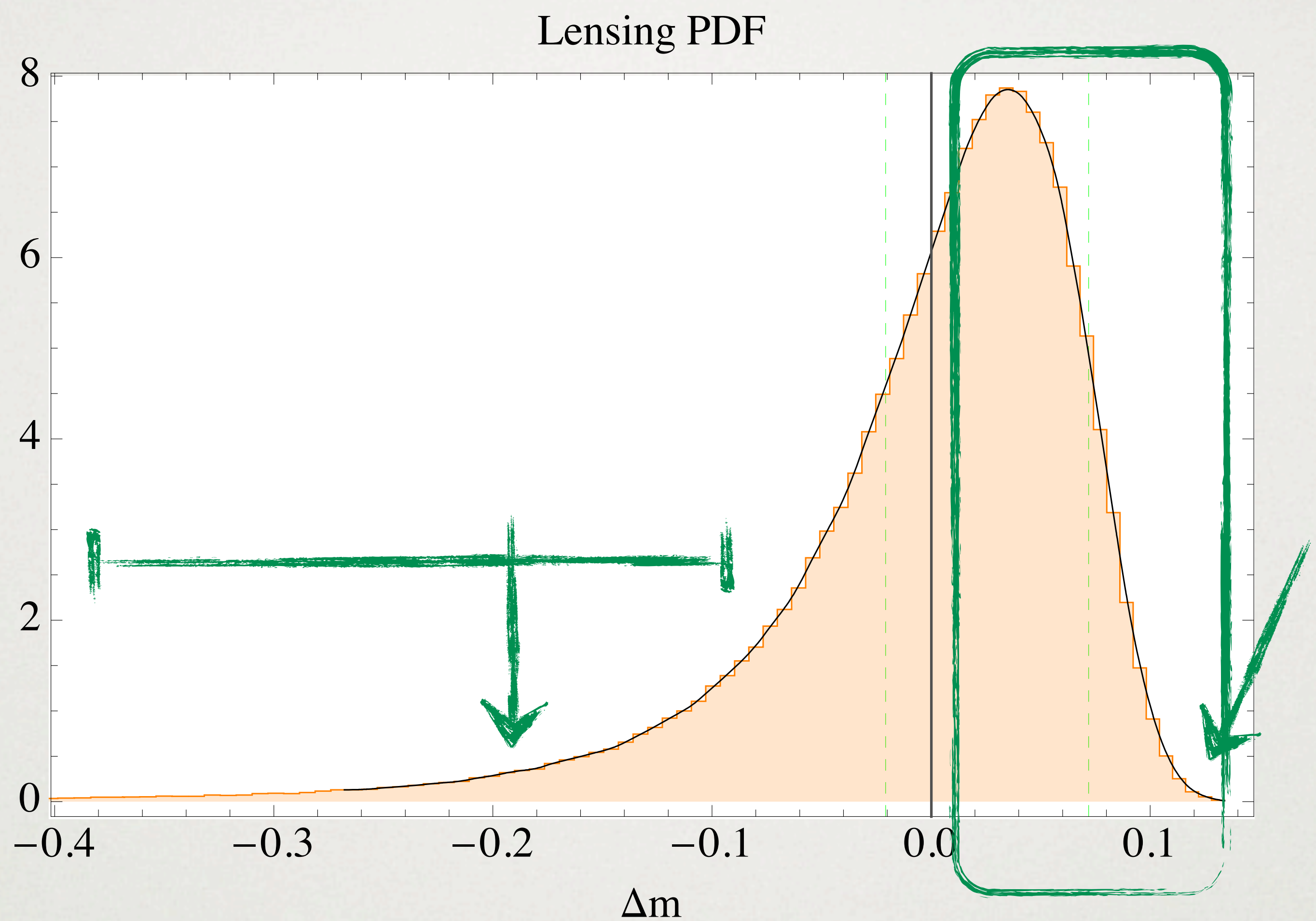
matter density contrast

overdensity $\delta_M > 0$ $\longrightarrow$ $\Delta m < 0$ magnification

underdensity $\delta_M < 0$ $\longrightarrow$ $\Delta m > 0$ demagnification

average density $\langle \delta_M \rangle = 0$ $\longrightarrow$ $\langle \Delta m \rangle = 0$ no lensing on average

So, what is interesting about lensing?



underdensities occupy most of the volume → most of the PDF is at demagnified values, the mode in particular

$\delta_M > -1$ → the low-magnification tail is "short"

overdensities are confined within filaments and walls → the high-magnification tail is low

δ_M is unbounded from above → the high-magnification tail is "long"

How to calculate the lensing PDF?

Current observations constrain *at most* the variance of the PDF

Theoretical approaches:

- "universal" PDFs with coefficient trained over N-body simulations → closed box: fast but not flexible
- ray-tracing techniques in N-body simulations → (slow)^2, but flexible
- ray-tracing techniques in toy models → slow, but flexible

sGL: fast and flexible!

Part II
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The sGL method

(stochastic Gravitational Lensing method)

Code available at turboGL.org

K. Kainulainen and V. Marra,
Phys. Rev. D 80, 123020 (2009),
Phys. Rev. D 83, 023009 (2011),
and arXiv:1101.4143

Step I

$$\kappa = \int_0^{r_s} dr\, G(r, r_s)\, \rho_{MC}\, \delta_M$$

binning the relevant parameters
in cells of volume ΔV_{iu}:
"a kind of Riemann integral"

halo surface
density

$$\kappa_{1iu} \equiv G_i\, \Sigma_{iu}$$

convergence due to a
halo in the bin (i, u)

***i* labels the z-slice**

$$\kappa(\{k_{iu}\}) = \sum_{iu} \kappa_{1iu} \Big(k_{iu} - \Delta N_{iu} \Big)$$

comoving
number density

configuration of
inhomogeneities

occupation
numbers

actual # of halos
in bin (i, u)

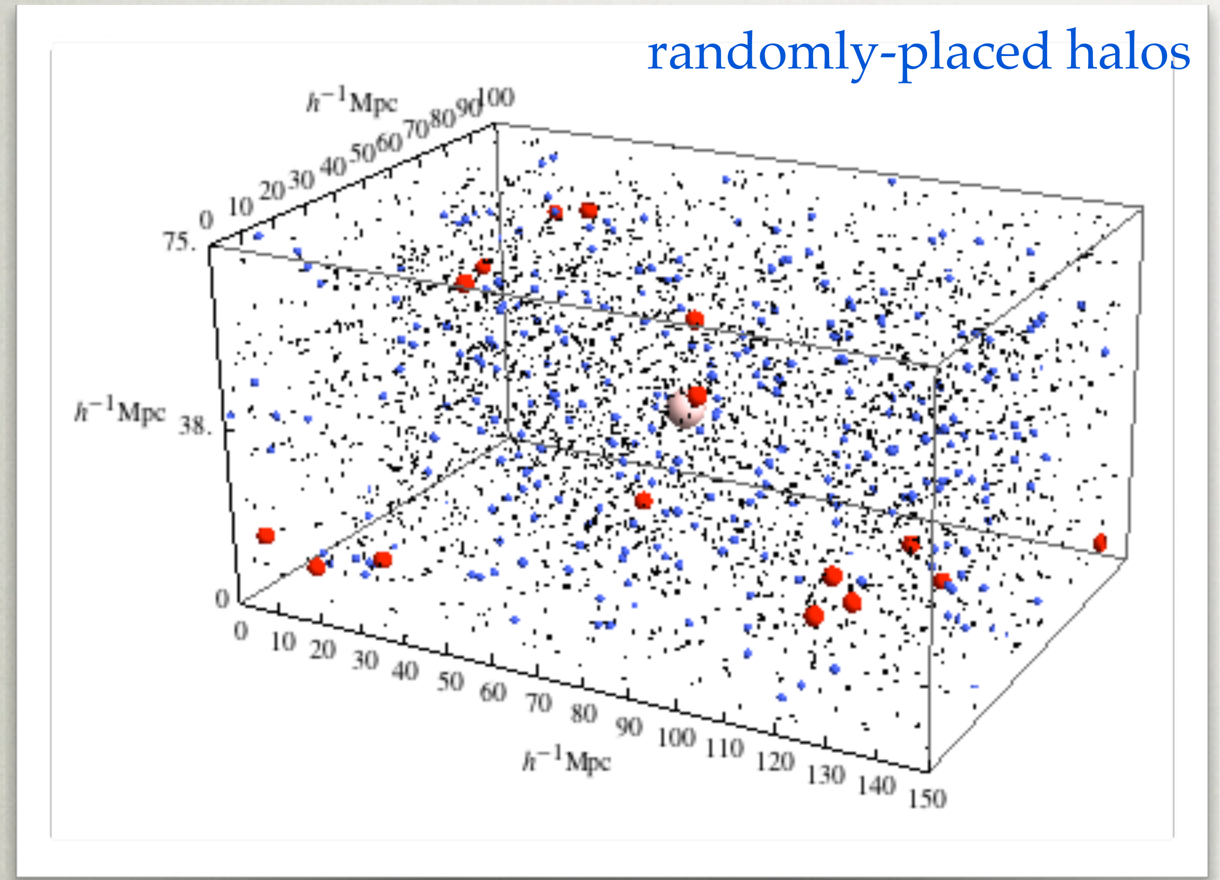
expected # of halos
in bin (i, u): $\Delta N_{iu} = \Delta n_{iu}\, \Delta V_{iu}$

Step II

$$\kappa(\{k_{iu}\}) = \sum_{iu} \kappa_{1iu}\Big(k_{iu} - \Delta N_{iu}\Big)$$

actual # of
halos in bin (i, u)

expected # of
halos in bin (i, u)



distributed as a
Poisson variable
of parameter ΔN_{im}
(and so $\langle \kappa \rangle = 0$)

to compute lensing PDF, enough to generate the numbers k_{iu}: FAST! ~1s

this modeling
is the so-called
"**Halo Model**"

Generalizing...

$$\kappa_{N_O}(\{k_{iu}\}) = \sum_{iu} \kappa_{1iu} \left(\frac{k_{iu,N_O}}{N_O} - \Delta N_{iu} \right)$$

survival probability: $\langle \kappa \rangle \neq 0$

where k_{iu,N_O} is distributed as a Poisson variable of parameter $N_O P_{iu}^{\text{sur}} \Delta N_{iu}$

observations in the data sample at given redshift:

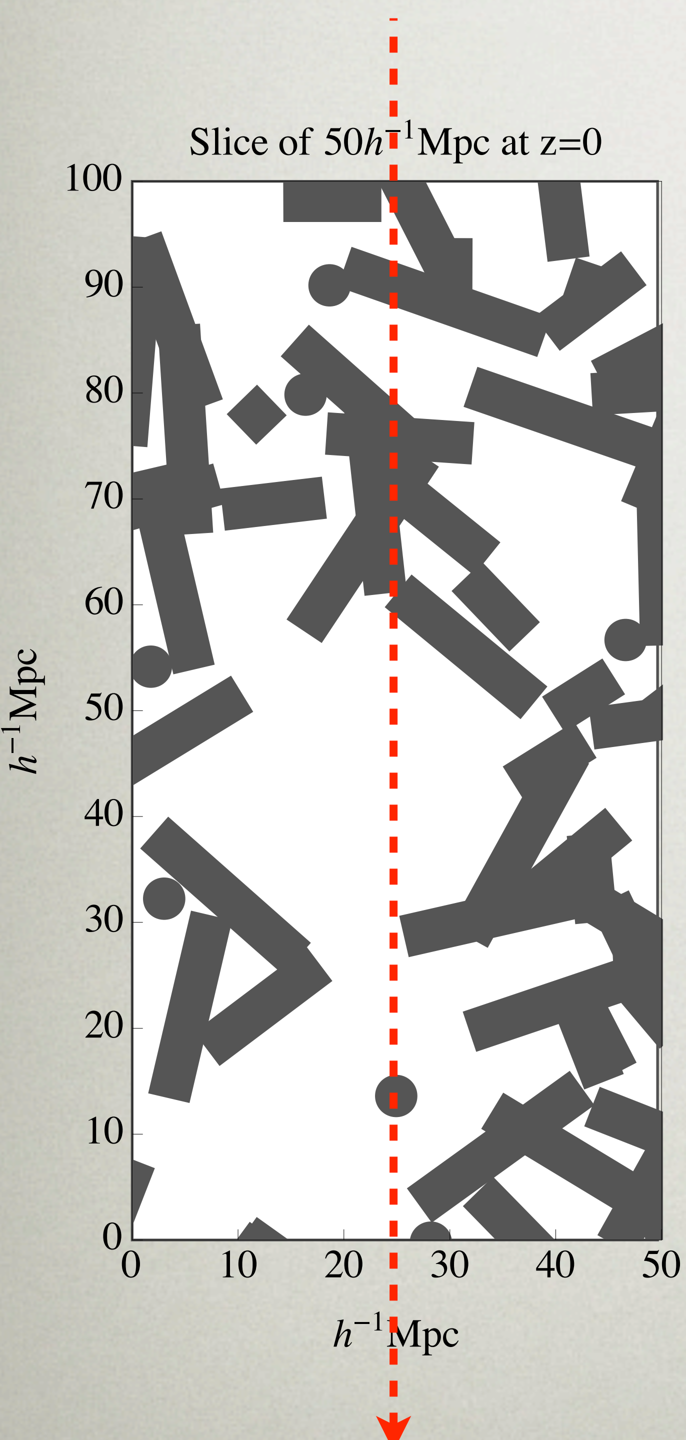
PDF approaches δ-function at $\kappa = 0$ for $N_O \to \infty$

This extremely simple formula describes both probabilistic and selection biases

...and can be further generalized

improved Halo Model (iHM: halos confined to filaments)

$$\kappa(\{k_{iu}\}) = \sum_{iu} \kappa_{1iu}\Big(k_{iu} - \Delta N_{iu}\Big)$$



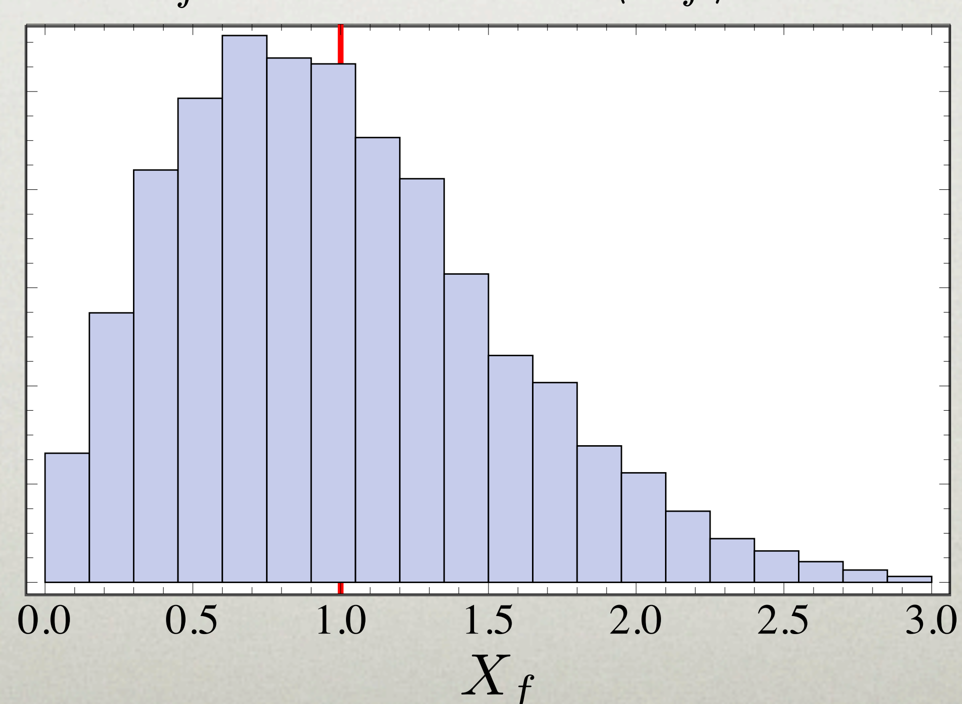
where k_{iu} is distributed as a Poisson variable of parameter

$$\Delta N^{fh}_{iu} = X_{fi}\,\Delta N^{h}_{iu} \text{ where } X_{fi} = \frac{\sum_v k^f_{iv}\Sigma^f_{iv}}{\sum_v \Delta N^f_{iv}\Sigma^f_{iv}}$$

actual distance travelled through filaments

corresponding expected value

X_f-PDF $\langle X_f \rangle = 1$

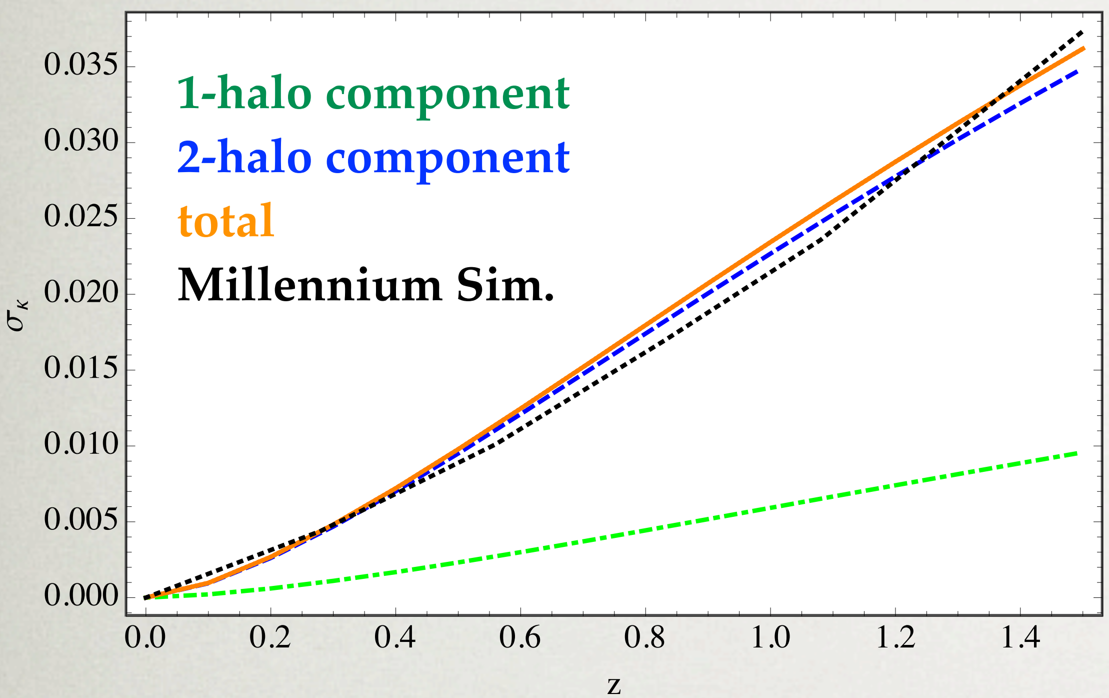


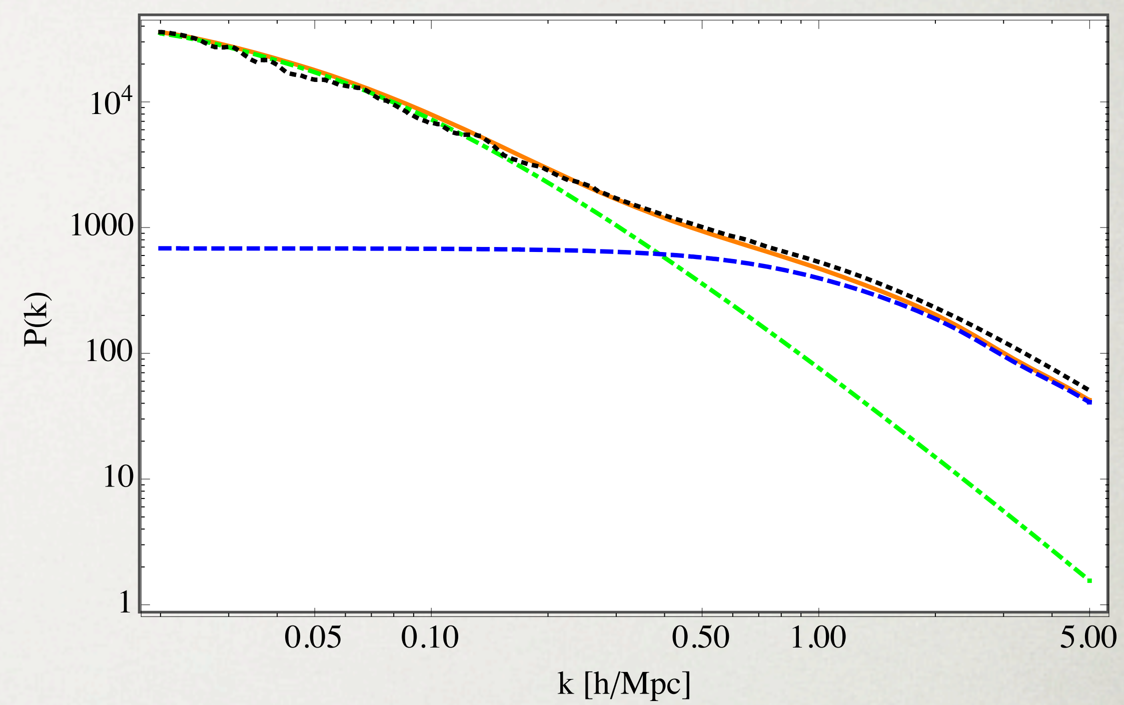
Parameter dependences

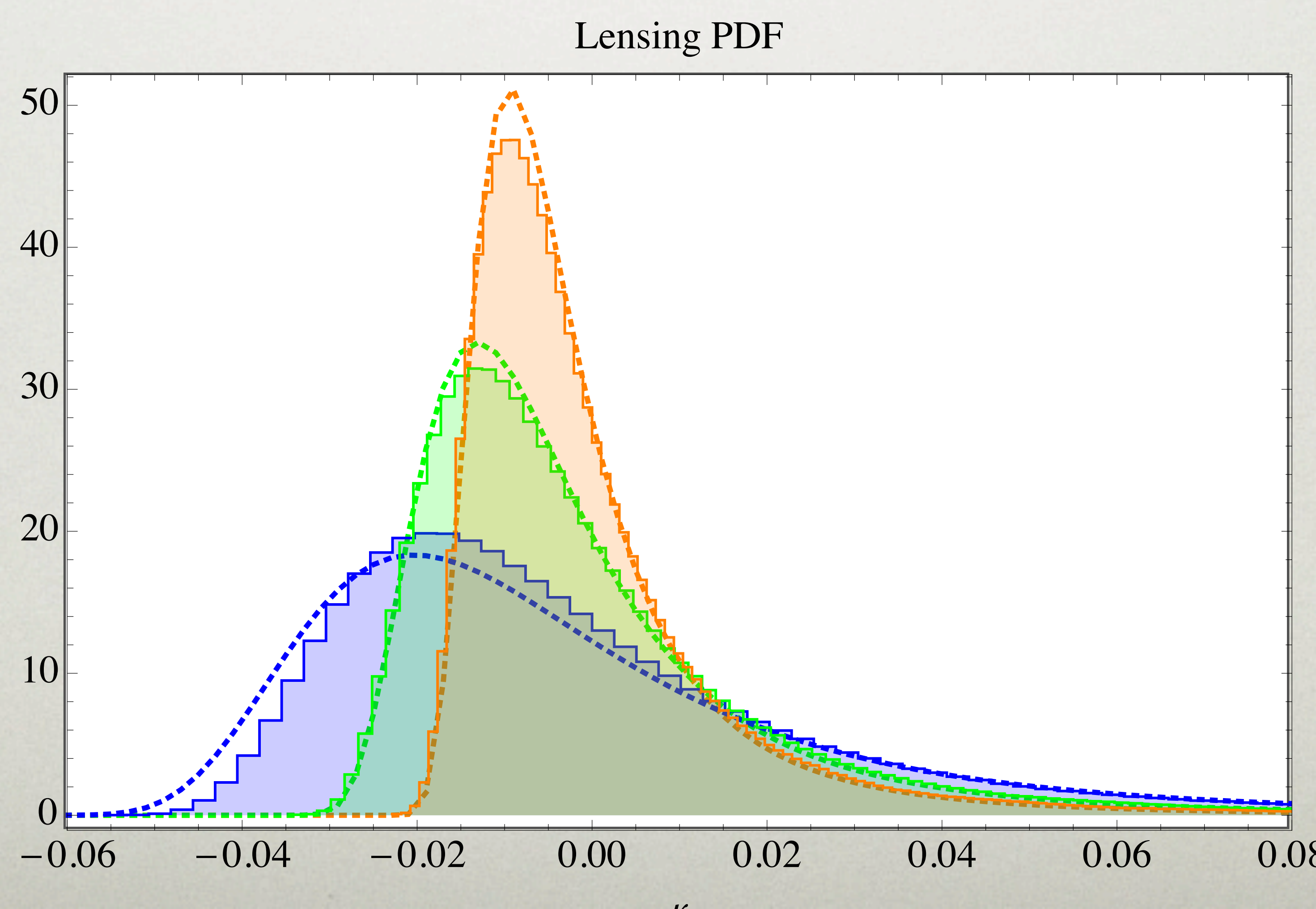
- the Halo Model part depends on its parameters (and their uncertainties) such as f(M,z), c(M,z), ...
- the parameters of the low-contrast structures (mass budget, sizes, ...) may be inferred from simulations or **by comparison** with observables such as lensing PDF, matter power spectrum, ...

sGL method gives the needed flexibility to do so!

Halo Model with sGL





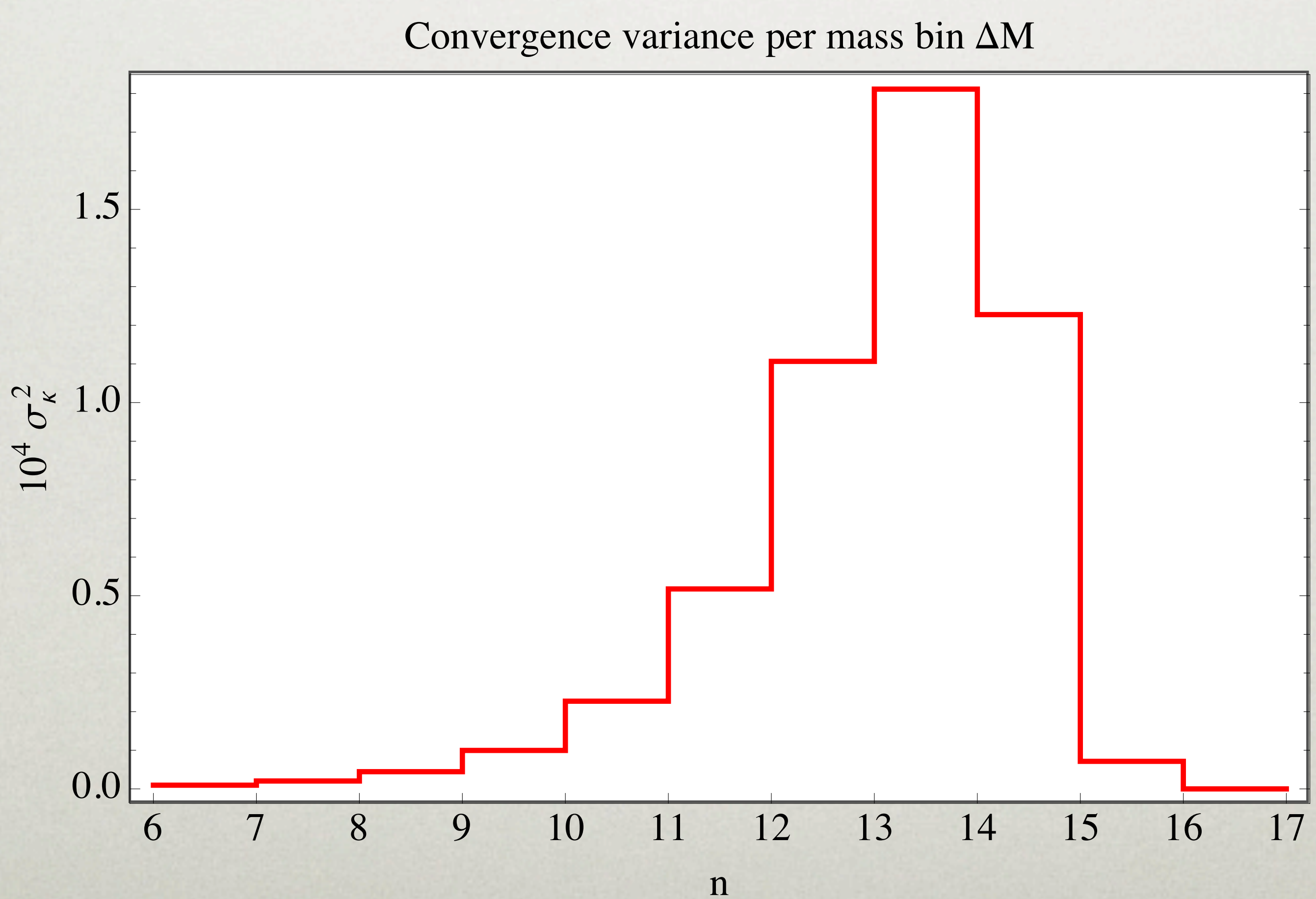


Kainulainen & Marra
arXiv:1101.4143v2

Halo Model with sGL

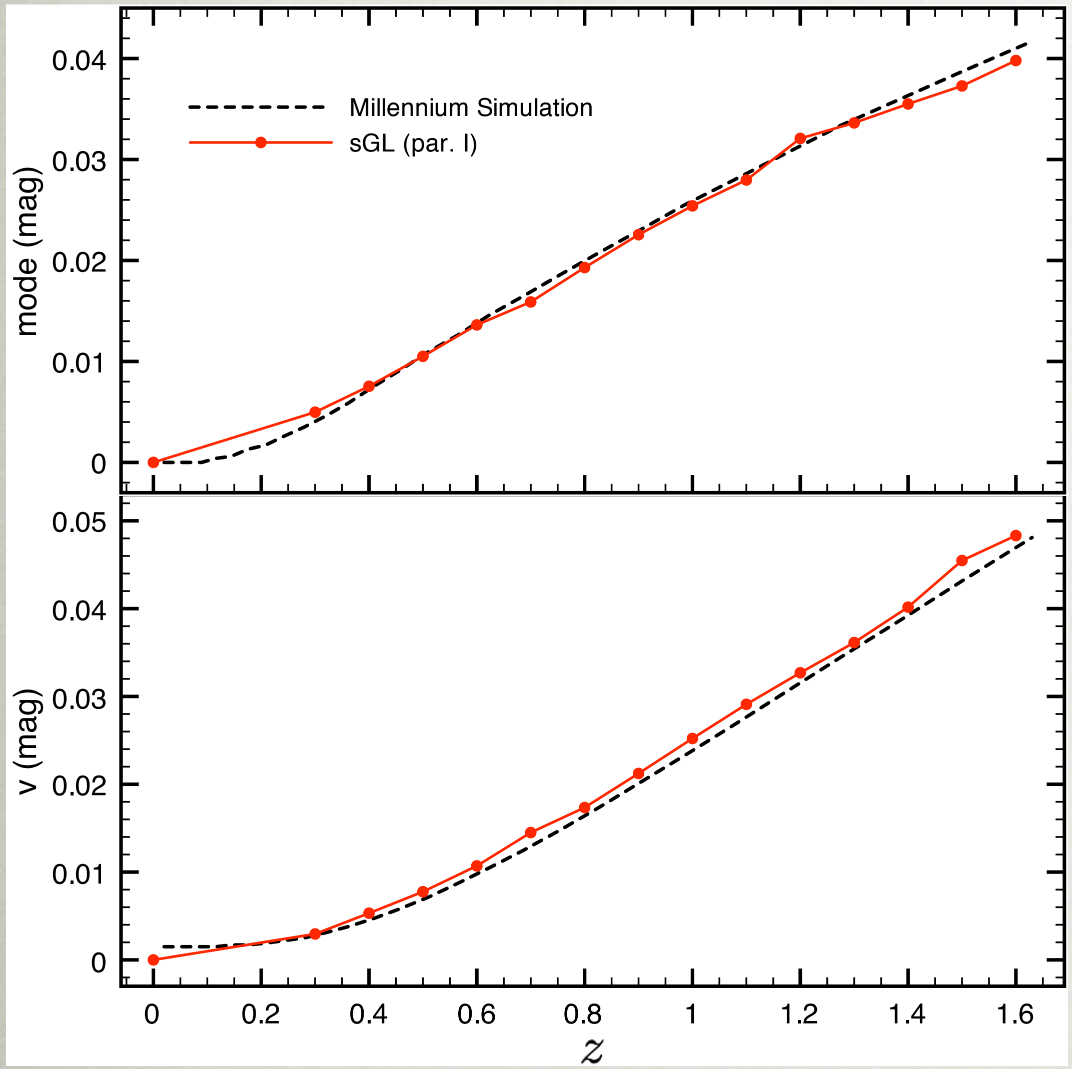
$$\langle \kappa_H \rangle = \int_0^{r_s} \mathrm{d}r\, G(r, r_s) \int_0^{\infty} \mathrm{d}n(M, z(r)) \times \\ \times \int_0^{R(M,z(r))} \mathrm{d}A(b)\, (P_{\mathrm{sur}} - 1)\, \Sigma(b, M, z(r))$$

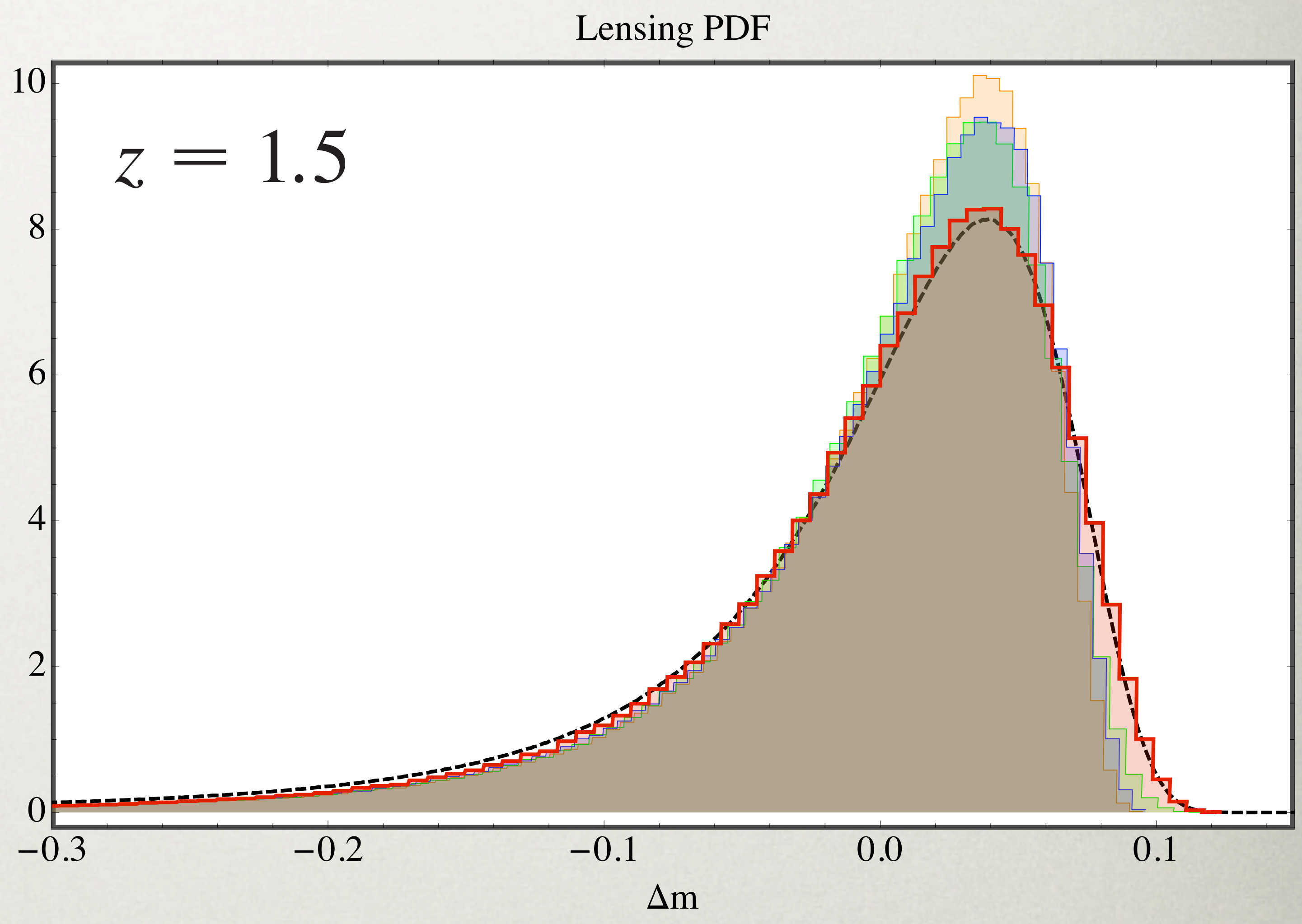
$$\sigma^2_{\kappa_H} = \int_0^{r_s} \mathrm{d}r\, G^2(r, r_s) \int_0^{\infty} \mathrm{d}n(M, z(r)) \times \\ \times \int_0^{R(M,z(r))} \mathrm{d}A(b)\, P_{\mathrm{sur}}\, \Sigma^2(b, M, z(r))$$



$$M_n = 10^n h^{-1} M_\odot$$

iHM vs Millennium Simulation





K. Kainulainen and V. Marra,
Phys. Rev. D 83, 023009 (2011)

Part III
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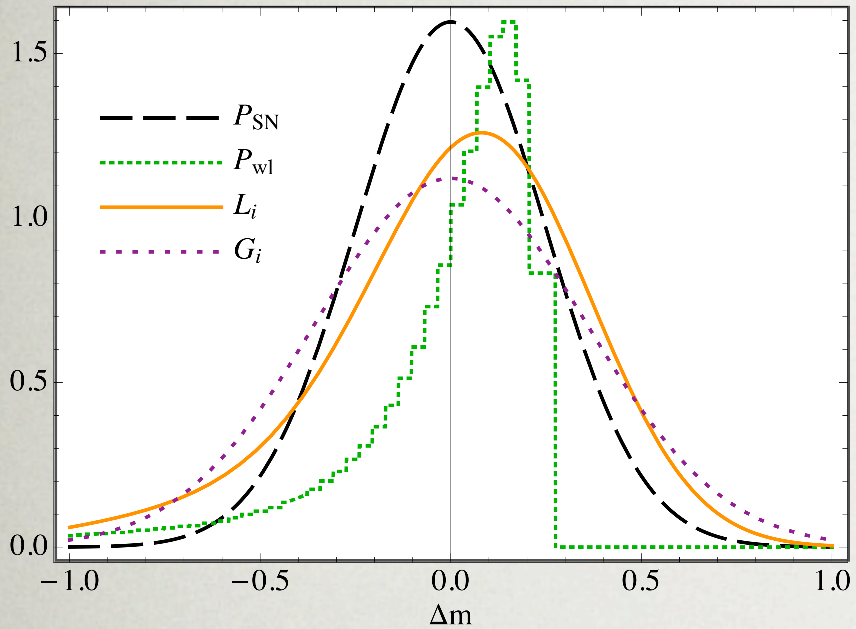
Applications

Key points for impact of lensing on parameter extraction

- lensing is cosmology dependent
- skewness of the PDF is important for small data sets
- selection effects survive in large data sets
- selection effects are degenerate with amount of LSS

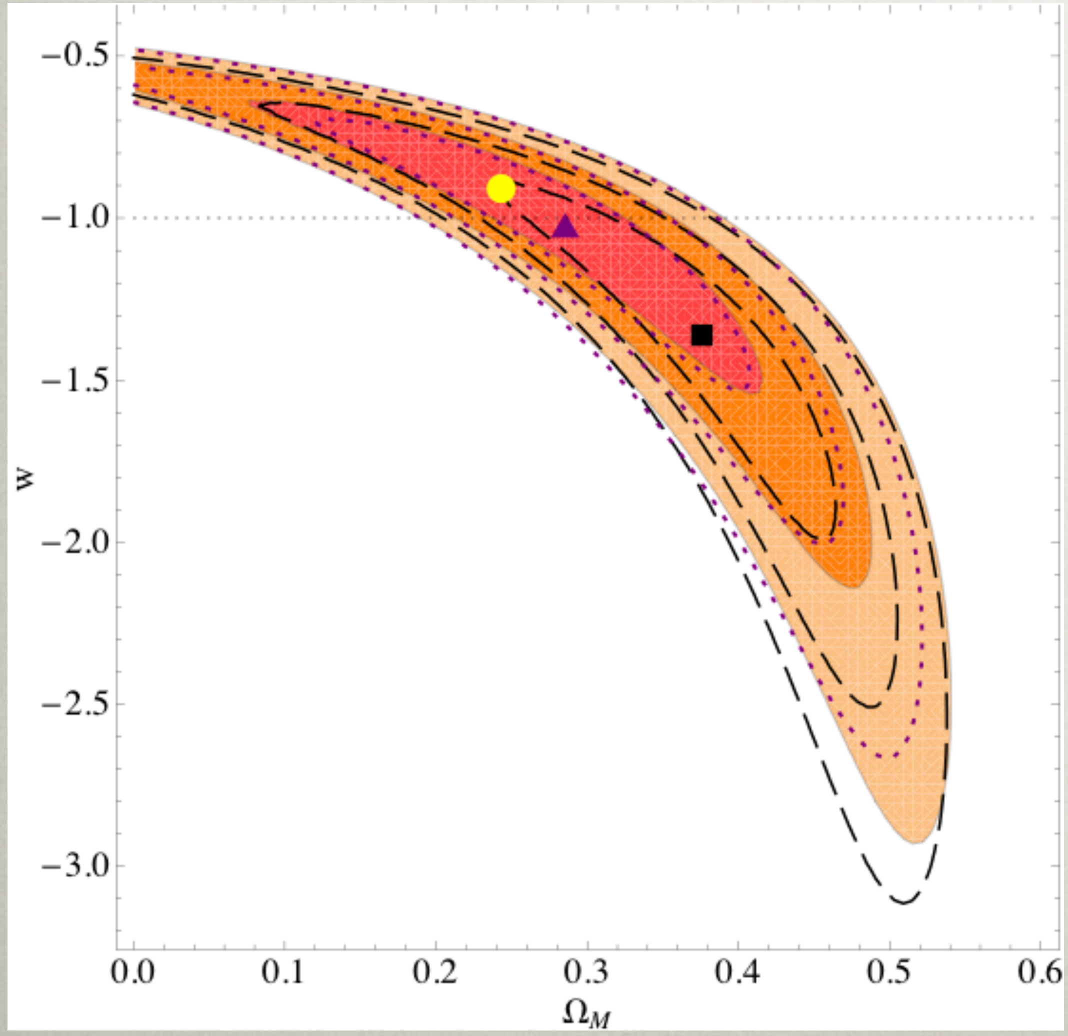
Union Compilation + sGL (single mass)



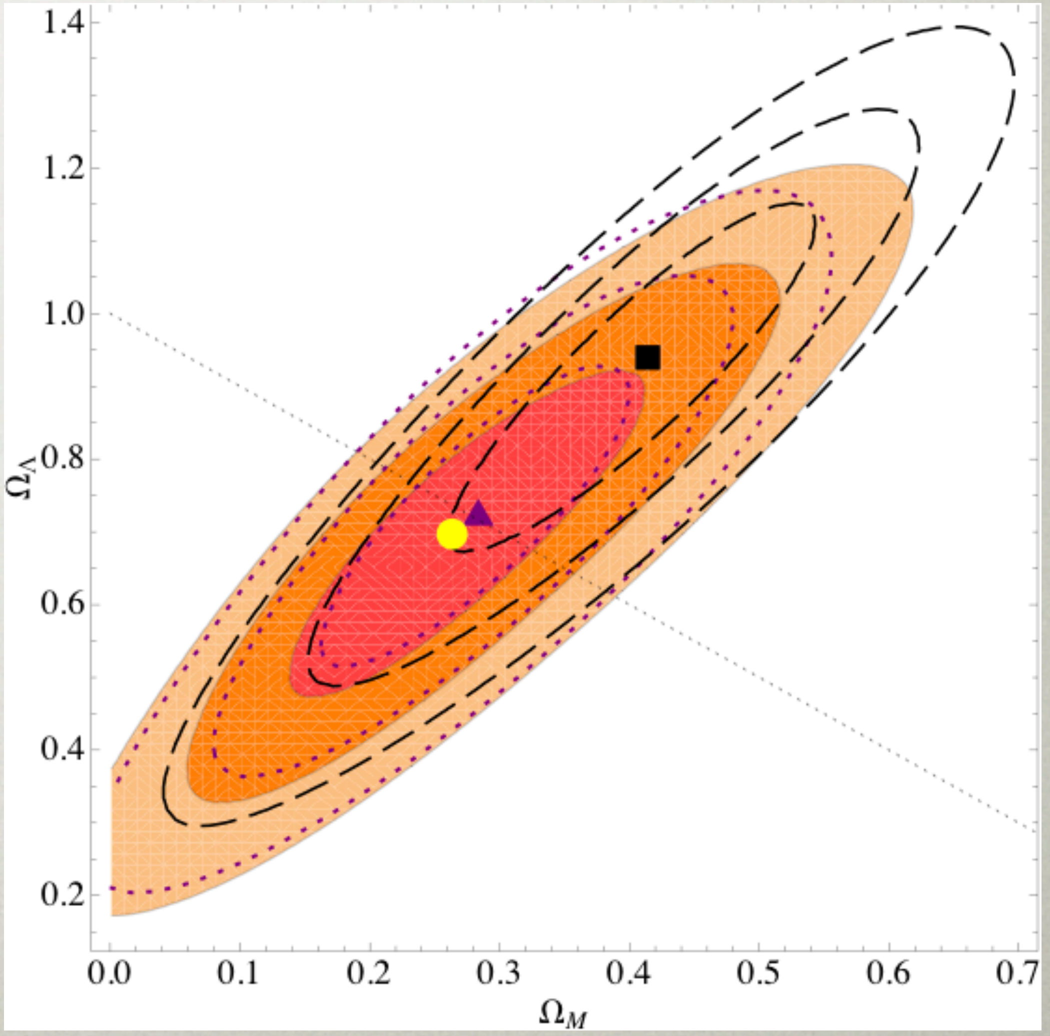
$$M = 5.6 \cdot 10^{14} h^{-1} \Omega_M \, M_\odot \qquad R_p^{\Lambda\mathrm{CDM}} \simeq 0.7 h^{-1} \text{ Mpc (NFW)}$$

$$L_i(\mu) = \int \mathrm{d}y \, P_{\mathrm{wl}}(y, z_i) P_{SN}(\Delta m_i - \mu - y, \sigma_i)$$

$$L(\Omega_M, \Omega_\Lambda, w) = \int d\mu \, \Pi_i L_i(\mu)$$



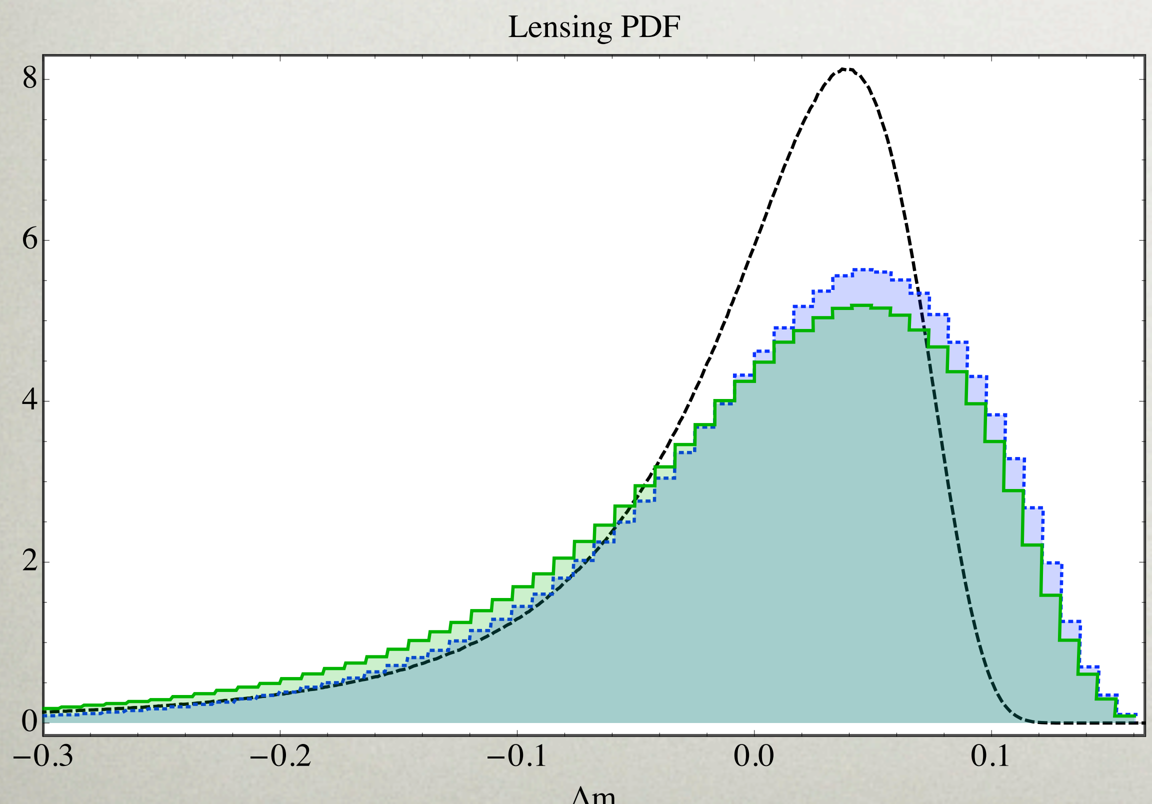
Amendola, Kainulainen, Marra and Quartin Phys. Rev. Lett 105, 121302 (2010)

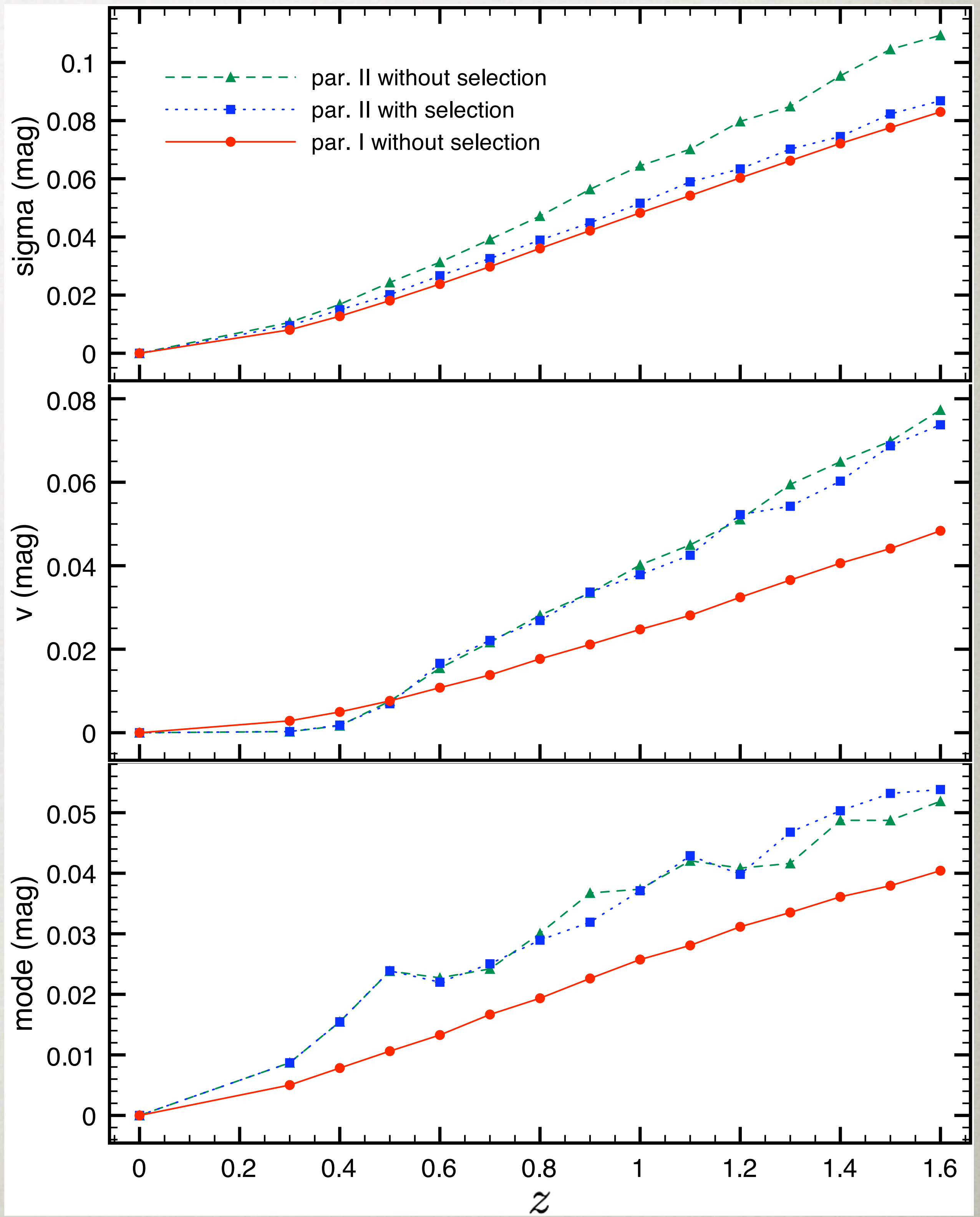


Selection effects vs LSS

Parameters	I	II
Δ_{f0}	4.5	9
R_{p0} $(h^{-1}\mathrm{Mpc})$	2	4
L_{p0} $(h^{-1}\mathrm{Mpc})$	20	25
profile	uniform	gaussian
M^{D}_{f0} $(h^{-1}\Omega_{M0}M_{\odot})$	9.3×10^{14}	9.3×10^{15}

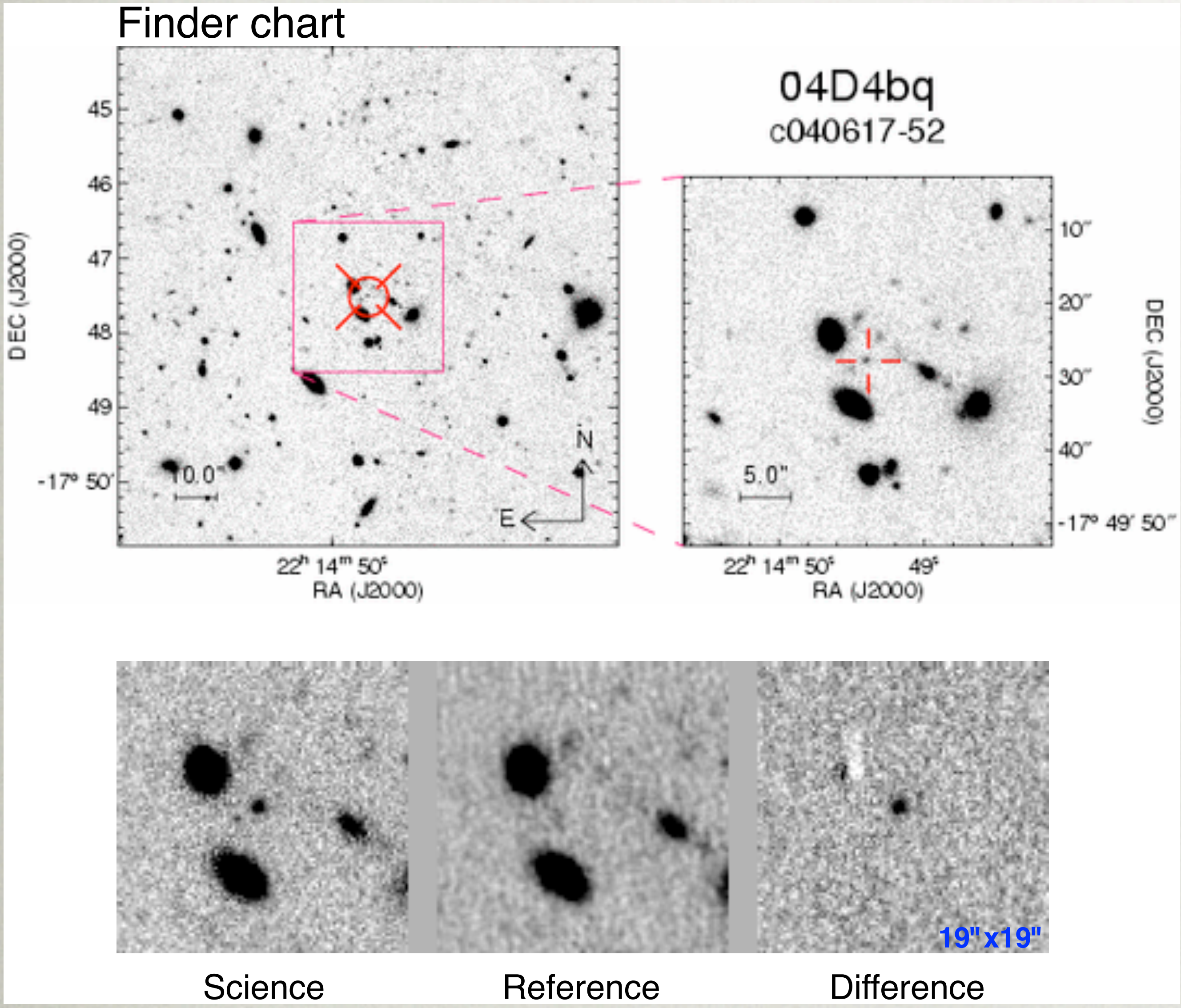
$$P_{\mathrm{sur}}(b) = \begin{cases} 0 & b/R < s_{\mathrm{cut}} \\ 1 & \text{otherwise} \end{cases} \qquad s_{\mathrm{cut}} = 10\%$$





Foreground light contamination

Supernova Legacy Survey



The SNLS consists of an imaging survey done on the 3.6m CFH telescope and a spectroscopic survey done on the 8m-class telescopes

$$\%\mathrm{inc} = \frac{f_{\mathrm{sci}} - f_{\mathrm{ref}}}{f_{\mathrm{ref}}} \times 100\%$$

%inc is used to rank the SNe. Candidates with a low percent increase may never be spectroscopically observed.

Work in progress...

- extend sGL to lensing excursion angle for CMB analysis
- extend sGL to finite smoothing angles
- foreground light contamination
- reanalyze Union Compilation with f(M,z) halos and biases
- extend sGL to double quasar counts
- ...

THANKS