Workshop:

Inhomogeneous Cosmologies

Jyväskylä
August 2011

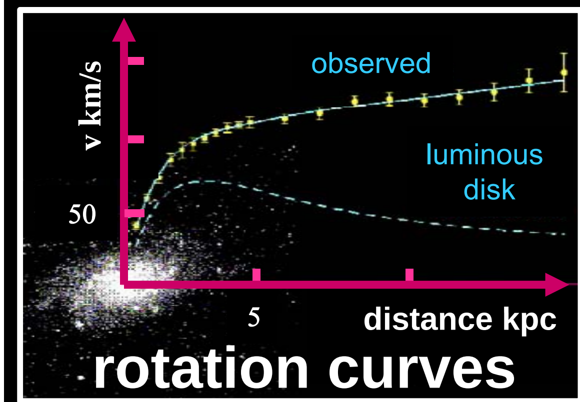
Rocky Kolb
University of Chicägo

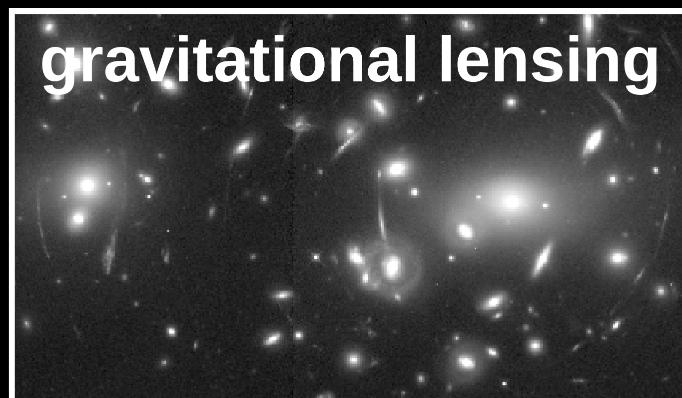
Dark Matter

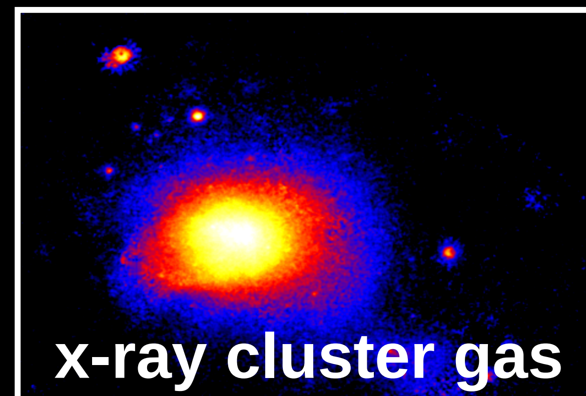
$$\Omega_M \sim 30\%$$

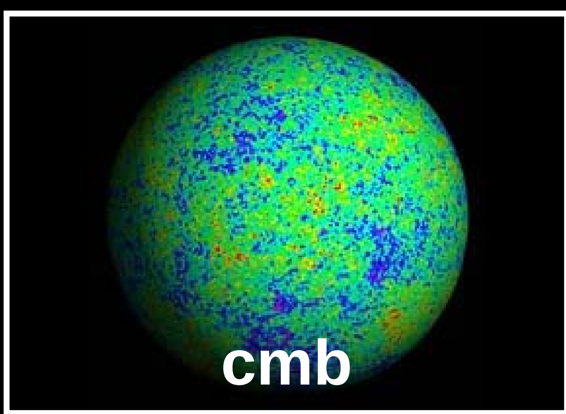
$$\Omega_B \sim 4\%$$

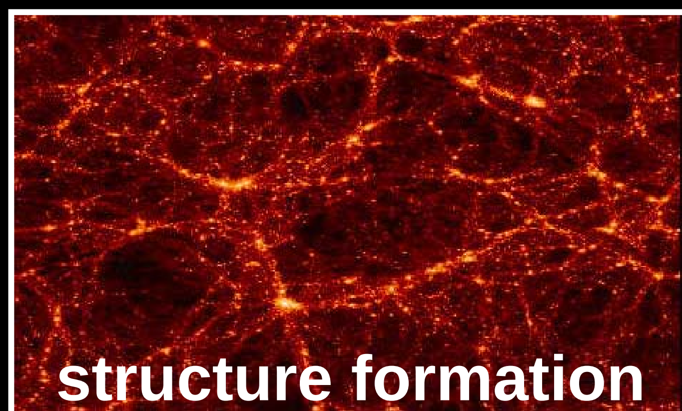
$$\Omega_i = \frac{\rho_i}{3H_0^2/8\pi G}$$











Dark Energy

Do not directly observe

- acceleration of the universe
- dark energy

We *infer* acceleration/dark energy by comparing *observations* with the predictions of a *model*

All evidence for dark energy/acceleration comes from measurements of the expansion history of the Universe

Expansion History of the Universe

Friedmann equation ($G_{00} = 8\pi G T_{00}$)

Hubble constant | curvature | matter | radiation

$$H^2(z) = H_0^2 \times \left[\Omega_k (1+z)^2 + \Omega_M (1+z)^3 + \Omega_R (1+z)^4\right]$$

- $\Omega_k + \Omega_M + \Omega_R = 1$

$$H^2(z) = H_0^2 \times \left[(1-\Omega_M-\Omega_R)(1+z)^2 + \Omega_M (1+z)^3 + \Omega_R (1+z)^4\right]$$

- radiation contribution (Ω_R) small for $z \lesssim 10^3$

$$H^2(z) = H_0^2 \times \left[(1-\Omega_M)(1+z)^2 + \Omega_M (1+z)^3\right]$$

- "All of observational cosmology is a search for two numbers." (H_0 and Ω_M) — Sandage, *Physics Today*, 1970

Expansion History of the Universe

Friedmann equation ($G_{00} = 8\pi G T_{00}$)

Hubble constant | cosmological constant | curvature | matter | radiation

$$H^2(z) = H_0^2 \times \left[\Omega_\Lambda (1+z)^0 + \Omega_k (1+z)^2 + \Omega_M (1+z)^3 + \Omega_R (1+z)^4\right]$$

- [Could add Ω_{walls} ($1+z$)^1]
- $1 = \Omega_\Lambda + \Omega_k + \Omega_M + \Omega_R$
- radiation contribution (Ω_R) small for $z \lesssim 10^3$
- Ω_k well determined (close to zero) from CMB
- Ω_M reasonably well determined

Expansion History of the Universe

Friedmann equation ($G_{00} = 8\pi G T_{00}$)

Hubble constant → H_0^2; dark energy → $\Omega_w (1+z)^{3(1+w)}$; curvature → $\Omega_k (1+z)^2$; matter → $\Omega_M (1+z)^3$; radiation → $\Omega_R (1+z)^4$

$$H^2(z) = H_0^2 \times \left[\Omega_w (1+z)^{3(1+w)} + \Omega_k (1+z)^2 + \Omega_M (1+z)^3 + \Omega_R (1+z)^4\right]$$

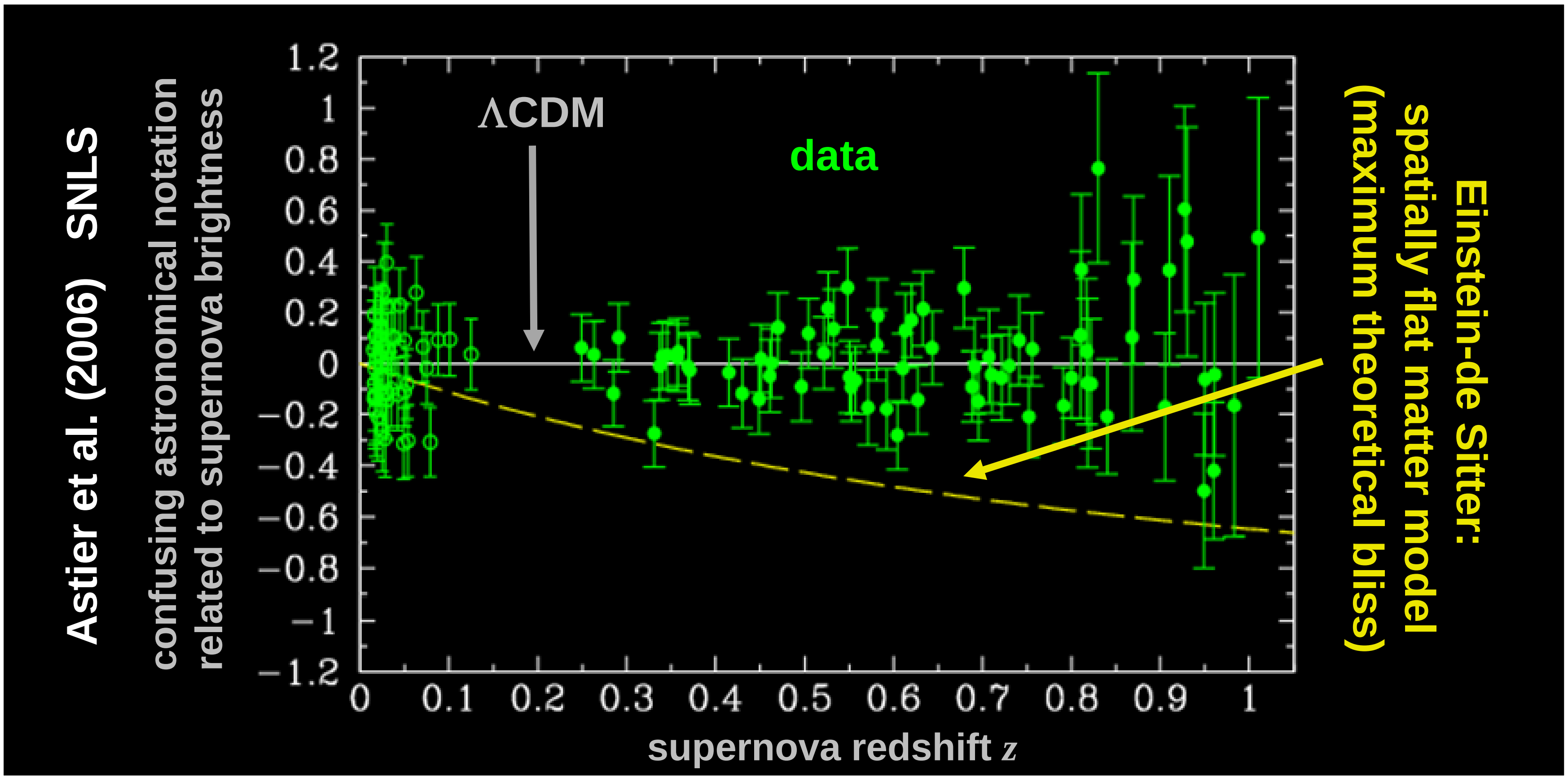
Equation of state parameter: $w = p / \rho \quad (w = -1$ for $\Lambda)$

if $w = w(z)$: $\quad (1+z)^{3(1+w)} \rightarrow \exp\left(-3\int_0^z \frac{dz'}{z'}\left[1 + w(z')\right]\right)$

parameterize: $w(z) = w_0 + w_a\, z / (1+z)$

Cosmology is a search for two numbers (w_0 and w_a).

The Cosmological Constant



The case for Λ:

1) Hubble diagram (SNe)
2) Cosmic Subtraction ($1 - 0.3 = 0.7$)
3) Baryon acoustic oscillations
4) Weak lensing
5) Galaxy clusters
6) Age of the universe
7) Structure formation

Taking Sides!

Can't hide from the data – ΛCDM too good to ignore

- SNe
- Subtraction: $1.0 - 0.3 = 0.7$
- Baryon acoustic oscillations
- Galaxy clusters
- Weak lensing
- …

$H(z)$ not given by Einstein–de Sitter

$$\boxed{G_{00}\,(\text{FLRW}) \neq 8\pi G\, T_{00}(\text{matter})}$$

Modify right-hand side of Einstein equations (ΔT_{00})

1. Constant ("just" a cosmological constant)
2. Not constant (dynamics described by a scalar field)

Modify left-hand side of Einstein equations (ΔG_{00})

3. Beyond Einstein (non-GR)
4. (Just) Einstein (back reaction of inhomogeneities)

The Cosmological Constant

The Unbearable Lightness of Nothing

$\rho_{\Lambda} = 10^{-30}$ g cm^{-3} ….. so small, and yet not zero!

Anthropic/Landscape/Cosmoillogical Constant

- Many sources of vacuum energy.
- String theory has many ($>10^{500}$?) vacua … the landscape.
- The multiverse could populate many (all?) vacua.
- Very, very rarely vacua have cancellations that yield a small Λ.
- While exponentially uncommon, they are preferred because …
  … more common values of Λ results in an inhospitable universe.

Anthropic principle requires $\Lambda \leq \Lambda_{\mathrm{OBS}}$.
Explains a $(10^{120}-1)\sigma$ result.

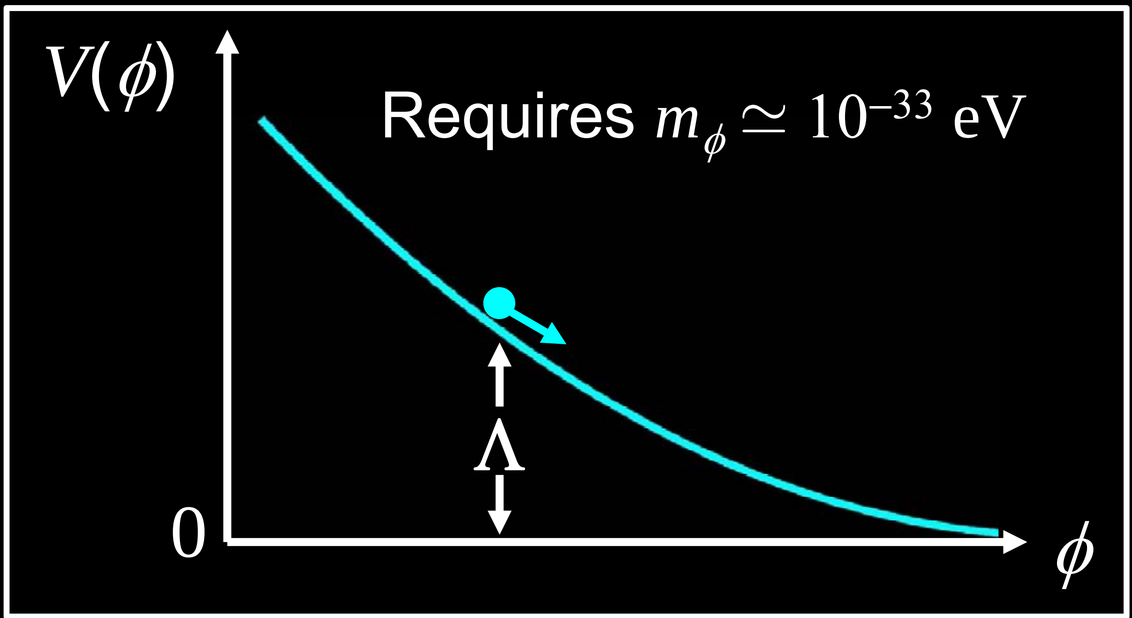
Anthropic/Landscape/Cosmoillogical Constant

- The anthropic “excuse” can explain the cosmoillogical constant.
- Perhaps there is no better idea than the anthropic principle …

  … (even without ideas, one can still have principles).
- But principles must not be applied selectively.
- What does this mean for particle physics?
  - Does it explain the weak scale/Planck scale hierarchy?
  - Who needs low-energy SUSY?
  - Give up searching for many answers (masses, etc.).
  - No dreams of a final theory.
- Is particle physics an environmental science?

Quintessence

- Many possible contributions.
- Why then is total so small?
- Perhaps some dynamics sets global vacuum energy to zero …
  … but we're not there yet!



- Can nature admit ultralight scalar fields?
- Long-range forces?

Tools to Modify the <u>Left-Hand</u> Side

- Braneworld modifies Friedmann equation

  Friedmann equation not from $G_{00} = 8\pi G\, T_{00}$ — Binetruy, Deffayet, Langois

- Gravitational force law modified at large distance

  Five-dimensional at cosmic distances — Deffayet, Dvali, Gabadadze

- Tired gravitons

  Gravitons unstable-leak into bulk — Gregory, Rubakov & Sibiryakov

- Gravity changes at distance $R \approx \mathrm{Gpc}$

  Becomes repulsive — Csaki, Erlich, Hollowood & Terning

- $n = 1$ KK graviton mode very light

  $m \approx (\mathrm{Gpc})^{-1}$ — Kogan, Mouslopoulos, Papazoglou, Ross & Santiago

- Einstein & Hilbert got it wrong

  $f(R)$ $S = (16\pi G)^{-1} \int d^4x \sqrt{-g}\left(R - \mu^4/R\right)$ — Carroll, Duvvuri, Turner & Trodden

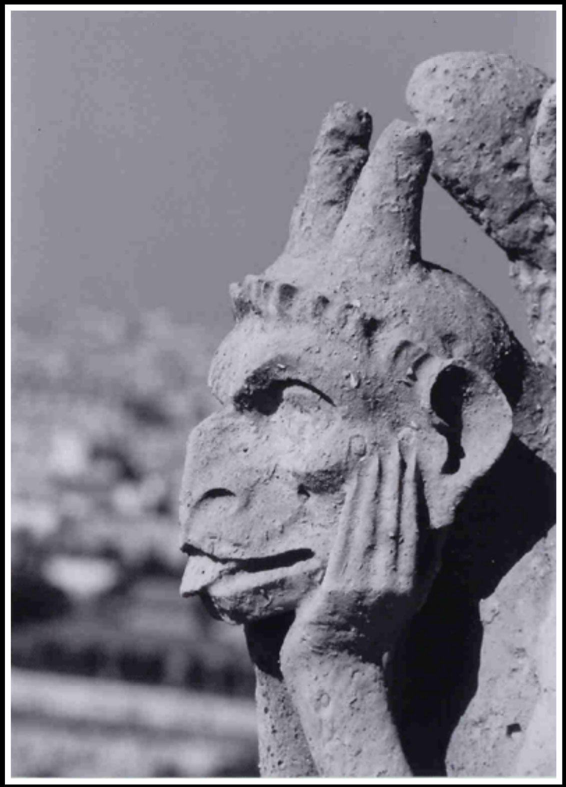
- "Backreaction" of inhomogeneities

  No dark energy — Why we're here

Backreaction of Inhomogeneities

ΛCDM is the approximate <u>phenomenological</u> model, but …

… there is no dark energy, gravity is not modified,
and the universe is not accelerating (in the usual sense).



Backreaction Causes Allergic Reaction

Backreaction of Inhomogeneities

1. Large-scale (Gpc) inhomogeneities (usually LTB)
   Célérier, Zibin, Yoo, Bołejko, Valkenburg, Pääkkönen, …

2. Small-scale (30 Mpc) inhomogeneities
   Wald, Green, Hotchkiss, …

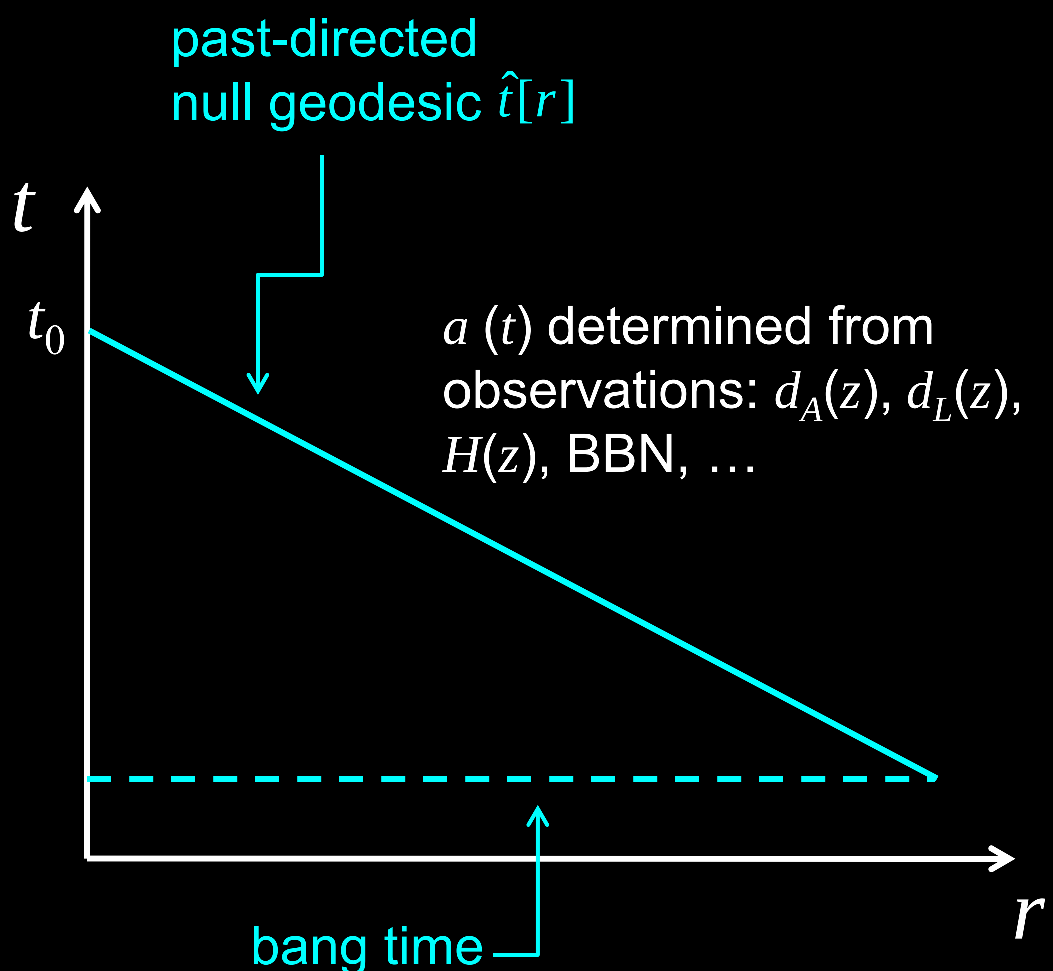
3. Swiss-cheese (garnished with spaghetti & meatballs)
   Marra, …

Homogeneous/Isotropic Models (FLRW)

Dynamical metric variable: $a\,(t)$

Evolution of $H(z)$ and $a\,(t)$ depend on curvature and $\rho\,(t)$

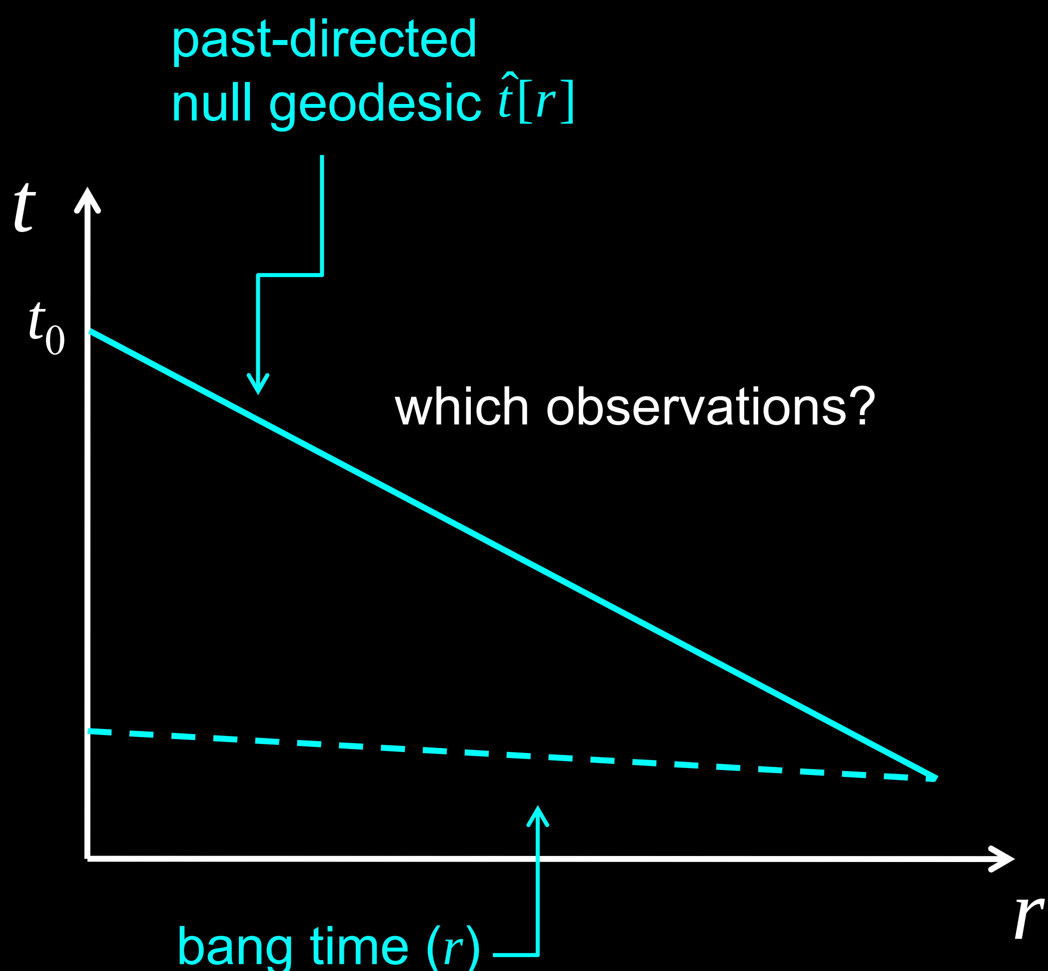
Flat FLRW: one function $\rho\,(t)$



- if homogeneous/isotropic
  $a(\hat{t}[r]) = a(t)$
- light-cone observations determine dynamical variable
- one degree of freedom $\Rightarrow$ one observable defines model
- consistency checks

Inhomogeneous/Isotropic Models (LTB)

Dynamical metric variables: $R(r,t)$, $R'(r,t)$: two expansion rates
Evolution of $R(r,t)$, R' depends on curvature (r), $\rho(r)$, bang time (r),
Coordinate choice determines one



Lottery approach

guess density (r) curvature (r) $\Rightarrow$ $d_L(z)$

$\Uparrow$ $\Downarrow$

no $\Leftarrow$ close to ΛCDM?

$\Downarrow$

write paper $\Leftarrow$ yes

Inhomogeneous/Isotropic Models (LTB)

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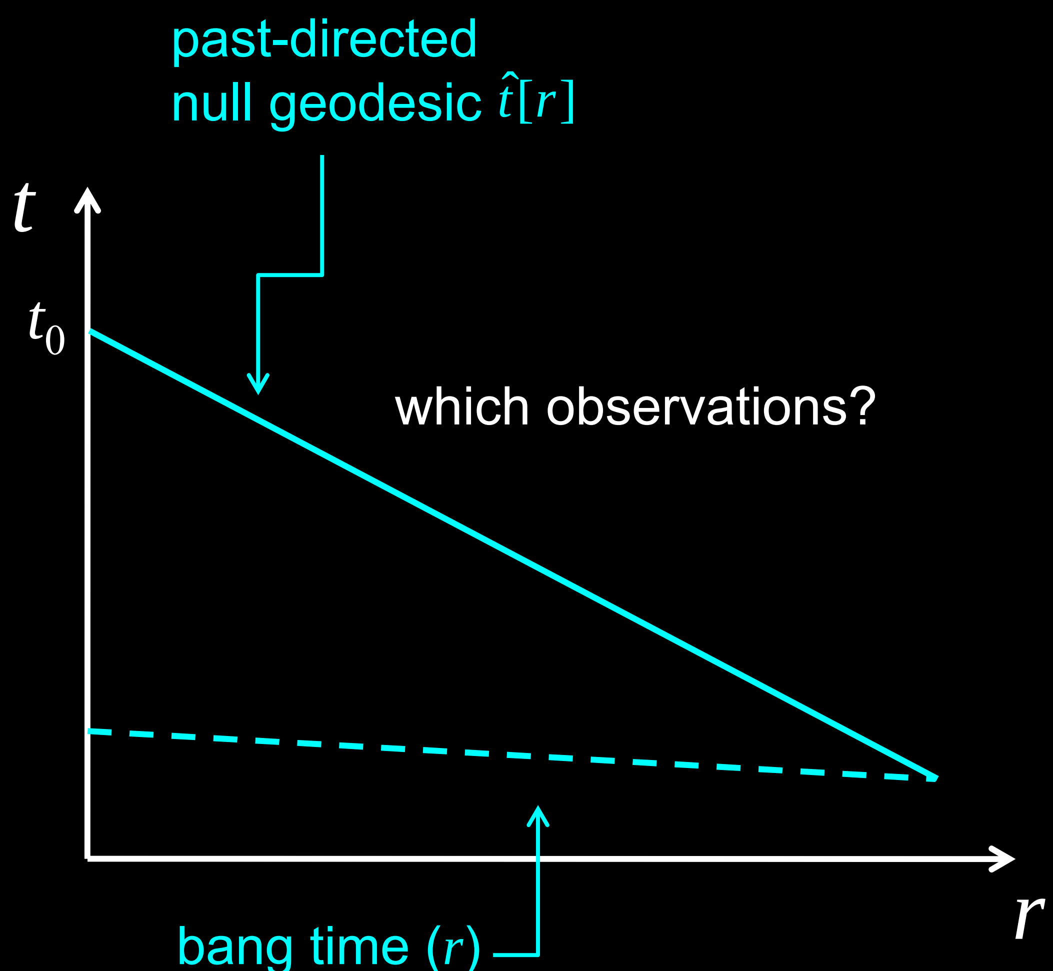

Inverse Problem

1. $d_L(z) + \rho(z)$
2. $d_L(z)$ + constant bang time

Inhomogeneous/ Isotropic Models (LTB)

Dynamical metric variables: $R(r,t)$, $R'(r,t)$: two expansion rates
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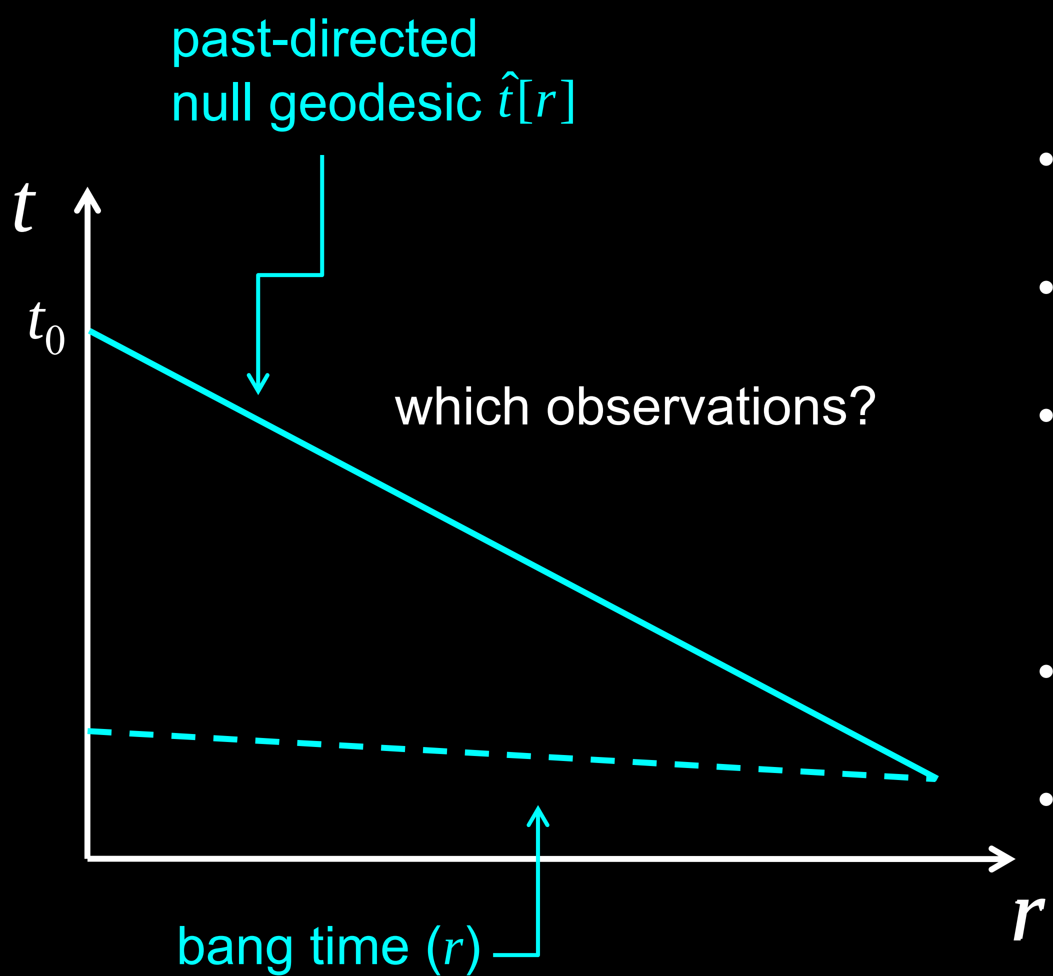
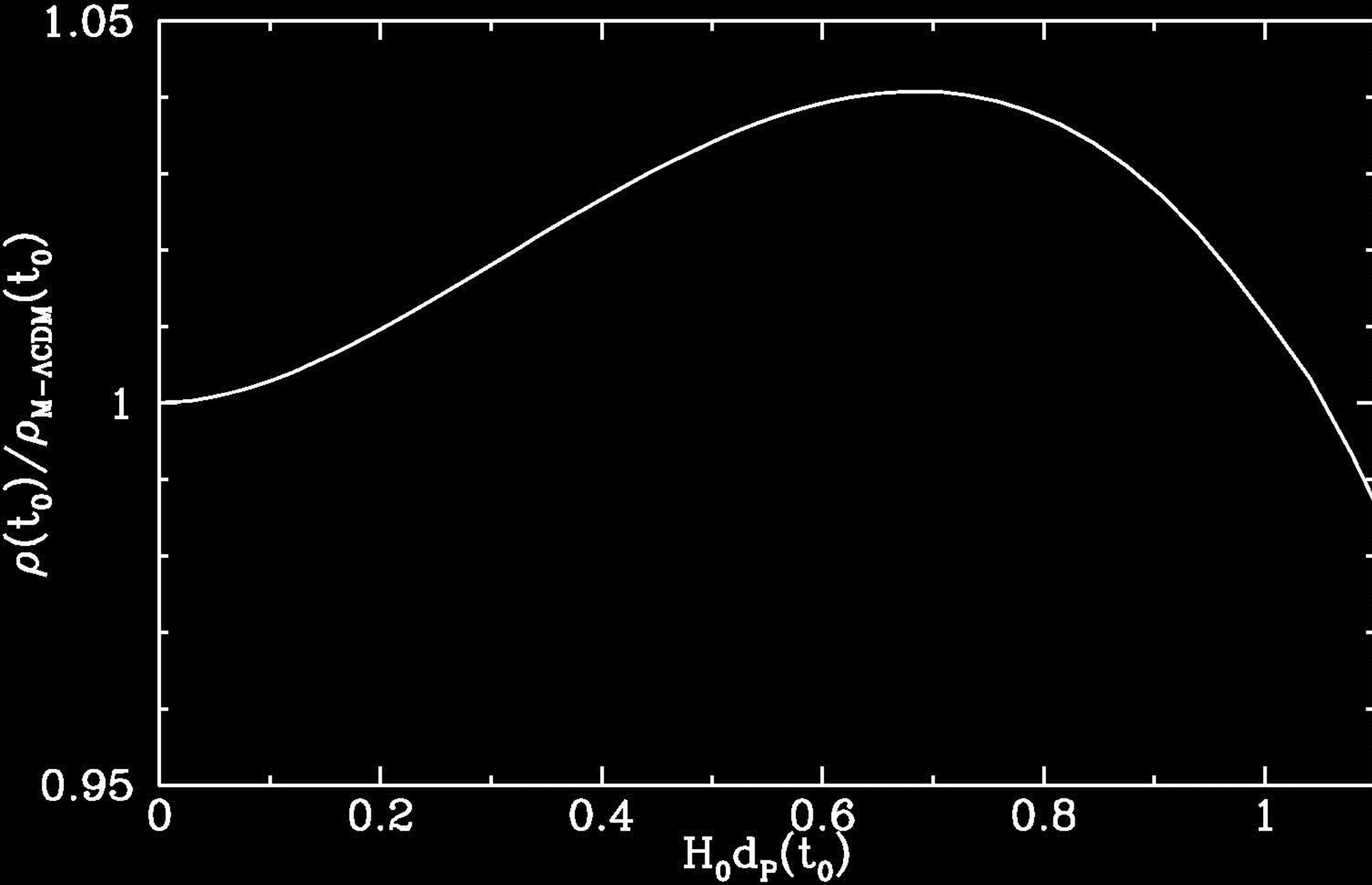
$$d_L(z) + \rho(z)$$



- Exactly fits LCDM $d_L(z) + \rho(z)$
- No “void”
- Bang time not constant (early-Universe evolution much different than FLRW
- Seems inconsistent with inflation
- Mixmaster-type behavior

Inhomogeneous/ Isotropic Models (LTB)



- No void, but a very mild overdensity out to the Hubble radius
- This is not observable … only observable is ρ on light cone

Inhomogeneous/ Isotropic Models (LTB)

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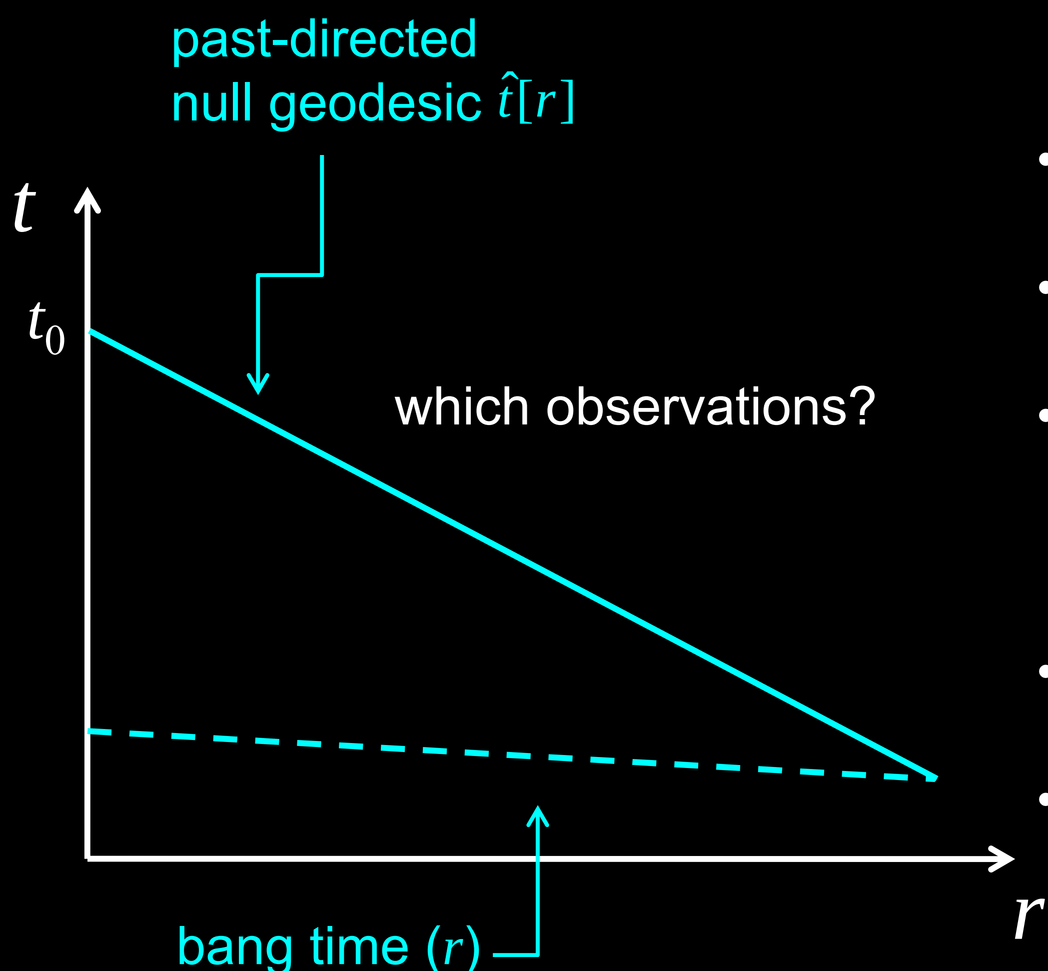

$$d_L(z) + \rho(z)$$

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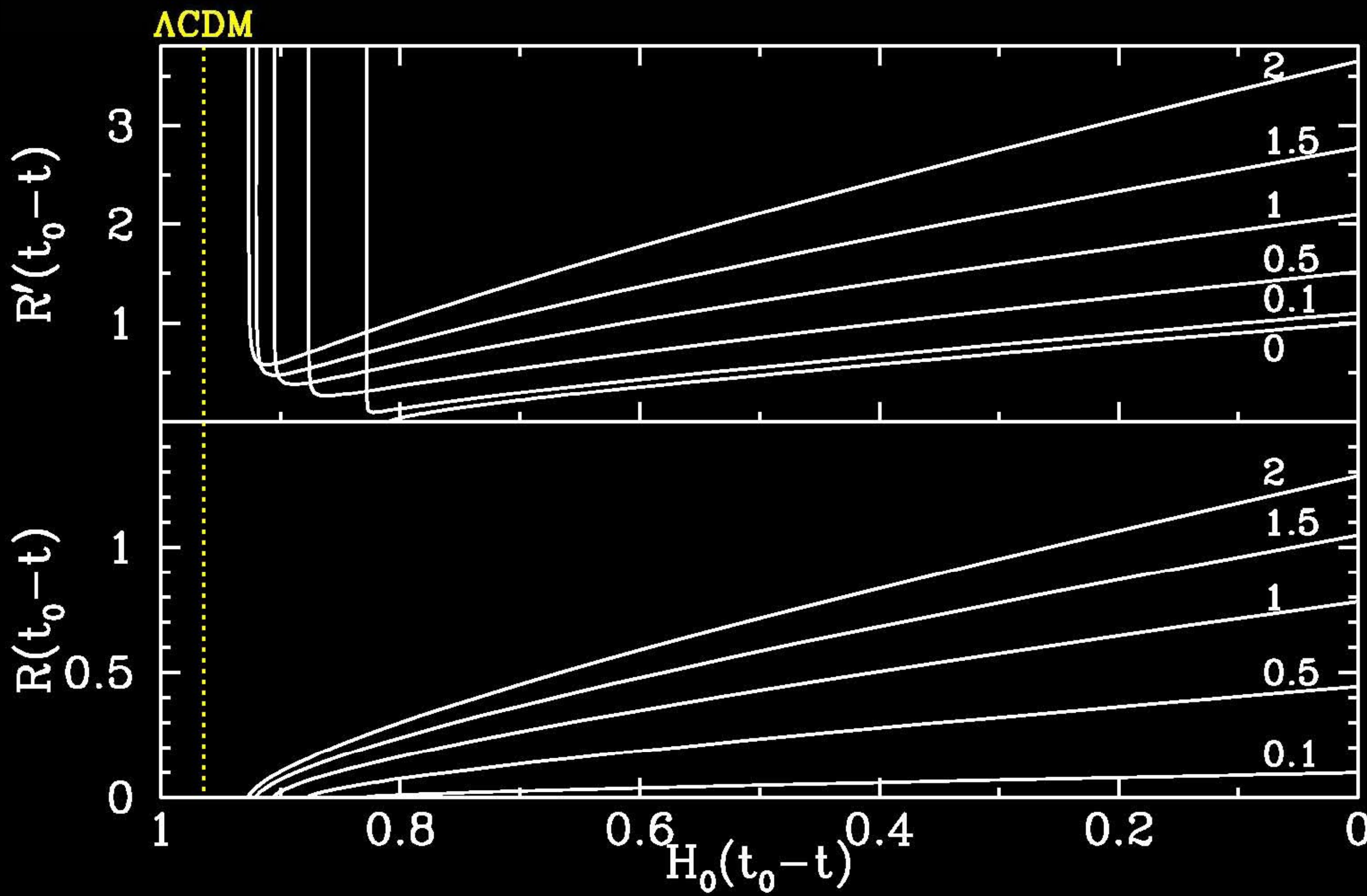
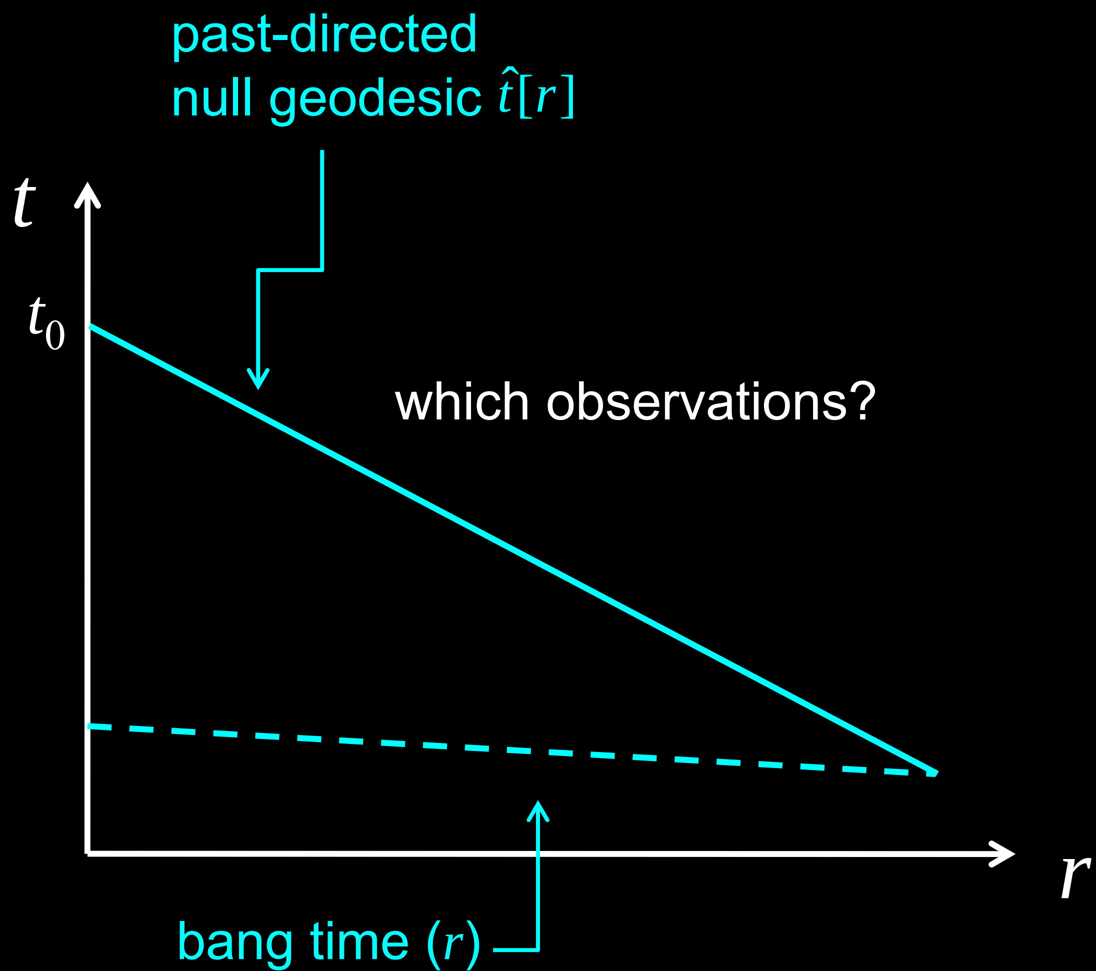
Inhomogeneous/ Isotropic Models (LTB)



- Bang-time is a function of r
- Mixmaster-like behavior

Inhomogeneous/ Isotropic Models (LTB)

Dynamical metric variables: $R(r,t)$, $R'(r,t)$: two expansion rates
Evolution of $R(r,t)$, R' depends on curvature (r), $\rho(r)$, bang time (r),
Coordinate choice determines one



CMB

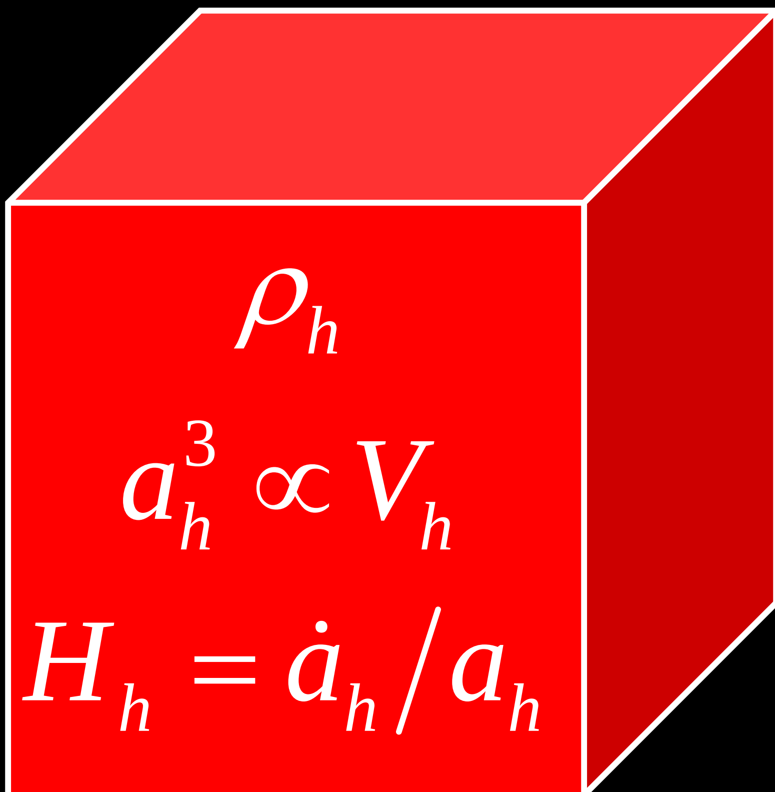
- isotropy
- kinematic S-Z
- late-time ISW
- …

Growth of Structure

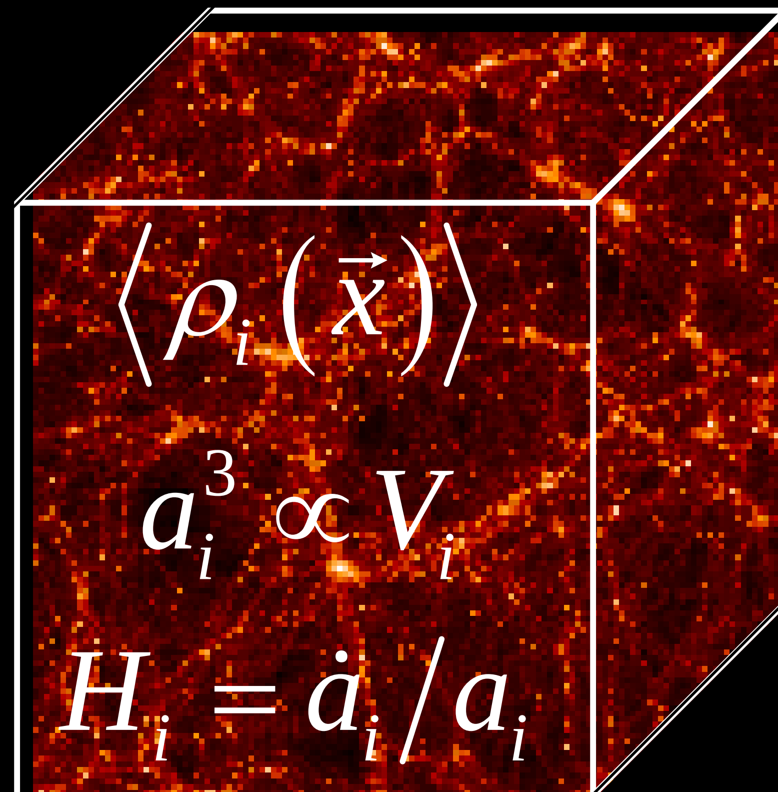
Redshift Drift

Small-Scale Inhomogeneities

Homogeneous model



Inhomogeneous model



(Buchert & Ellis)

$$\rho_h = \langle \rho_i(\vec{x}) \rangle \Rightarrow H_h = H_i \ ?$$

Small-Scale Inhomogeneities

Averaging Procedure

Buchert

- Define a coarse-grained scale factor:

$$a_D \equiv \left(V_D / V_{D0}\right)^{1/3} \qquad V_D = \int_D d^3x \sqrt{h}$$

- Coarse-grained Hubble rate:

$$H_D = \frac{\dot{a}_D}{a_D} = \tfrac{1}{3}\left\langle \Theta \right\rangle_D$$

- Effective evolution equations:

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G}{3}\left(\rho_{\text{eff}} + 3p_{\text{eff}}\right) \qquad \rho_{\text{eff}} = \left\langle \rho \right\rangle_D - \frac{Q_D}{16\pi G} - \frac{\left\langle {}^3R \right\rangle_D}{16\pi G}$$

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G}{3}\rho_{\text{eff}} \qquad 3p_{\text{eff}} = -\frac{3Q_D}{16\pi G} + \frac{\left\langle {}^3R \right\rangle_D}{16\pi G}$$

- Kinematical back reaction: $Q_D = \tfrac{2}{3}\left(\left\langle \Theta^2 \right\rangle_D - \left\langle \Theta \right\rangle_D^2\right) - 2\left\langle \sigma^2 \right\rangle_D$

Small-Scale Inhomogeneities

In ΛCDM $ds^2 = -\,dt^2 + a^2(t)\,dx^2$

with $a(t)$ from FLRW w/ matter $+\ \Lambda\ +$ whatever it takes

But perhaps $ds^2 = -\,dt^2 + a_D^2(t)\,dx^2$

with $a_D(t)$ from an averaging procedure and just matter

Small-Scale Inhomogeneities

Some thoughts on cosmological background solutions

Global Background Solution: FLRW solution generated using $\rho = \langle \rho \rangle_H$, ${}^3\mathcal{R} = \langle {}^3\mathcal{R} \rangle_H$ (sub-$H \to$ Hubble volume average), and the *local* equation of state (e.o.s.).

Average Background Solution: FLRW solution that describes volume expansion of our past light cone. Energy content, curvature, and e.o.s. that generates the *ABS* need not be $\langle \rho \rangle_H$, $\langle {}^3\mathcal{R} \rangle_H$, nor *local* e.o.s. (Buchert formalism)

Phenomenological Background Solution: FLRW model that best describes the observations on our light cone. Energy content, curvature, and e.o.s. that generates the *PBS* need not be $\langle \rho \rangle_H$, $\langle {}^3\mathcal{R} \rangle_H$, and *local* e.o.s. (Swiss-cheese example)

Small-Scale Inhomogeneities

Backreaction: the three backgrounds do not coincide

Strong Backreaction:

Global Background Solution does not describe expansion history (hence does not describe observations) (Buchert formalism)

Weak Backreaction:

Global Background Solution describes global expansion, but *Phenomenological Background Solution* differs (Swiss Cheese)

Small-Scale Inhomogeneities

• 2nd-order perturbation theory in $\phi(x)$ (Newtonian potential):

$$\frac{\langle \Theta - H \rangle}{H} = \cancel{-\frac{20\tau^2}{9}\langle \nabla^2\phi \rangle - \frac{23\tau^4}{54}\langle \nabla^2\phi \rangle \langle \nabla^2\phi \rangle} \quad \text{mean of} \quad \nabla^2\phi = 0$$

$$+\frac{130\tau^2}{27}\langle \phi^{,i}\phi_{,i} \rangle + \frac{4\tau^4}{27}\left(\langle \nabla^2\phi\nabla^2\phi \rangle - \langle \phi^{,ij}\phi_{,ij} \rangle \right)$$

Post-Newtonian Newtonian

– Each derivative accompanied by conformal time $\tau = 2/aH$

– Each factor of τ accompanied by factor of c.

– Highest derivative is highest power of $\tau \propto c$: “Newtonian”

– Lower derivative terms $\propto c^{-n}$: “Post-Newtonian”

– ϕ and its derivatives can be expressed in terms of $\delta\rho/\rho$

Small-Scale Inhomogeneities

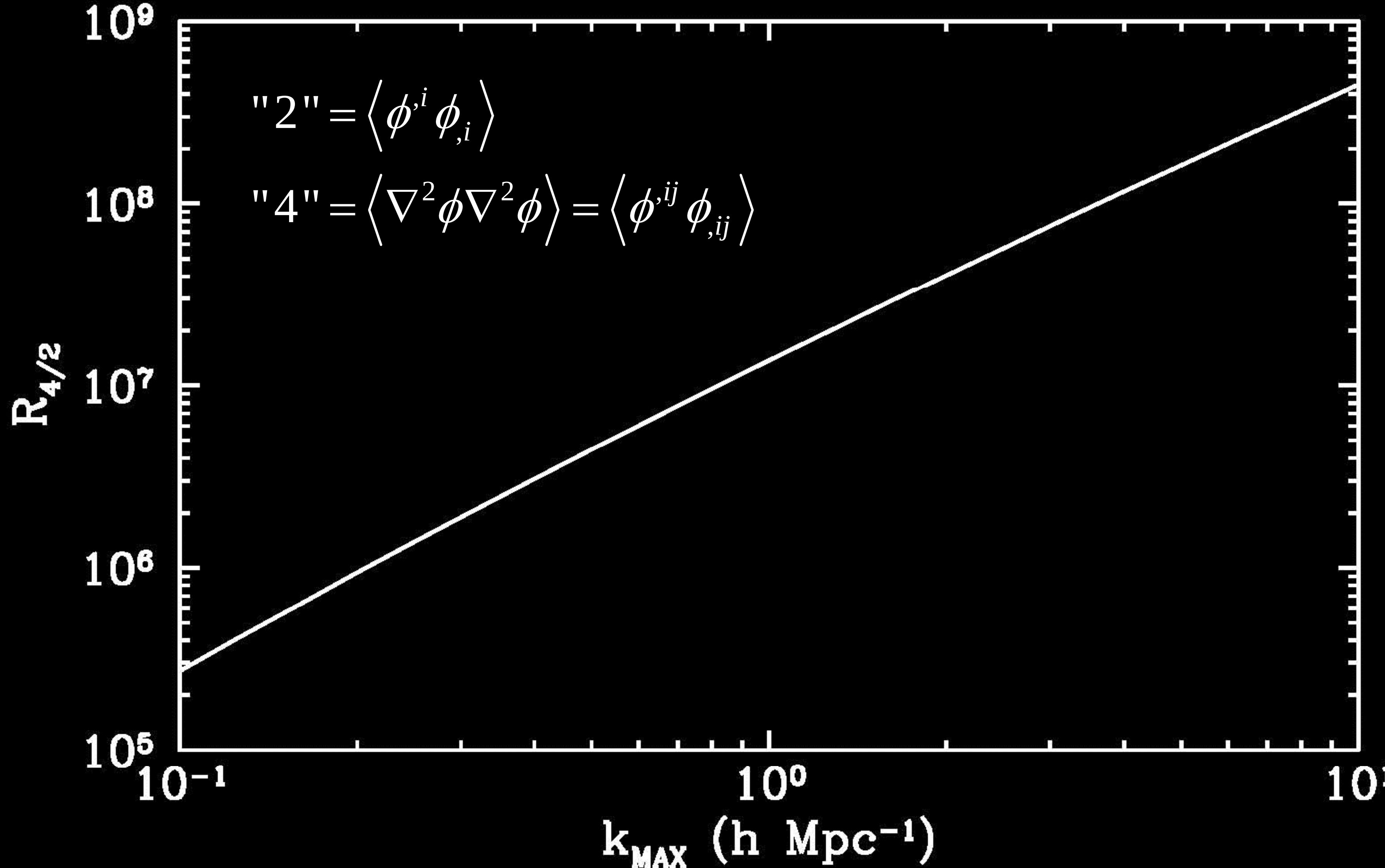
- $$\tau^2 \left\langle \nabla\phi \cdot \nabla\phi \right\rangle \simeq A^2 \frac{1}{a^2 H^2} \int_0^{k_H} dk\, k\, T^2(k) \sim 10^{-5} \frac{a}{a_0}$$
- $$\tau^4 \left\langle \nabla^2\phi \nabla^2\phi \right\rangle \simeq A^2 \frac{1}{a^4 H^4} \int_0^{k_H} dk\, k^3\, T^2(k) \sim 10^{0} \left(\frac{a}{a_0}\right)^2$$

– Individual Newtonian terms large, *i.e.*, $\langle \nabla^2\phi \nabla^2\phi \rangle = \mathcal{O}(1)$

– <u>*But*</u> total Newtonian term vanishes $\langle \nabla^2\phi \nabla^2\phi \rangle = \langle \phi^{,ij} \phi_{,ij} \rangle$

– Post-Newtonian: $\langle \nabla\phi \cdot \nabla\phi \rangle = \mathcal{O}(10^{-5})$ ***huge!*** (large k^2/a^2H^2)

Small-Scale Inhomogeneities



Small-Scale Inhomogeneities

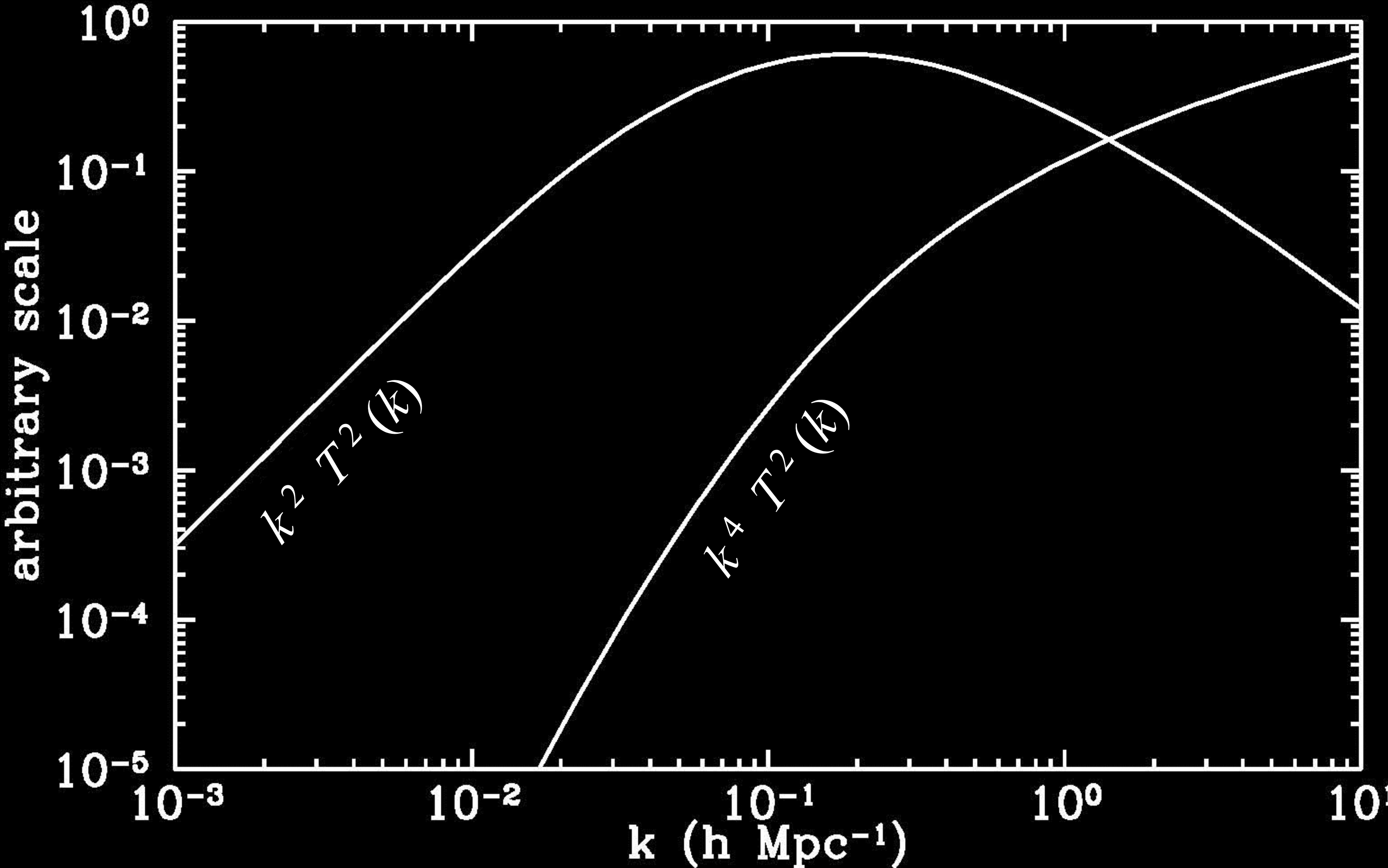
$$\Delta H \propto \mathcal{O}_{n,m} \equiv \tau^{n} \mathcal{D}^{n} \phi^{2m}$$

- $\mathcal{D}^{n} \phi^{2m} \rightarrow \left(\int d^{3}k \; k^{n/m} \; P_{\phi}(k)\right)^{m}$
- $\tau^{n} \rightarrow (a_0 H_0)^{-n} \; (a/a_0)^{n/2}$
- $\Delta H \propto (H_0^{-1} h \,\mathrm{Mpc}^{-1})^{n} A^{2m} (a/a_0)^{n/2} \left[\int (dk/k)\, k^{n/m} \; T^{2}(k)\right]^{m}$
- Higher terms are
  - numerically larger

$$\mathcal{O}_{2,1} \sim \tau^{2} \langle \phi^{,i} \phi_{,i} \rangle \sim 10^{-3} (a/a_0)$$
$$\mathcal{O}_{4,1} \sim \tau^{4} \langle \nabla^{2}\phi \nabla^{2}\phi \rangle \sim 10^{+4} (a/a_0)^{2}$$
$$\mathcal{O}_{6,2} \sim \mathcal{O}_{2,1} \times \mathcal{O}_{4,1} \sim 10^{+1} (a/a_0)^{3}$$

  - progressively peaked in the ultraviolet
  - must be cutoff

Small-Scale Inhomogeneities



Small-Scale Inhomogeneities

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- Higher terms are
  - numerically larger

$$\begin{aligned} \mathcal{O}_{2,1} &\sim \tau^{2} \langle \phi^{,i} \phi_{,i} \rangle &&\sim 10^{-3} (a/a_0) \\ \mathcal{O}_{4,1} &\sim \tau^{4} \langle \nabla^2\phi \nabla^2\phi \rangle &&\sim 10^{+4} (a/a_0)^{2} \\ \mathcal{O}_{6,2} &\sim \mathcal{O}_{2,1} \times \mathcal{O}_{4,1} &&\sim 10^{+1} (a/a_0)^{3} \end{aligned}$$

  - progressively peaked in the ultraviolet
  - must be cutoff

Small-Scale Inhomogeneities

inflation-produced perturbations $z \sim 10^{25}$ → matter-dominated growth $z \sim 10^4$ → dark energy dominates $z \sim 1$ → metric coefficients small

Standard Lore

$$ds^2 = -\,(1+2\,\psi)\,dt^2 + a^2(t)\,(1-2\phi)\,dx^2$$

$a(t)$ is the ΛCDM background solution

inflation-produced perturbations $z \sim 10^{25}$ → matter-dominated growth $z \sim 10^4$ → scales become nonlinear $z \sim 1$ → require averaging procedure

Backreaction

$$ds^2 = -\,(1+2\,\psi)\,dt^2 + a_D{}^2(t)\,(1-2\phi)\,dx^2$$

$a_D(t)$ is the averaged scale factor

Why the Allergic Reaction?

- We have been driven to consider some remarkable possibilities
  - 10^{500} ground states in the landscape, anthropic rationalization
  - Modification of GR *in the infrared*
  - Lorentz violation
  - 10^{-33} eV scalar fields
  - Extra dimensions
- There should be some effort in rethinking some basic old things
  - Is there a global background solution?
  - Is ΛCDM just a phenomenological background solution?
  - Could it revolutionize something in the early universe (e.g, inflation)?
- Backreactions can potentially do three remarkable things
  - Explains “why now”
  - Expresses “dark energy” parameters in terms of observables
  - Potentially predict “Ω_Λ”

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