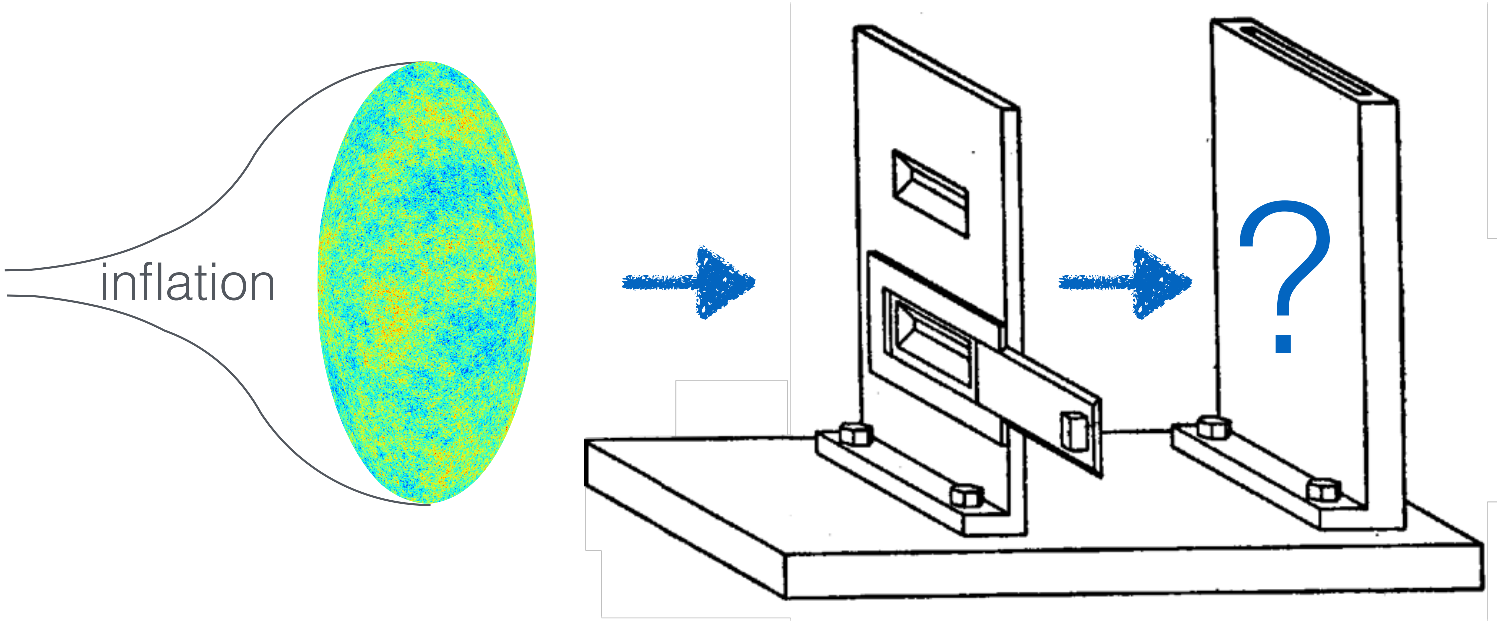
Can we show that cosmological structures are of quantum-mechanical origin?



Vincent Vennin

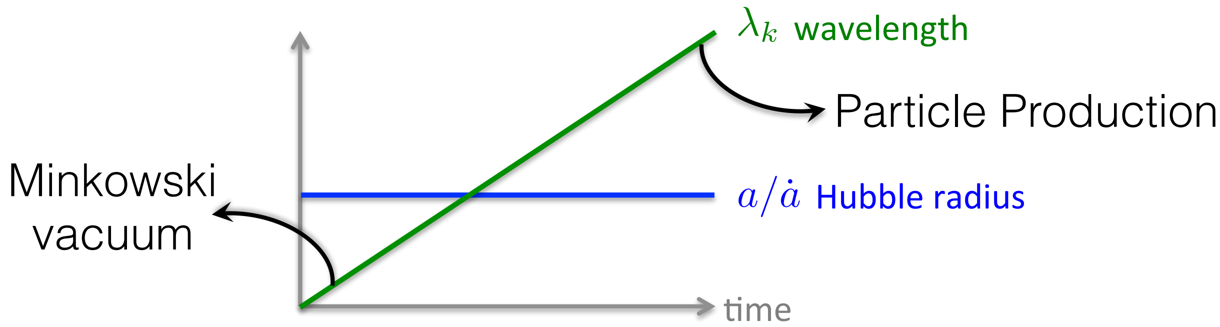
Workshop: Inflation and the dark sector — Current challenges and future perspectives

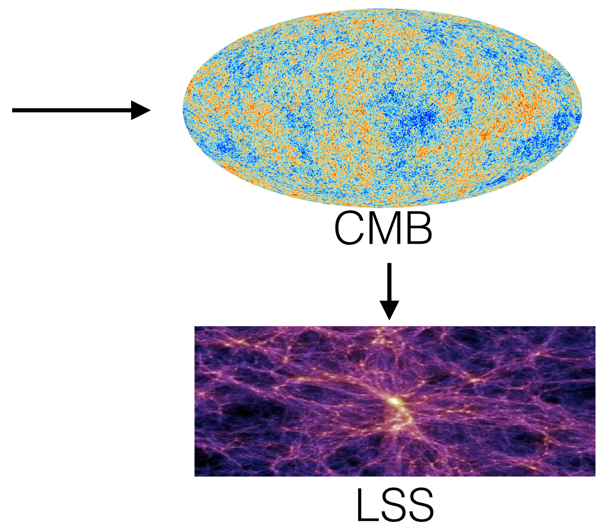
University of Jyväskylä

7 June 2019

Inflation

Quantum vacuum fluctuations of the gravitational and scalar fields amplified by gravitational instability and stretched by cosmic expansion





Quantum mechanics on cosmological scales!

- Strong statement (extraordinary statement requires extraordinary evidence)
- The consequences that can be inferred from this idea are consistent with observations
- This gives an indirect confirmation that cosmological structures have an quantum-mechanical origin

Any direct evidence?

Inflationary perturbations

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(t, \boldsymbol{x})$$

$$\phi = \bar{\phi}(t) + \widehat{\delta\phi}(t, \boldsymbol{x})$$

Scalar perturbations are described by a single combination of metric and field fluctuations that directly determines CMB temperature anisotropies $\longrightarrow \hat{\zeta}(t, \boldsymbol{x})$

Expansion of Einstein-Hilbert + scalar field action at second order:

$$\hat{H} = \int \mathrm{d}^3\boldsymbol{k} \left[\frac{k}{2}\left(\hat{c}_{\boldsymbol{k}}\hat{c}^\dagger_{\boldsymbol{k}} + \hat{c}_{-\boldsymbol{k}}\hat{c}^\dagger_{-\boldsymbol{k}}\right) - \frac{i}{2}\frac{\left(a\sqrt{\epsilon_1}\right)'}{a\sqrt{\epsilon_1}}\left(\hat{c}_{\boldsymbol{k}}\hat{c}_{-\boldsymbol{k}} - \hat{c}^\dagger_{-\boldsymbol{k}}\hat{c}^\dagger_{\boldsymbol{k}}\right)\right]$$

Free term

Interaction term between the quantum fluctuations and the classical background

Creation / annihilation of pairs of particles

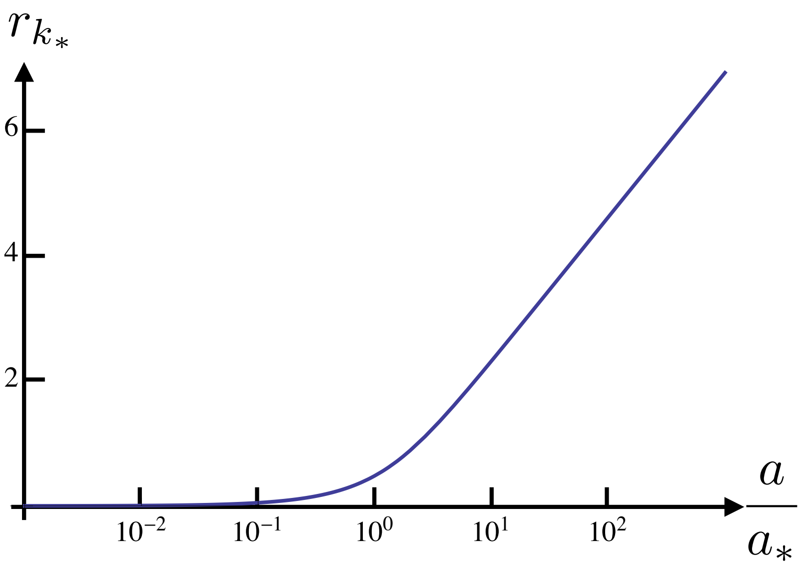
Pump field: time-dependent coupling constant
Depends only on the scale factor and its derivative
Vanishes if *a* is constant

Quantum state of cosmological perturbations

Two-mode squeezed sate

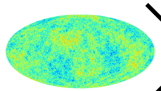
$$|\Psi_{\mathrm{CMB}}\rangle = \bigotimes_{\boldsymbol{k}\in\mathbb{R}^{3+}} |\Psi_{\boldsymbol{k}}\rangle \quad \text{with} \quad |\Psi_{\boldsymbol{k}}\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{2in\varphi_k}(-1)^n \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$$

Entangled state (correlations between modes *k* and -*k*)



Strongest squeezed state produced in nature (*r*=50)

Large-squeezing limit: goes to an Einstein-Podolski-Rosen state $|\uparrow,\uparrow\rangle + |\downarrow,\downarrow\rangle$

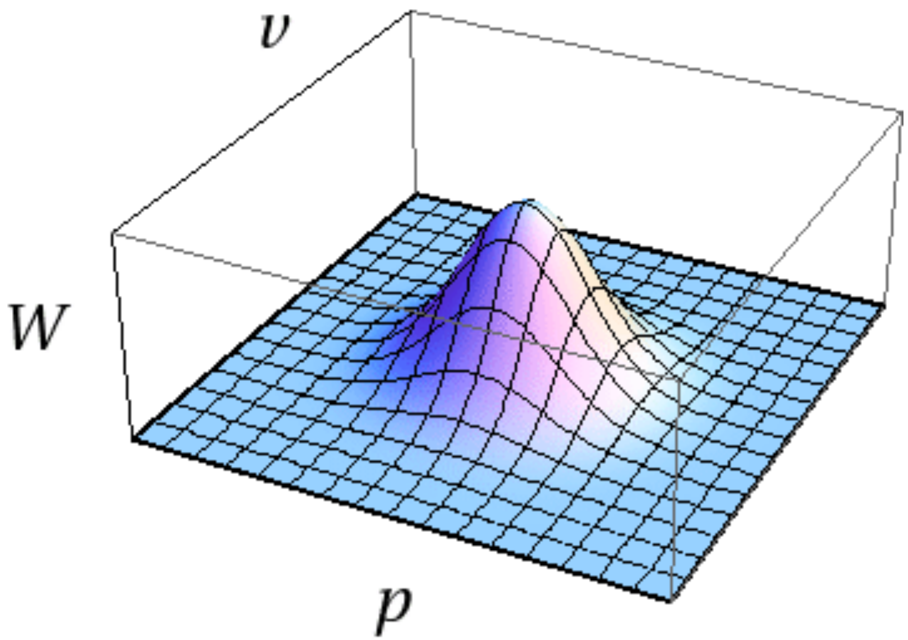
|  $\rangle$ = highly-non classical state? → Can we violate Bell's inequalities with the CMB?

Classicality in the Wigner approach

Wigner function $$W\left(q,p\right)=\int\Psi^{*}\left(q-\frac{u}{2}\right)e^{-ipu}\Psi\left(q+\frac{u}{2}\right)\frac{\mathrm{d}u}{2\pi}$$

Evolution equation: $$\frac{\partial}{\partial t}W\left(q,p,t\right)=-\left\{W(q,p,t),H(q,p,t)\right\}_{\mathrm{Poisson\ \ bracket}}$$

for quadratic Hamiltonians



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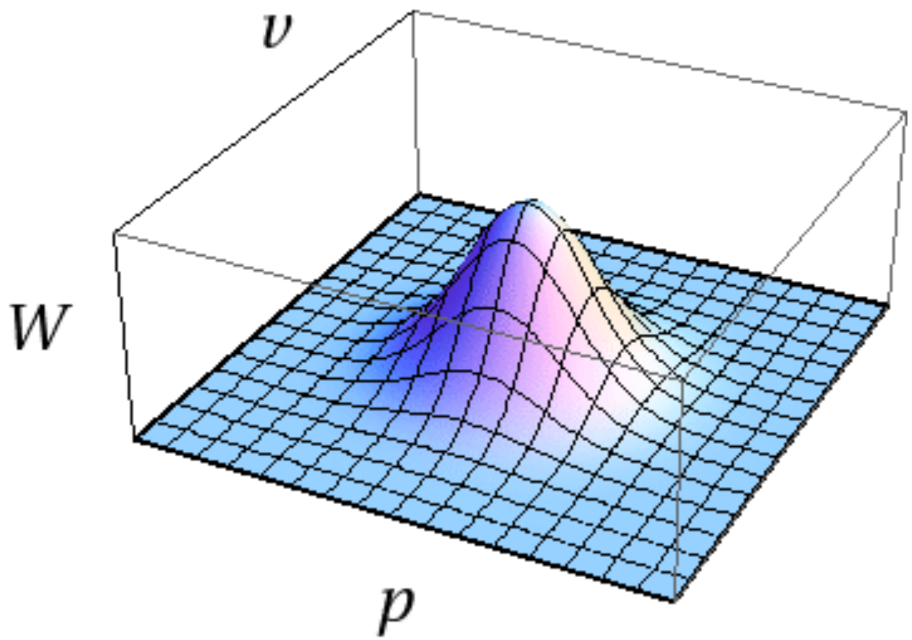

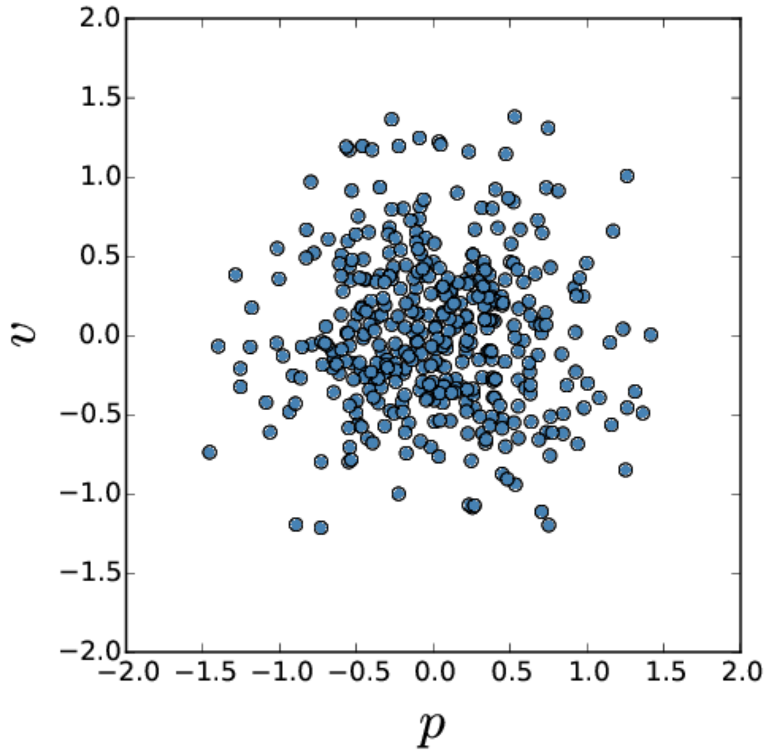

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for quadratic Hamiltonians

Weyl Transform $$\widetilde{A}(q,p)=\int\mathrm{d}ue^{-ipu}\left\langle q+\frac{u}{2}\right|\hat{A}\left|q-\frac{u}{2}\right\rangle$$

(with this definition: $W=\dfrac{\widetilde{\rho}}{2\pi}$)

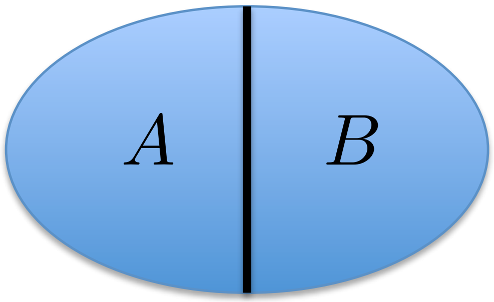
Expectation value of quantum operators $$\langle\hat{A}\rangle=\int\widetilde{A}\left(q,p\right)W\left(q,p\right)\mathrm{d}q\mathrm{d}p$$

$W>0$ ⟶ "quasi-probability distribution"

John Bell 1986, *EPR correlations and EPW distributions*:
Bell inequality violation requires non-positive Wigner function

Quantum discord

Henderson and Vedral 2001; Ollivier and Zurek 2001



Idea: Find two ways to calculate the mutual information between A and B that coincide for classical correlations but may differ in quantum systems

$\mathcal{I} = S(A) + S(B) - S(A,B)$

$\mathcal{J} = S(A) - S(A|B)$ with respect to measurements $\hat{\Pi}_j$

$\hat{\Pi}_j$: complete set of projectors defined on $\mathcal{E}_B$

$\hat{\rho} \to \hat{\rho}\hat{\Pi}_j/p_j$ with probability $p_j = \mathrm{Tr}\left(\hat{\rho}\hat{\Pi}_j\right)$ and $\rho_{A;\hat{\Pi}_j} = \mathrm{Tr}_B\left(\rho\hat{\Pi}_j/p_j\right)$

$$S(A|B) = \sum_j p_j S\left(\rho_{A;\hat{\Pi}_j}\right)$$

$$\delta(A,B) = \min_{\{\hat{\Pi}_j\}}\left(\mathcal{I} - \mathcal{J}\right)$$

Quantum discord

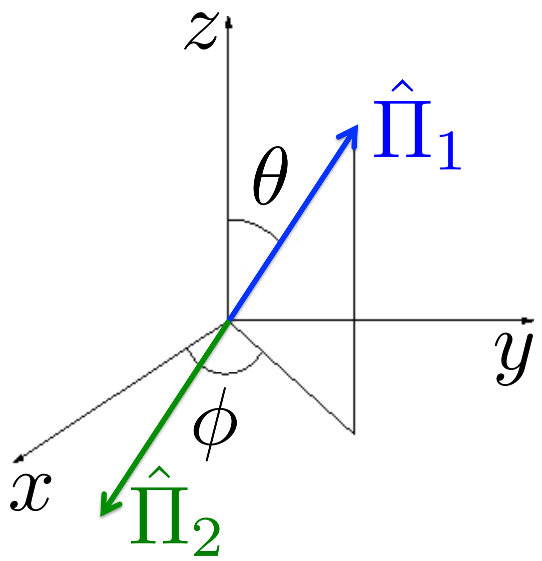
Example: $|\Psi\rangle = \frac{|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle}{\sqrt{2}}$

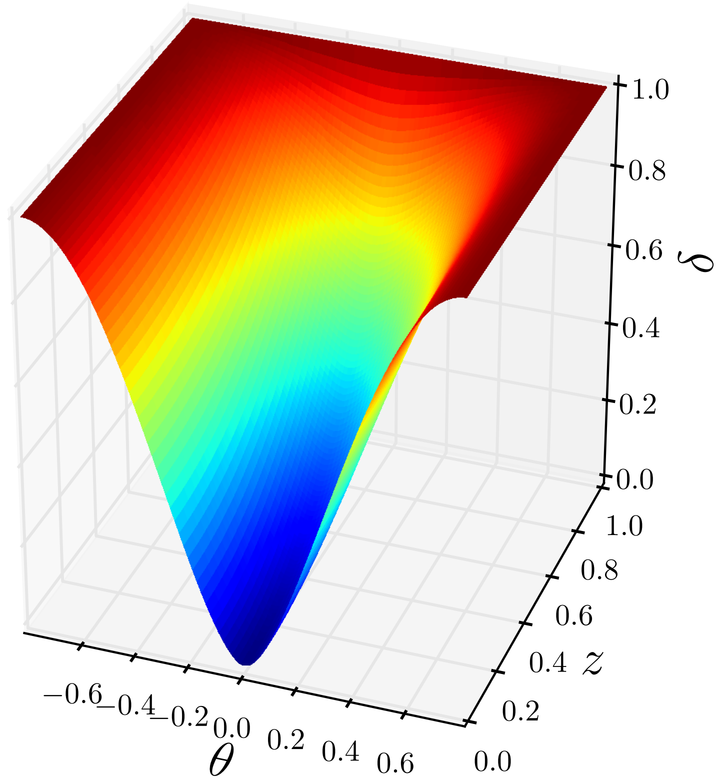
$$\rho = \frac{1}{2}|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + \frac{1}{2}|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + \frac{1}{2}|\downarrow\downarrow\rangle\langle\uparrow\uparrow| + \frac{1}{2}|\uparrow\uparrow\rangle\langle\downarrow\downarrow|$$

Quantum discord

Example: $|\Psi\rangle = \frac{|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle}{\sqrt{2}}$

$$\rho = \frac{1}{2}|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + \frac{1}{2}|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + \frac{z}{2}|\downarrow\downarrow\rangle\langle\uparrow\uparrow| + \frac{z}{2}|\uparrow\uparrow\rangle\langle\downarrow\downarrow|$$



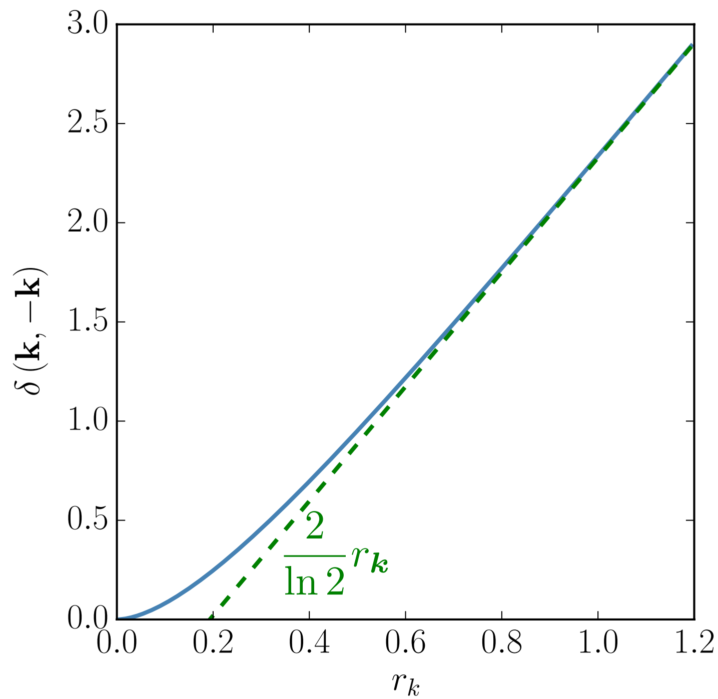


Quantum discord

For the two-mode squeezed state of inflation: J. Martin, V.V., 1510.04038

$$\delta(\boldsymbol{k}, -\boldsymbol{k}) = \cosh^2 r_k \log_2\left(\cosh^2 r_k\right) - \sinh^2 r_k \log_2\left(\sinh^2 r_k\right)$$

~ 150 at the end of inflation



So is the CMB very classical or very quantum?

You have reached the point of maximum confusion

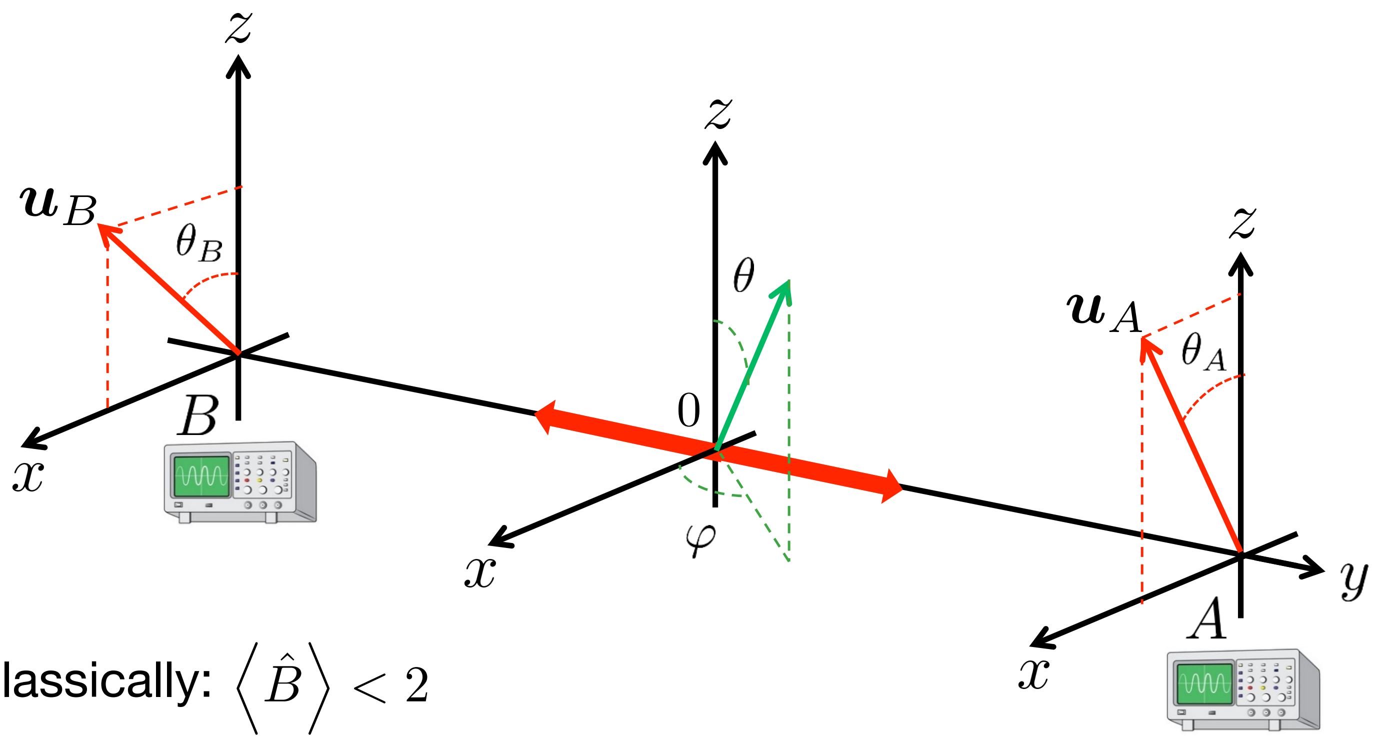
Revzen 2006

Bell inequalities can be violated even when *W*>0 with improper operators

Proper operator: $\widetilde{A}$ takes values within the spectrum of $\hat{A}$.

Bell inequalities

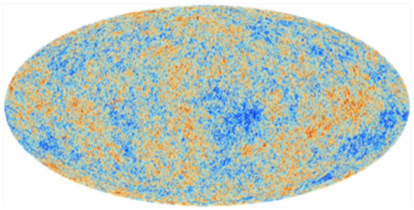
$$\hat{B} = \left(\boldsymbol{u}_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}_B \cdot \hat{\boldsymbol{S}}_B\right) + \left(\boldsymbol{u}_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}'_B \cdot \hat{\boldsymbol{S}}_B\right) + \left(\boldsymbol{u}'_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}_B \cdot \hat{\boldsymbol{S}}_B\right) - \left(\boldsymbol{u}'_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}'_B \cdot \hat{\boldsymbol{S}}_B\right)$$



Classically: $\left\langle \hat{B} \right\rangle < 2$

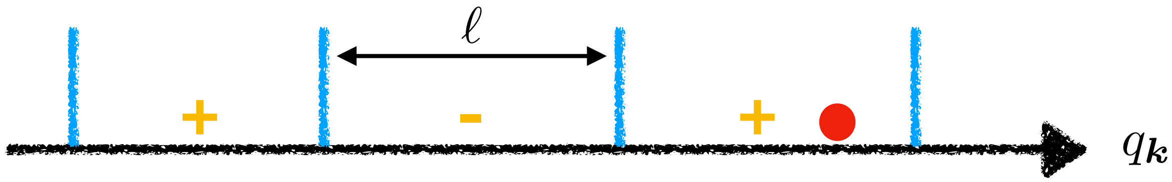
- Bipartite system: *k* and -*k*
- Entangled system: two-mode squeezed state
- improper, spin-like operators

Bell inequalities



continuous variable

$$\hat{q}_{\boldsymbol{k}} = \frac{\hat{c}_{\boldsymbol{k}} + \hat{c}_{\boldsymbol{k}}^{\dagger}}{\sqrt{2k}} = \hat{q}_{\boldsymbol{k}}^{\dagger}$$



continuous spectrum

- Divide the real axis into intervals $[n\ell, (n+1)\ell]$
- Perform a measurement of $q_{\boldsymbol{k}}$
- Return $S_z(\ell) = (-1)^n$

Larsson 2004

$$\hat{S}_z(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{n\ell}^{(n+1)\ell} \mathrm{d}q_{\boldsymbol{k}} \left|q_{\boldsymbol{k}}\right\rangle \left\langle q_{\boldsymbol{k}}\right| \longrightarrow \hat{S}_z^2(\ell) = 1$$

$$\hat{S}_+(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{2n\ell}^{(2n+1)\ell} \mathrm{d}q_{\boldsymbol{k}} \left|q_{\boldsymbol{k}}\right\rangle \left\langle q_{\boldsymbol{k}} + \ell\right|$$

$$\longrightarrow \hat{S}_x(\ell) = \hat{S}_+(\ell) + \hat{S}_+^{\dagger}(\ell)$$

$$\longrightarrow \hat{S}_y(\ell) = -i\left[\hat{S}_+(\ell) - \hat{S}_+^{\dagger}(\ell)\right]$$

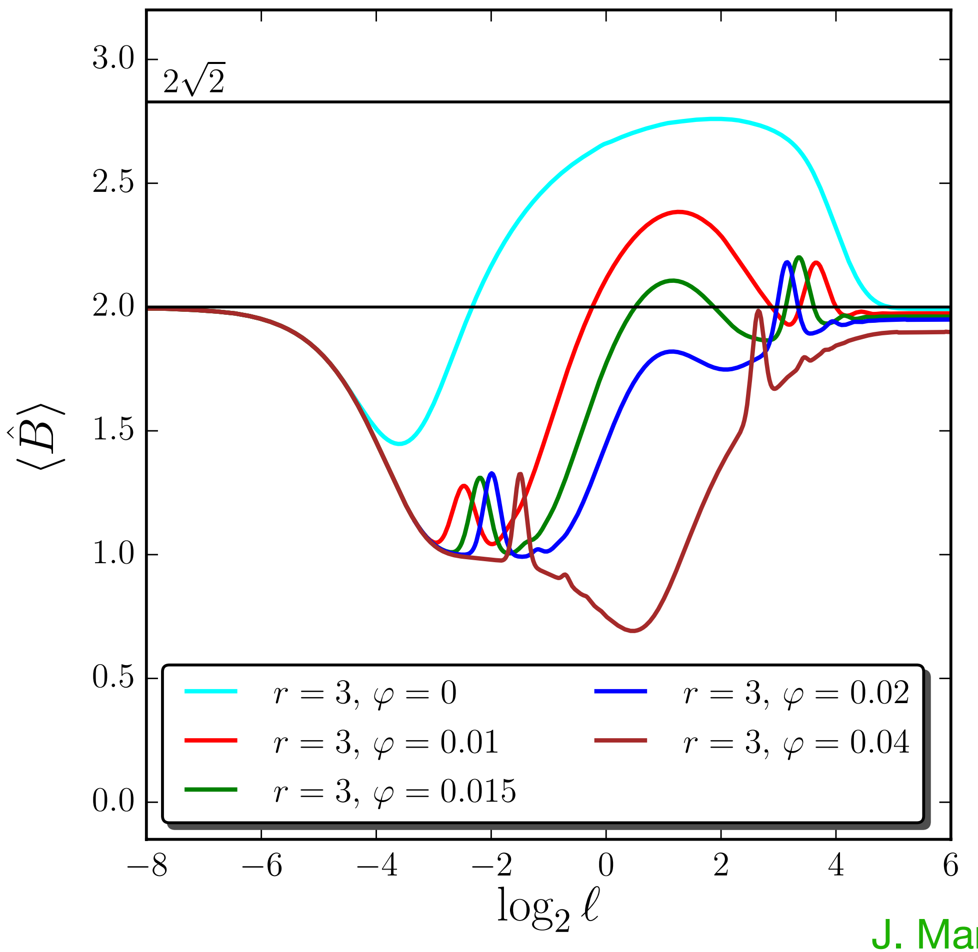
$$\longrightarrow \left[\hat{S}_i(\ell), \hat{S}_j(\ell)\right] = 2i\epsilon_{ijk}\hat{S}_k(\ell)$$

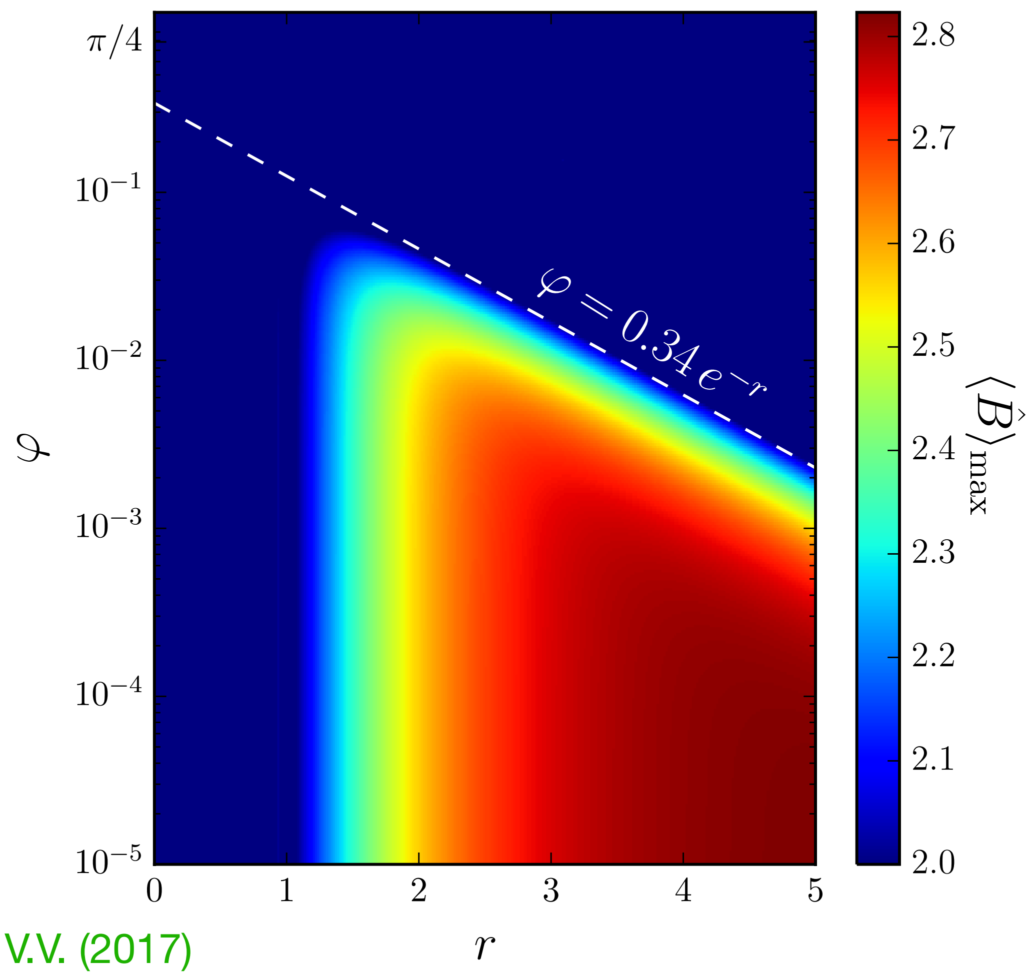
obey spin algebra

Bell inequalities in the CMB

$$\begin{aligned}\hat{B}(\ell) \quad &= \left[\mathbf{n}\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]+\left[\mathbf{n}\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}'\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]\\ &+\left[\mathbf{n}'\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]-\left[\mathbf{n}'\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}'\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]\end{aligned}$$

classically: $\left\langle \hat{B}\left(\ell\right)\right\rangle < 2$





J. Martin, V.V. (2017)

Bell inequalities in the CMB

How to measure $\hat{S}_+(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{2n\ell}^{(2n+1)\ell} \mathrm{d}q_{\boldsymbol{k}} \, |q_{\boldsymbol{k}}\rangle \langle q_{\boldsymbol{k}} + \ell|$?

requires to access phase information

conjugated momentum $\pi_{\boldsymbol{k}}$

decaying mode

$\zeta'_{\boldsymbol{k}} \sim e^{-r_{\boldsymbol{k}}} \zeta_{\boldsymbol{k}}$

Can we detect quantum correlations using “position” measurements only?

$\widetilde{\zeta}_{\boldsymbol{k}} = \zeta_{\boldsymbol{k}}$ and $\widetilde{f}(\zeta_{\boldsymbol{k}}) = f(\zeta_{\boldsymbol{k}})$ so according to Revzen’s theorem: not with Bell inequalities!

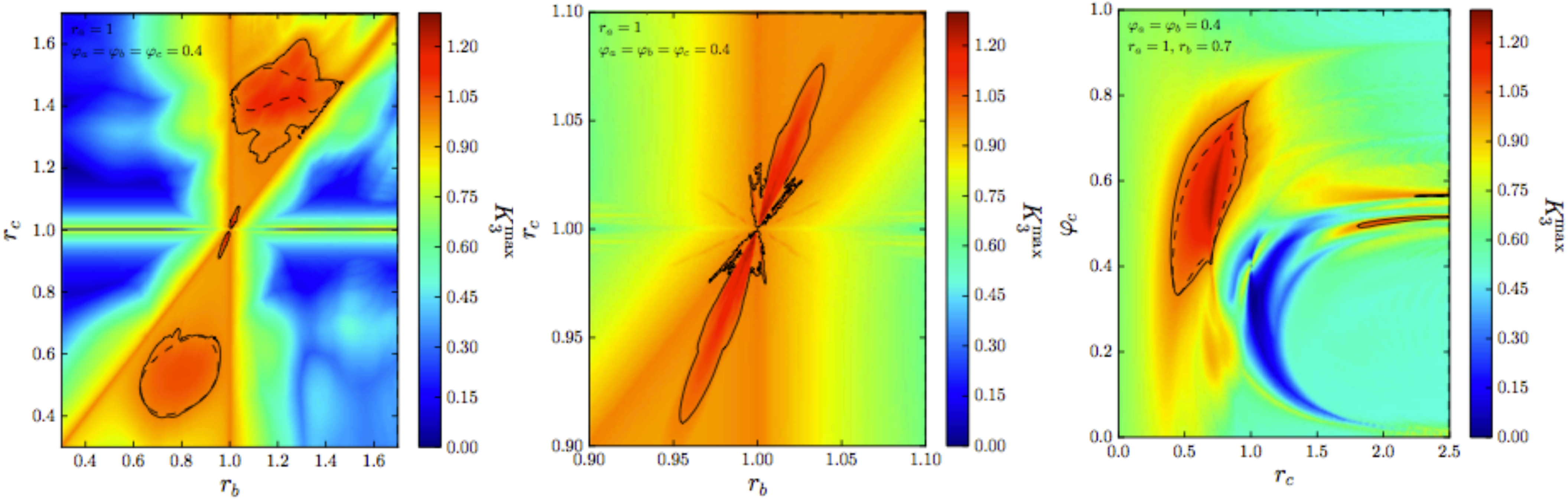
Leggett-Garg inequalities

Two-time correlators $C_{ab} = \left\langle \hat{S}_z(t_a,\ell)\hat{S}_z(t_b,\ell)\right\rangle$

Leggett-Garg three strings $K_3 = C_{ab} + C_{bc} - C_{ac}, \quad K_3' = -C_{ab} - C_{bc} - C_{ac}$

Classically: $-3 \leq K_3, K_3' \leq 1$

J. Martin, V.V. (2016)



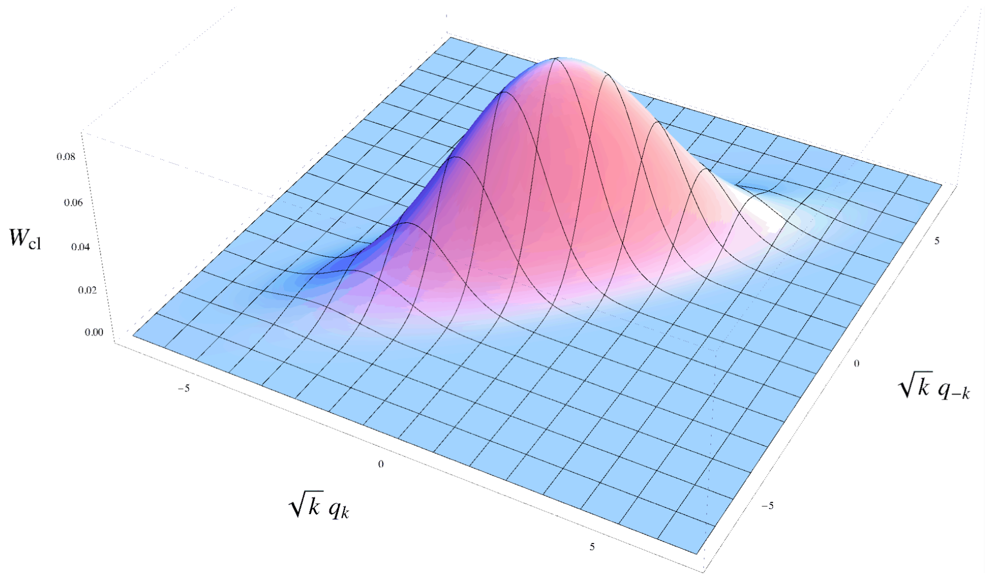
But requires to measure zeta at three different times …

Non-discordant states

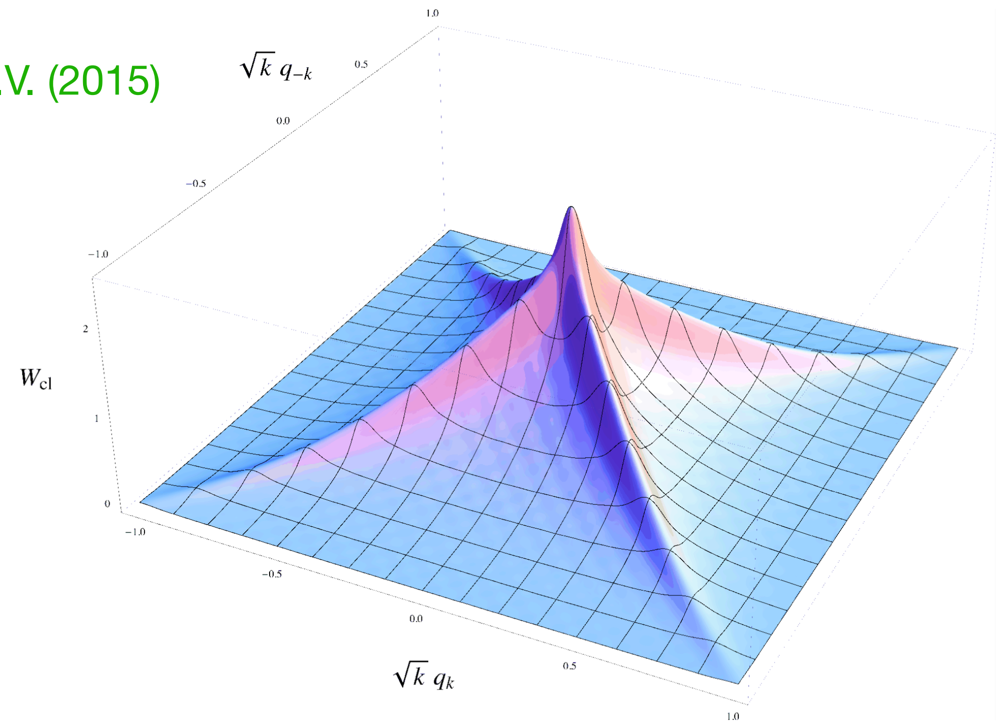
Can we detect quantum correlations using single-time, “position” measurements only?

Classical states = non-discordant states $\delta\left(\boldsymbol{k},-\boldsymbol{k}\right)=0$

J. Martin, V.V. (2015)



Two-mode squeezed state



Non-discordant state sharing the same two-point functions as the two-mode squeezed state

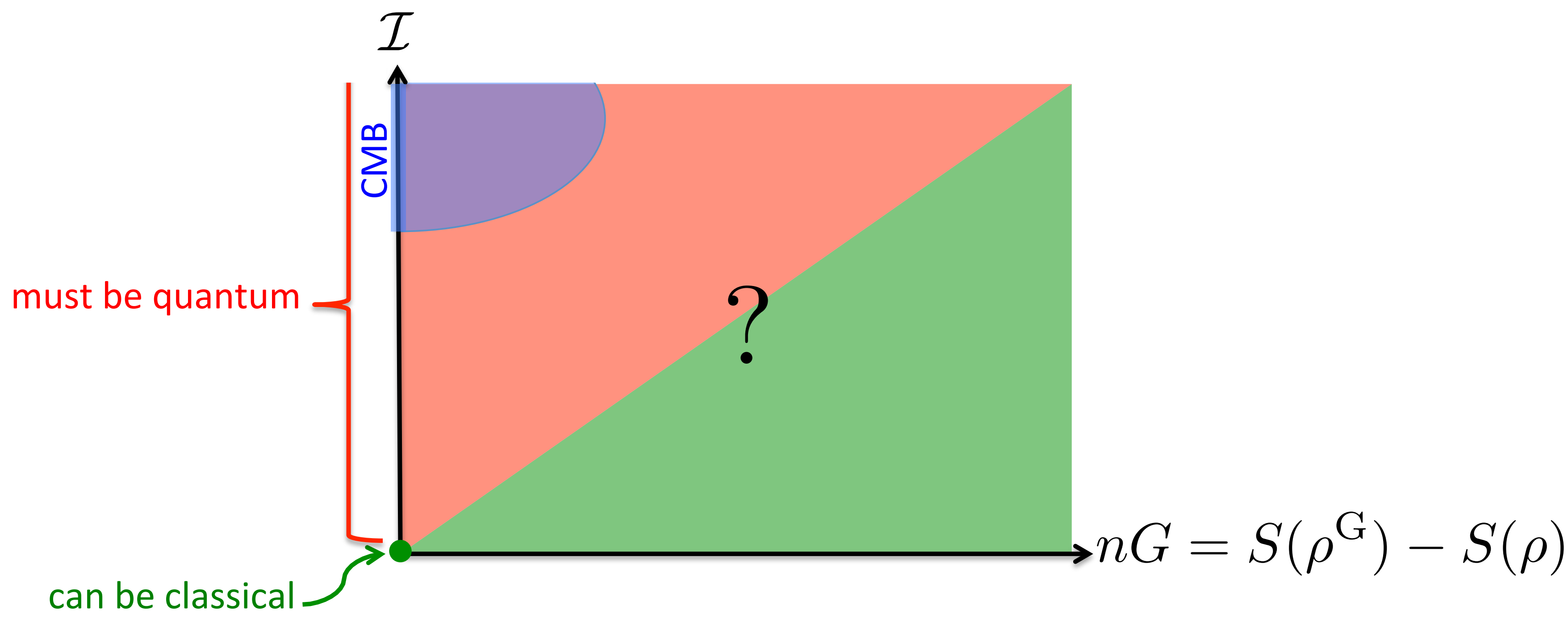
Non-discordant states

Can we detect quantum correlations using single-time, “position” measurements only?

Classical states = non-discordant states $\delta\left(\boldsymbol{k},-\boldsymbol{k}\right)=0$

Theorem: the only classical Gaussian states are product states

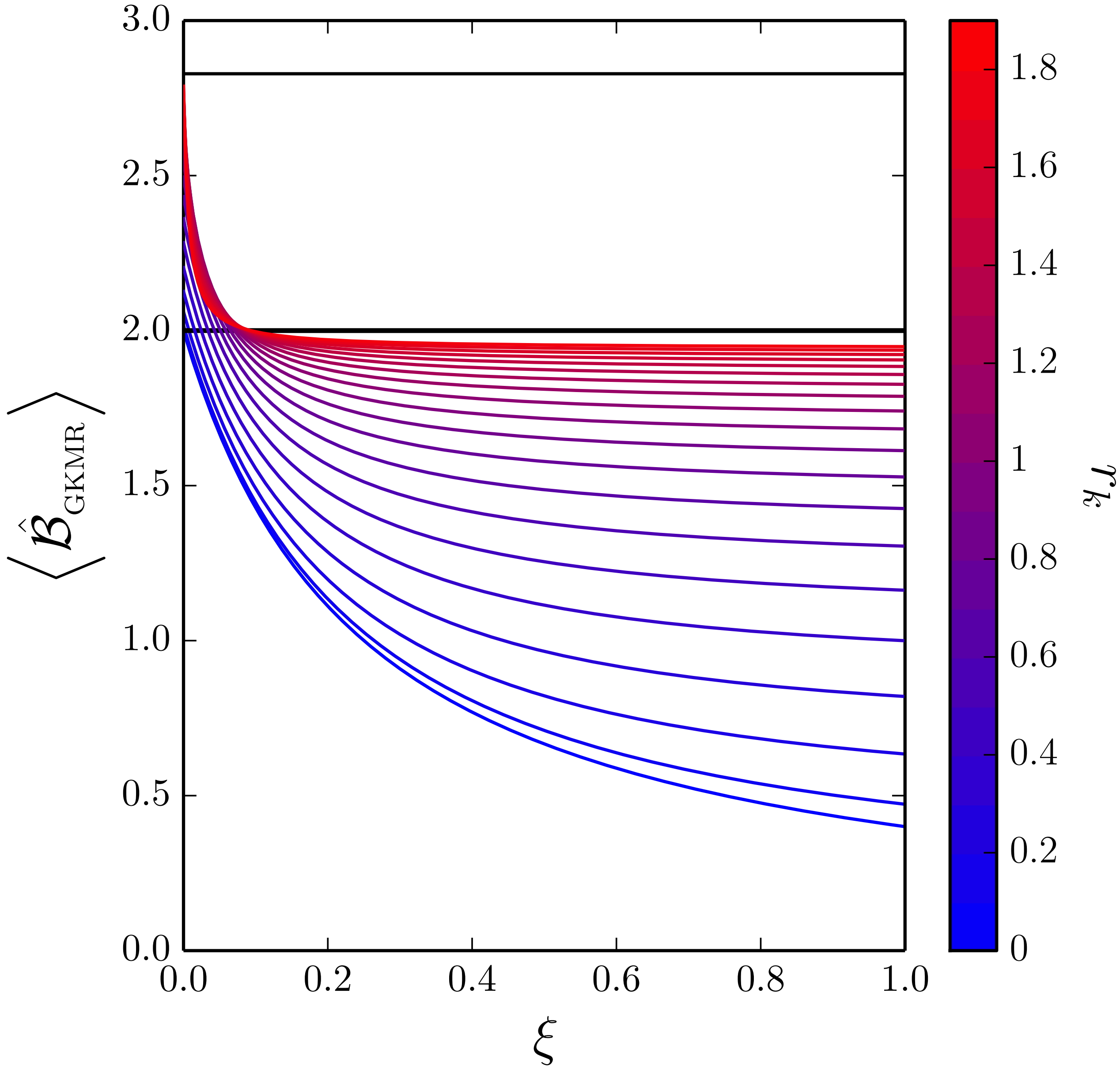
Adesso, Datta 2010; Rahimi-Keshari, Calves, Ralph 2013; Mista, Mc Nulty 2014



Conclusions

- **Cosmological perturbations** are placed in a two-mode highly **squeezed state** in the very early Universe
- Such a state has a large **quantum discord**, denoting the presence of **large quantum correlations** between particles created with opposite wave momenta
- In principle, **Bell experiments** can therefore be constructed that would prove that CMB anisotropies are of quantum mechanical origin
- In practice, these experiments require to measure exponentially small quantities ($\propto$ **decaying mode**), at least in the standard setup
- **Legget-Garg** inequalities evade this issue but require to measure perturbations at **different times**
- The CMB cannot have been placed in a **classical Gaussian state.** Current constraints on non-Gaussianities may be already sufficient to exclude non-discordant states!
- Role of decoherence?

Decoherence



Decoherence