The gravity track of Higgs inflation

(work with Vera-Maria Enckell, Kari Enqvist, Eemeli Tomberg and Lumi-Pyry Wahlman)

Syksy Räsänen

University of Helsinki

Department of Physics and Helsinki Institute of Physics

Using what you have

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2 + \xi h^2}{2} R - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right]$$

$$V(h) = \frac{\lambda}{4}(h^2 - v^2)^2 \simeq \frac{\lambda}{4} h^4$$

- Non-minimal coupling $\xi h^2 R$ is the only new dimension 4 term for the combined Einstein-Hilbert + SM action.
- The coupling ξ is generated by renormalisation, even if it is classically zero.
- Non-minimal coupling enables Higgs inflation, which uses the only known scalar field that may be elementary. (Bezrukov and Shaposhnikov: 0710.3755)

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When the action is not enough

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{M_{\mathrm{Pl}}^2 + \xi h^2}{2} R - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right]$$

$$V(h) = \frac{\lambda}{4}(h^2 - v^2)^2 \simeq \frac{\lambda}{4} h^4$$

- Complication: classical low-energy action is not enough to specify the theory.

- Two sources of ambiguity.
  - Quantum theory: how to calculate loop corrections?

    (c.f. Eemeli's talk on black holes as dark matter)
  - General relativity: what are the gravitational degrees of freedom?

The many faces of Einstein gravity

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1+\xi h^2}{2} g^{\alpha\beta} R_{\alpha\beta}(g, \partial g, \partial^2 g) - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right]$$

- Usually the gravitational degrees of freedom are taken to be the metric and its derivatives.
- In the Palatini formulation, the metric and the connection are independent degrees of freedom.
- In the Einstein-Hilbert case, the metric and the Palatini formulation are equivalent.
  - No need to add the York-Gibbons-Hawking boundary term.
- With a non-minimally coupled scalar field, they are different physical theories. (Bauer and Demir: 0803.2664)

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To the Einstein frame

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- The Einstein frame is reached with the conformal transformation $g_{\alpha\beta} \to (1+\xi h^2)^{-1}g_{\alpha\beta}$
- In the Palatini case, the conformal transformation does not affect the Ricci tensor.

- To recover canonical kinetic term, define new field χ:
  - metric: $$\frac{\mathrm{d}\chi}{\mathrm{d}h} = \sqrt{\frac{1+\xi h^2+6\xi^2h^2}{(1+\xi h^2)^2}} \simeq \frac{\sqrt{6}}{h} \Rightarrow h \propto e^{\chi/\sqrt{6}}$$
  - Palatini: $$\frac{\mathrm{d}\chi}{\mathrm{d}h} = \sqrt{\frac{1+\xi h^2}{(1+\xi h^2)^2}} \simeq \frac{1}{\sqrt{\xi}h} \Rightarrow h \propto e^{\sqrt{\xi}\chi}$$

- Polynomial potential is transformed into exponential potential.

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To the Einstein frame

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Higgs potential in Metric vs Palatini

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$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi - U(\chi) \right]$$

- We get a different Einstein frame potential depending on the gravitational degrees of freedom:
  - metric: $U(\chi) \equiv \dfrac{V[h(\chi)]}{[1+\xi h(\chi)^2]^2} \simeq \dfrac{\lambda}{4\xi^2}(1 - 2e^{-\frac{2}{\sqrt{6}}\chi})$
  - Palatini: $U(\chi) \simeq \dfrac{\lambda}{4\xi^2}(1 - 8e^{-2\sqrt{\xi}\chi})$
- The potential is exponentially flat.

Predictions of Higgs inflation on the plateau

- On the exponentially flat plateau, we get:
  - metric: $n_s = 1 - \frac{2}{N}\ ,\ r = \frac{12}{N^2}\ ,\ \frac{U}{\epsilon} = \frac{\lambda}{3\xi^2}N^2$

    $$n_s = 0.96\ ,\ r = 5\times 10^{-3},\ \xi = 4\times 10^{4}\sqrt{\lambda}$$
  - Palatini: $n_s = 1 - \frac{2}{N}\ ,\ r = \frac{2}{\xi N^2}\ ,\ \frac{U}{\epsilon} = \frac{2\lambda}{\xi}N^2$

    $$n_s = 0.96,\ r = \frac{8\times 10^{-4}}{\xi} = \frac{8\times 10^{-14}}{\lambda}\quad \xi = 10^{10}\lambda$$
- Reheating is fixed: *N*≈50. (Garcia-Bellido, Figueroa and Rubio: 0812.4624, Rubio and Tomberg: 1902.10148)

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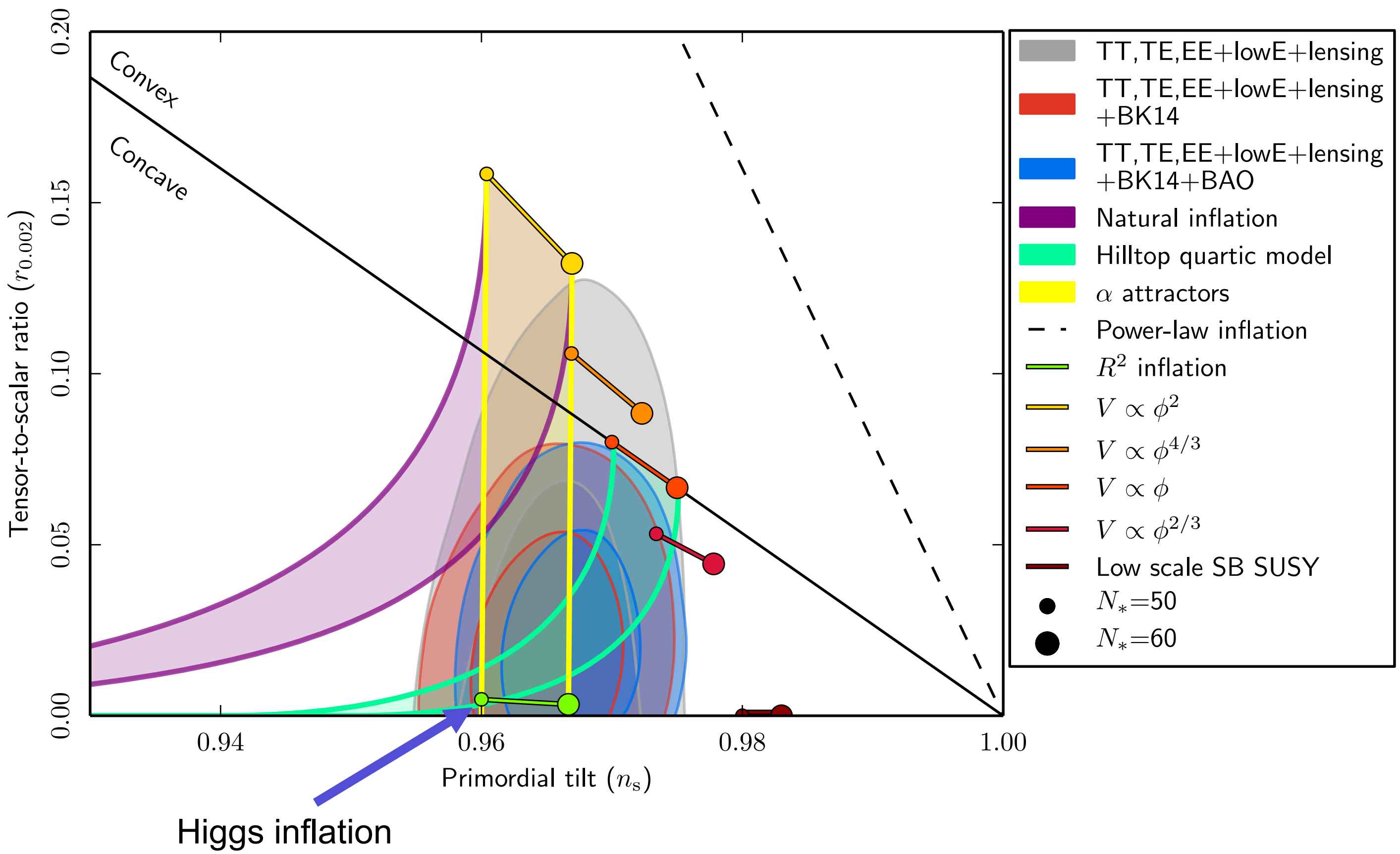
The data likes Higgs inflation



Planck: arXiv:1807.06211

Metric vs Palatini: R^2 term

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2}(1+\xi h^2) g^{\alpha\beta} R_{\alpha\beta} + \alpha R^2 - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right]$$

- Loop corrections generate an R^2 term in the action.

- Its effect is completely different in the metric and in the Palatini formulation.

R^2 à la metric: two-field model

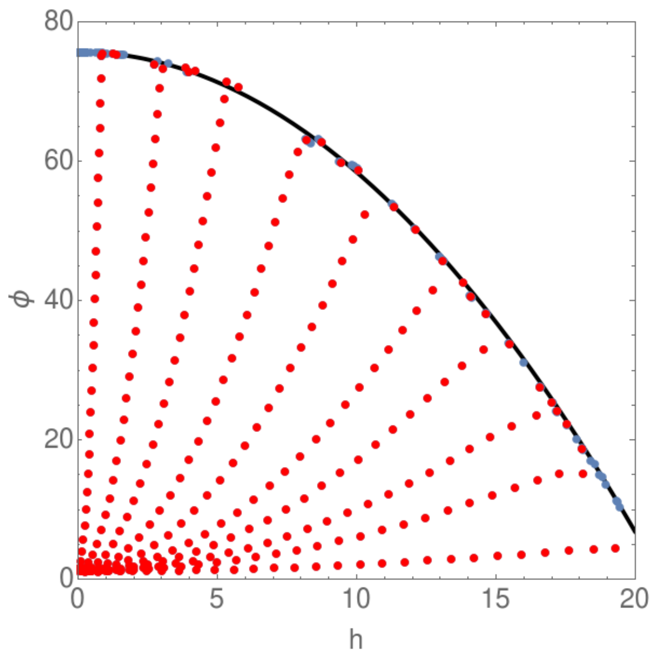
- In the metric case, the R^2 term adds a scalar degree of freedom, so we get a two-field model. (Wang and Wang: 1701.06636, Ema: 1701.07665, Zhang, Huang and Sasaki: 1712.09896, He, Starobinsky and Yokoyama: 1804.00409, Gundhi and Steinwachs: 1810.10546, Enckell, Enqvist, SR, Wahlman: 1812.08754)
- The action becomes

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{\varphi + \xi h^2 + 6\xi^2 h^2}{2(\varphi + \xi h^2)^2} g^{\alpha\beta}\partial_\alpha h \partial_\beta h - \frac{3\xi h}{(\varphi+\xi h^2)^2} g^{\alpha\beta}\partial_\alpha h \partial_\beta \varphi \right.$$
$$\left. - \frac{3}{4(\varphi + \xi h^2)^2} g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - \hat{V}(h,\varphi)\right]$$

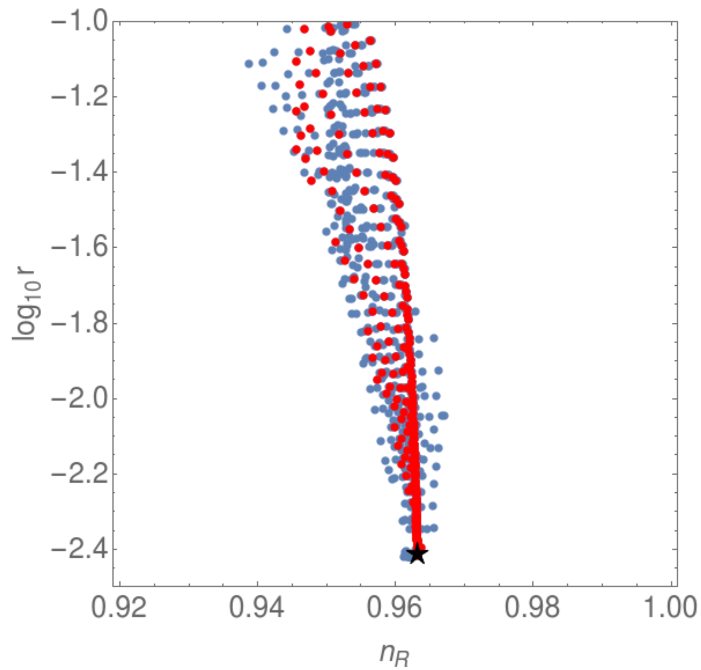
$$\hat{V}(h,\varphi) = \frac{\lambda}{4}\frac{(h^2 - v^2)^2}{(\varphi + \xi h^2)^2} + \frac{1}{8\alpha}\frac{(\varphi - 1)^2}{(\varphi + \xi h^2)^2}$$

Destabilising Higgs inflation

- The R^2 term destabilises Higgs inflation. (Enckell, Enqvist, SR, Wahlman: 1812.08754)



Inflation in field space.



Results for observables.

R^2 à la Palatini: saving your favourite model

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$$U(\chi) = \frac{V[h(\chi)]}{[1+\xi h(\chi)^2]^2}$$

- In the Palatini case, the new scalar field does not get a kinetic term and can be integrated out. (Enckell, Enqvist, SR, Wahlman: 1810.05536)

- The action reduces to

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial^\alpha\chi\partial_\alpha\chi + \frac{\alpha}{2}(1+8\alpha U)(\partial^\alpha\chi\partial_\alpha\chi)^2 - \frac{U}{1+8\alpha U}\right] .$$

- The only effect is to change the tensor power spectrum, giving $r \to r/(1+8\alpha U)$.

- This can be used to rescue any scalar field model where *r* was excluded by the data.

Features unique to Palatini

- In the metric case, the metric and the Riemann tensor are the only geometrical tensors.
- In the Palatini case, we have two new geometrical tensors:
  - non-metricity $Q_{\alpha\beta\gamma} \equiv \nabla_\alpha g_{\beta\gamma}$
  - torsion $T^\alpha{}_{\beta\gamma} \equiv 2\Gamma^\alpha{}_{[\beta\gamma]}$

Non-minimal gravity from non-minimal coupling

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{F(h)}{2}g^{\alpha\beta}R_{\alpha\beta}(\Gamma,\partial\Gamma) - \frac{1}{2}g^{\alpha\beta}\partial_\alpha h\partial_\beta h - V(h)\right]$$

$$Q_{\alpha\beta\gamma} \equiv \nabla_\alpha g_{\beta\gamma}$$

$$T^\alpha{}_{\beta\gamma} \equiv 2\Gamma^\alpha{}_{[\beta\gamma]}$$

- The non-minimal coupling *F(h)* generates non-metricity and/or torsion. We get either

  $$Q_{\alpha\beta\gamma} = -g_{\beta\gamma}\partial_\alpha \ln F$$, or

  $$T_{\alpha\beta\gamma} = g_{\alpha[\beta}\partial_{\gamma]} \ln F$$

- In the Einstein frame, the non-minimal connection is mapped onto the Higgs kinetic term and/or potential.

Kinetic terms for the metric

- With non-metricity and/or torsion we have many new scalars, with no correspondence in the metric case.
- We can simplify by demanding the new tensors only appear via the connection (i.e. the covariant derivative and the Riemann tensor).
- We then only have new kinetic terms, such as

$$g_{\alpha\beta}\nabla_\gamma g^{\gamma\alpha}\nabla_\delta g^{\delta\beta} \qquad h\nabla_\alpha h\nabla_\beta g^{\beta\alpha}$$

- Mixing of Higgs and metric kinetic terms sources non-metricity and/or torsion (SR: 1811.09514).

Kinetic terms for the metric

- Considering terms with only up to 2 derivatives (and demanding the equations of motion can be derived without adding boundary terms), the action is (SR: 1811.09514)

$$Q_{\alpha\beta\gamma} \equiv \nabla_\alpha g_{\beta\gamma}$$

$$Q_\gamma \equiv g^{\alpha\beta} Q_{\alpha\beta\gamma}$$

$$\hat{Q}_\alpha \equiv g^{\beta\gamma} Q_{\alpha\beta\gamma}$$

$$\begin{aligned} S = \int \mathrm{d}^4x \sqrt{-g} \bigg[& \frac{1}{2} F(h) g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - \frac{1}{2} g^{\alpha\beta} \nabla_\alpha h \nabla_\beta h - V(h) \\ & + A_1(h) \nabla_\alpha h \nabla_\beta g^{\beta\alpha} + A_2(h) g^{\alpha\beta} g_{\gamma\delta} \nabla_\alpha h \nabla_\beta g^{\gamma\delta} + B_1(h) g^{\alpha\beta} g_{\gamma\delta} g_{\epsilon\eta} \nabla_\alpha g^{\gamma\epsilon} \nabla_\beta g^{\delta\eta} \\ & + B_2(h) g_{\gamma\delta} \nabla_\alpha g^{\beta\gamma} \nabla_\beta g^{\alpha\delta} + B_3(h) g_{\alpha\beta} \nabla_\gamma g^{\gamma\alpha} \nabla_\delta g^{\delta\beta} \\ & + B_4(h) g^{\alpha\beta} g_{\gamma\delta} g_{\epsilon\eta} \nabla_\alpha g^{\gamma\delta} \nabla_\beta g^{\epsilon\eta} + B_5(h) g_{\gamma\delta} \nabla_\alpha g^{\alpha\beta} \nabla_\beta g^{\gamma\delta} \bigg] \\ = \int \mathrm{d}^4x \sqrt{-g} \bigg[& \frac{1}{2} F(h) g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - \frac{1}{2} g^{\alpha\beta} \nabla_\alpha h \nabla_\beta h - V(h) \\ & - A_1(h) \nabla_\alpha h \hat{Q}^\alpha - A_2(h) \nabla_\alpha h Q^\alpha + B_1(h) Q_{\gamma\alpha\beta} Q^{\gamma\alpha\beta} + B_2(h) Q_{\gamma\alpha\beta} Q^{\beta\gamma\alpha} \\ & + B_3(h) \hat{Q}_\alpha \hat{Q}^\alpha + B_4(h) Q_\alpha Q^\alpha + B_5(h) Q_\alpha \hat{Q}^\alpha \bigg] \; . \end{aligned}$$

- Non-minimal coupling and kinetic mixing source non-metricity and/or torsion.

Coordinates in field space

$$U(\chi) = \frac{V[h(\chi)]}{[1+\xi h(\chi)^2]^2}$$

- The action can be simplified by choosing suitable coordinates in field space.

$$h \to \chi(h)$$
$$g_{\alpha\beta} \to \Omega(h)^{-1} g_{\alpha\beta}$$
$$\Gamma^{\gamma}_{\alpha\beta} \to \Gamma^{\gamma}_{\alpha\beta} + \Sigma^{\gamma}{}_{\alpha\beta} = \Gamma^{\gamma}_{\alpha\beta} + g^{\gamma\delta}\left[\Sigma_1(h) g_{\alpha\beta}\partial_\delta h + 2\Sigma_2(h) g_{\delta(\alpha}\partial_{\beta)} h + 2\Sigma_3(h) g_{\delta[\alpha}\partial_{\beta]} h\right]$$

- All effects of the non-minimal connection are mapped onto the potential:

$$S = \int d^4x \sqrt{-g}\left[R - \frac{1}{2} g^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi - U(\chi)\right] .$$

- The field transformation is $\frac{d\chi}{dh} = \pm\sqrt{K(h)}$, where *K(h)* is a complicated function.

The Higgs case

$$U(\chi) = \frac{V[h(\chi)]}{[1+\xi h(\chi)^2]^2}$$

$$Q_{\alpha\beta\gamma} \equiv \nabla_\alpha g_{\beta\gamma}$$

$$Q_\gamma \equiv g^{\alpha\beta} Q_{\alpha\beta\gamma}$$
$$\hat{Q}_\alpha \equiv g^{\beta\gamma} Q_{\alpha\beta\gamma}$$

- Consider the Higgs case with only dimension 4 operators.

$$S = \int d^4x \sqrt{-g} \Big[\frac{1}{2}(f+\xi h^2) g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - g^{\alpha\beta}\partial_\alpha h \partial_\beta h - \frac{\lambda}{4} h^4$$
$$- a_1 h \partial_\alpha h \hat{Q}^\alpha - a_2 h \partial_\alpha h Q^\alpha + (b_{10} + b_{11} h^2) Q_{\gamma\alpha\beta} Q^{\gamma\alpha\beta} + (b_{20} + b_{21} h^2) Q_{\gamma\alpha\beta} Q^{\beta\gamma\alpha}$$
$$+ (b_{30} + b_{31} h^2) \hat{Q}_\alpha \hat{Q}^\alpha + (b_{40} + b_{41} h^2) Q_\alpha Q^\alpha + (b_{50} + b_{51} h^2) Q_\alpha \hat{Q}^\alpha \Big]$$

- Here *f*, *k*, a_i, b_{i0} and b_{i1} are constants. We get

$$\frac{d\chi}{dh} = \pm\sqrt{K(h)} = \pm\sqrt{\frac{\sum_{n=0}^{6} c_n h^{2n}}{\sum_{n=0}^{7} d_n h^{2n}}}$$

- By tuning the constants, we can generate an inflection point, α-attractor or

$$U \propto 1 - a\chi^2, U \propto 1 - a\chi^{-2/3}, U \propto \chi^2, U \propto \chi^{4/3}$$

Higgs as a window to gravity

- Higgs inflation is a conservative possibility using only known degrees of freedom.
- Different formulations of general relativity become inequivalent theories when the matter couples to the connection.
- Higgs opens a window to gravitational degrees of freedom.
- In the Palatini case, r is suppressed compared to the metric case.
- In the metric case, R^2 term leads to a complicated two-field model. In the Palatini case, it is harmless.
- In the Palatini case, new gravitational terms can completely change the effective potential. (More to come: teleparallel, Ashtekar, ...)