Thermal phase transitions, resummations and the lattice: Multi-scalar edition

Lauri Niemi

In collaboration with:
K. Kainulainen, V. Keus, K. Rummukainen, T.V.I. Tenkanen and V. Vaskonen

University of Helsinki and Helsinki Institute of Physics

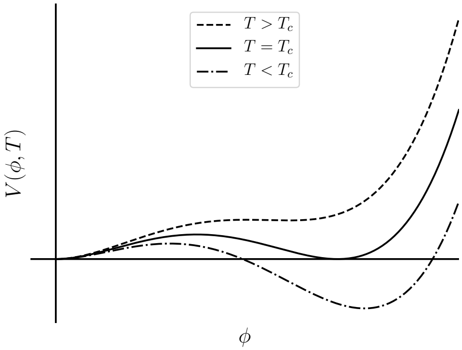
Based on arXiv:1904.01329 (preprint)

E-mail: lauri.b.niemi@helsinki.fi

Contents

Review: First-order phase transitions in QFT

- $T = 0$ effective potential + thermal corrections:



- Potential barrier separating the energy minima. Heat released when tunneling.

Review: Bubble nucleation

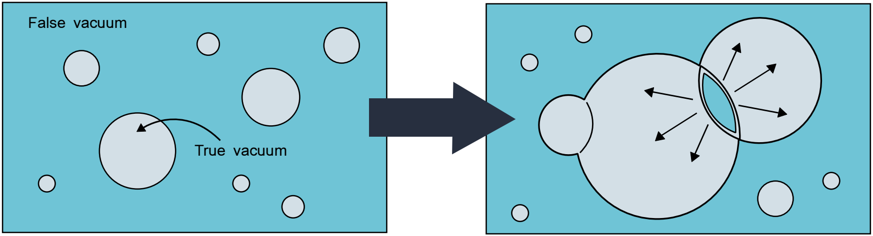
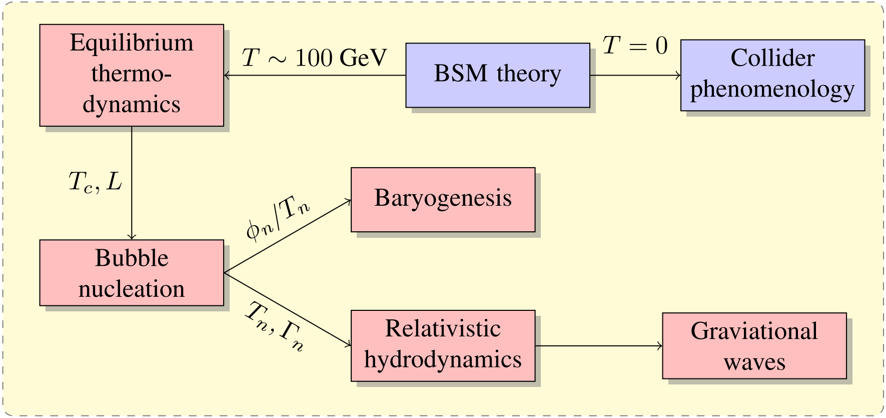


Figure by A. Kormu

- Deviation from thermal equilibrium $\rightarrow$ venue for interesting physics!
- Baryogenesis with sphaleron transitions, gravitational waves...

Pipeline for electroweak phase transition (EWPT)

- Can have strongly first-order EWPT in many (scalar) extensions.



- Goal for now: accurate determination of equilibrium quantities $T_c, L, \ldots$

Obtaining a first-order EWPT with additional fields

- Through radiative corrections to the Higgs:

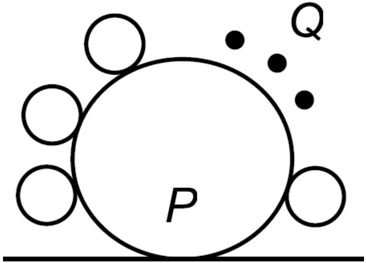
$$V_{\text{eff}}(\phi, T) \sim m^2\phi^2 + \lambda\phi^4 - \frac{1}{12\pi}\lambda_{\text{BSM}}\phi^3 T + \dots$$

- Often in BSM theories, strong phase transitions obtained with very large couplings, $\lambda_{\text{BSM}} \sim \mathcal{O}(1)$.

- Alternatively, introduce sufficiently light fields $\rightarrow$ can have multiple phase transitions.

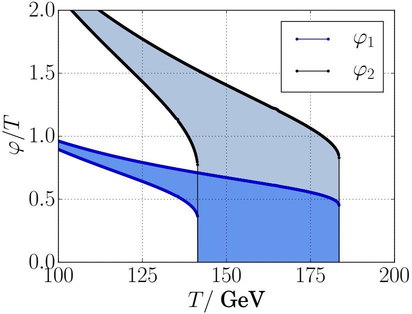
Standard perturbative analysis in a nutshell

- Track minimum of 1-loop Coleman-Weinberg potential + thermal corrections.
- Account for thermal screening with (1-loop) daisy resummation.



Try standard analysis with two Higgs doublets

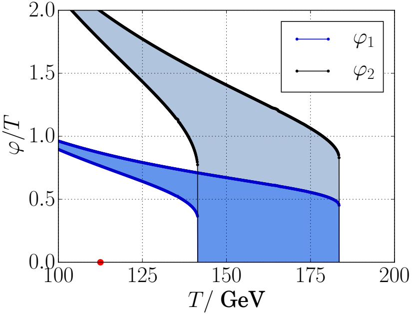
- Assume a typical EWPT scenario with $\lambda_3 \sim 4$ and track the global minimum.
- Colored band illustrates uncertainty from RG scale variation ($\sim$ size of higher-order corrections)



- Large uncertainty from residual RG scale dependence!

Try standard analysis with two Higgs doublets

- Assume a typical EWPT scenario with $\lambda_3 \sim 4$ and track the global minimum.
- Colored band illustrates uncertainty from RG scale variation ($\sim$ size of higher-order corrections)



- Large uncertainty from residual RG scale dependence! ... and is wrong?

Issues with perturbation theory

- Linde problem: high-T phase is nonperturbative.

$$\text{expansion parameter} \sim \frac{\lambda_{\text{BSM}} T}{\pi m_W} = \left(\frac{T}{\phi}\right) \frac{2\lambda_{\text{BSM}}}{\pi g} \xrightarrow{\phi \to 0} \infty$$

  Note also that the Higgs mass $m \approx 0$ at T_c.

- In the broken phase, T/ϕ is small for strong transitions, **but convergence may be still be bad if λ_{BSM} is large.**

- Gauge problem: $\langle \phi^\dagger \phi \rangle$ is gauge invariant, $\langle \phi \rangle$ is not. Solution: expand the potential consistently in powers of $\hbar$.

Contents

Obstacles for lattice simulations

- Hard to simulate (chiral) fermions.
- Heat bath and screening produce many separate mass scales: $g^2T, gT, \pi T$, heavy BSM fields...
- Relating to continuum physics cumbersome in BSM theories containing many parameters.

Example: 2HDM $\quad m_1^2, m_2^2, m_{12}^2, \lambda_{1-5}$

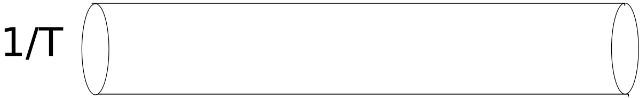
Make use of the thermal mass hierarchy

- Matsubara decomposition:

$$\phi(\tau, \mathbf{x}) = T \sum_n \tilde{\phi}(\omega_n, \mathbf{p}) e^{i\omega_n \tau}, \ \omega_n = \begin{cases} 2\pi n T & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$$

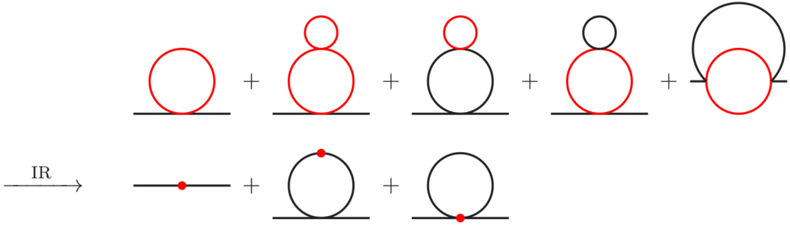
- Propagators: $\frac{1}{\mathbf{p}^2 + m^2 + \omega_n^2}$

- Modes with $\omega_n \neq 0$ are heavy and decouple at distances $\gg 1/T \to$ can integrate out!



Dimensional reduction 1/2

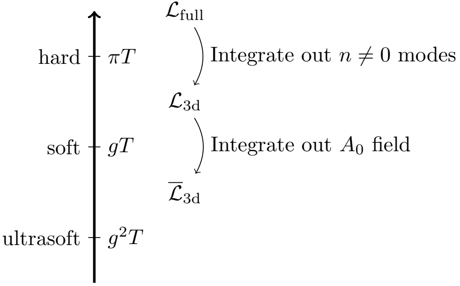
- Construct effective theory by matching Green's functions. Perturbative but IR safe!
- Assumption: BSM masses ligher than $\sim \pi T$; if not, integrate out first.
- Zero modes are screened by the plasma. Efficient way to incorporate thermal resummations!



Dimensional reduction 2/2

- Gauge field temporal components $A_0, B_0, \ldots$ get screened and can be integrated out as well: $m_D \sim gT$.

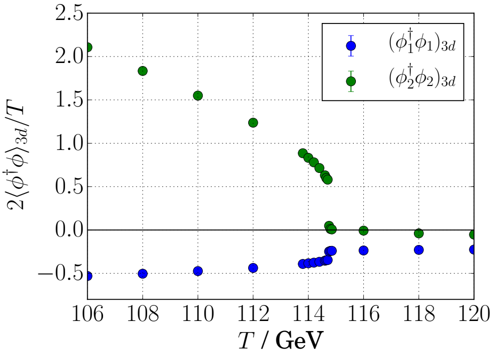
- Final EFT: gauge fields + scalar zero modes **only**, with T-dependent parameters.



Study nonperturbatively on the lattice

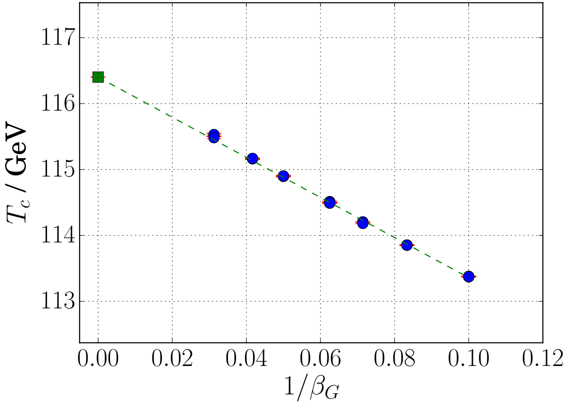
$$S_{\text{latt}} = \sum_{\mathbf{x}} \left[\beta_G \sum_{i<j} \left(1 - \frac{1}{2} P_{ij}(x) \right) - 2a \sum_{i} \phi_1^\dagger(x) U_i(x) \phi_1(x+i) \right] + \ldots$$

- Calculate $\langle \phi_1^\dagger \phi_1 \rangle, \langle \phi_2^\dagger \phi_2 \rangle, \ldots$ with multicanonical Monte Carlo, vary T and find discontinuity.



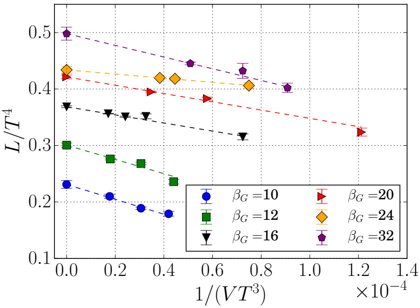
Extrapolating to the continuum: critical temperature

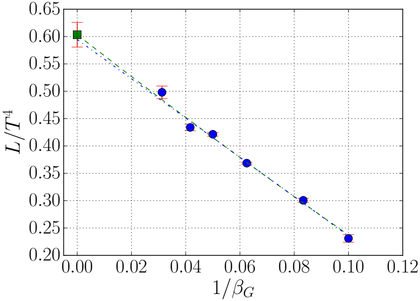
- Lattice spacing varied by changing $\beta_G = 4/(a\bar{g}^2)$. Take limit $1/\beta_G \propto a \to 0$:



Extrapolating to the continuum: latent heat

- Infinite volume limit important for transition strength.



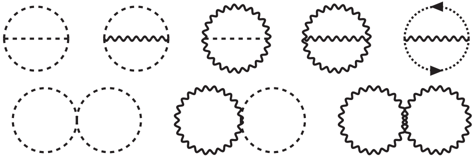


Contents

Effective potential within the effective theory

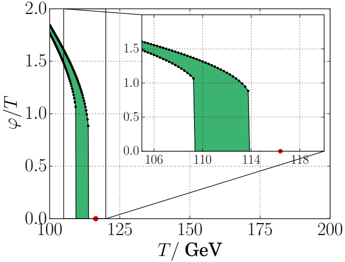
$$V_{\text{eff}}^{\text{3d}} = V_{\text{tree}}^{\text{3d}} + V_{\text{1-loop}}^{\text{3d}} + V_{\text{2-loop}}^{\text{3d}}$$

- Calculate as usual in background-field gauge, but now the loop integrals are 3d.
- Two loop is straightforward: thermal corrections and resummations already included in dimensional reduction.
- Can check the nonperturbative IR effects by comparing to lattice!

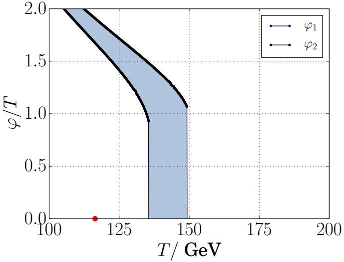


Comparing the different methods in 2HDM: point 1

- Inert ϕ_1, $m_H = 66$ GeV, $m_A = m_{H^\pm} = 300$ GeV, $\max(\lambda_{\text{BSM}}) \sim 3$.



(a) 2-loop potential in the EFT

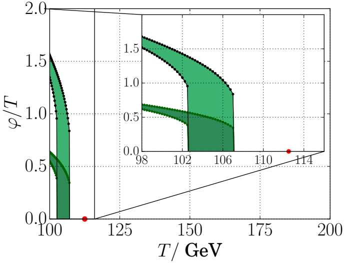


(b) Resummed 1-loop potential in 4d

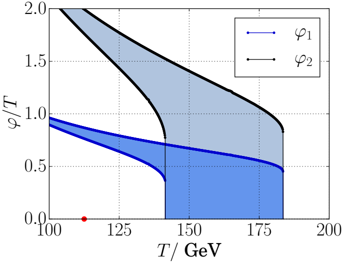
Method	T_c/GeV	L/T_c^4	ϕ_c/T_c
1-loop resum.	142.4 ± 6.88	0.33 ± 0.02	1.00 ± 0.07
2-loop V_{eff} in 3d	111.6 ± 2.30	0.57 ± 0.10	0.98 ± 0.09
3d lattice	116.40 ± 0.005	0.60 ± 0.02	1.08 ± 0.02

Comparing the different methods in 2HDM: point 2

- Alignment limit: $\tan\beta = 2.75$, $m_H = 150$ GeV, $m_A = m_{H^\pm} = 350$ GeV, $\max(\lambda_{\text{BSM}}) \sim 4$.



(a) 2-loop potential in the EFT



(b) Resummed 1-loop potential in 4d

Method	T_c/GeV	L/T_c^4	ϕ_c/T_c
1-loop resum.	162.5 ± 21.0	0.20 ± 0.03	0.88 ± 0.05
2-loop V_{eff} in 3d	104.9 ± 2.30	0.61 ± 0.10	0.97 ± 0.06
3d lattice	112.5 ± 0.01	0.81 ± 0.05	1.09 ± 0.03

Performance of perturbation theory

- We estimate $\sim 20\%$ error in dimensional reduction: neglected $\mathcal{O}(\lambda^3), \mathcal{O}(g^6)$ effects.
- To get stronger transitions we could increase the couplings, but the errors would only get bigger!

  $\Rightarrow$ Many strong-EWPT scenarios in the literature outside the reach of all current state-of-art methods?!?!

Conclusions and outlook

- Large couplings are bad for predictivity. Focus instead on 2-step transitions?
- Dimensional reduction helps with resummations and should be preferred even for purely perturbative calculations.
- Lattice simulations eventually required for best results of (at least) the equilibrium quantities.
- Future prospects: Simulations of bubble nucleation in the EFT!

Backup: Properties of the effective theory

Example: Two Higgs doublet model.

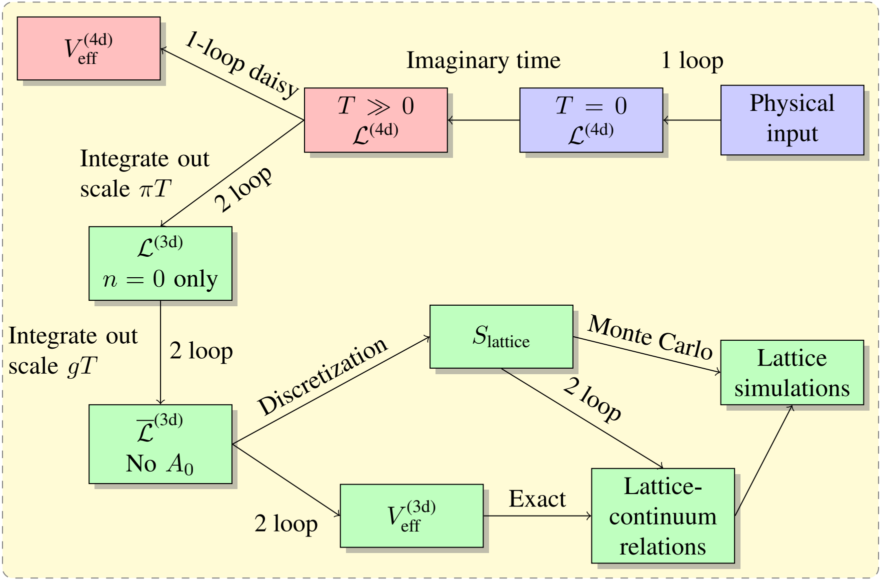
$$V(\phi_1, \phi_2) = m_1^2 \phi_1^\dagger \phi_1 + m_2^2 \phi_2^\dagger \phi_2 + m_{12}^2 \phi_1^\dagger \phi_2 + \cdots + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \ldots$$

The resulting EFT:

$$\begin{aligned}\bar{\mathcal{L}}_{\text{3d}} =& \frac{1}{4}(F_{rs})^2 + (D_r\phi_1)^\dagger (D_r\phi_1) + (D_r\phi_2)^\dagger (D_r\phi_2) + \overline{m}_1^2(T)\phi_1^\dagger\phi_1 \\ &+ \overline{m}_2^2(T)\phi_2^\dagger\phi_2 + \overline{m}_{12}^2(T)\phi_1^\dagger\phi_2 + \cdots + \overline{\lambda}_3(T)(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \ldots\end{aligned}$$

- Super-renormalizable $\rightarrow$ **exact** relations to lattice parameters.
- Very accurate in the weak coupling regime. Error in the SM is $\sim 1\%$.

Backup: Summary of the calculation(s)



Backup: Finding the transition point from histograms

- Find precise T_c by reweighting the histograms to equal weight.