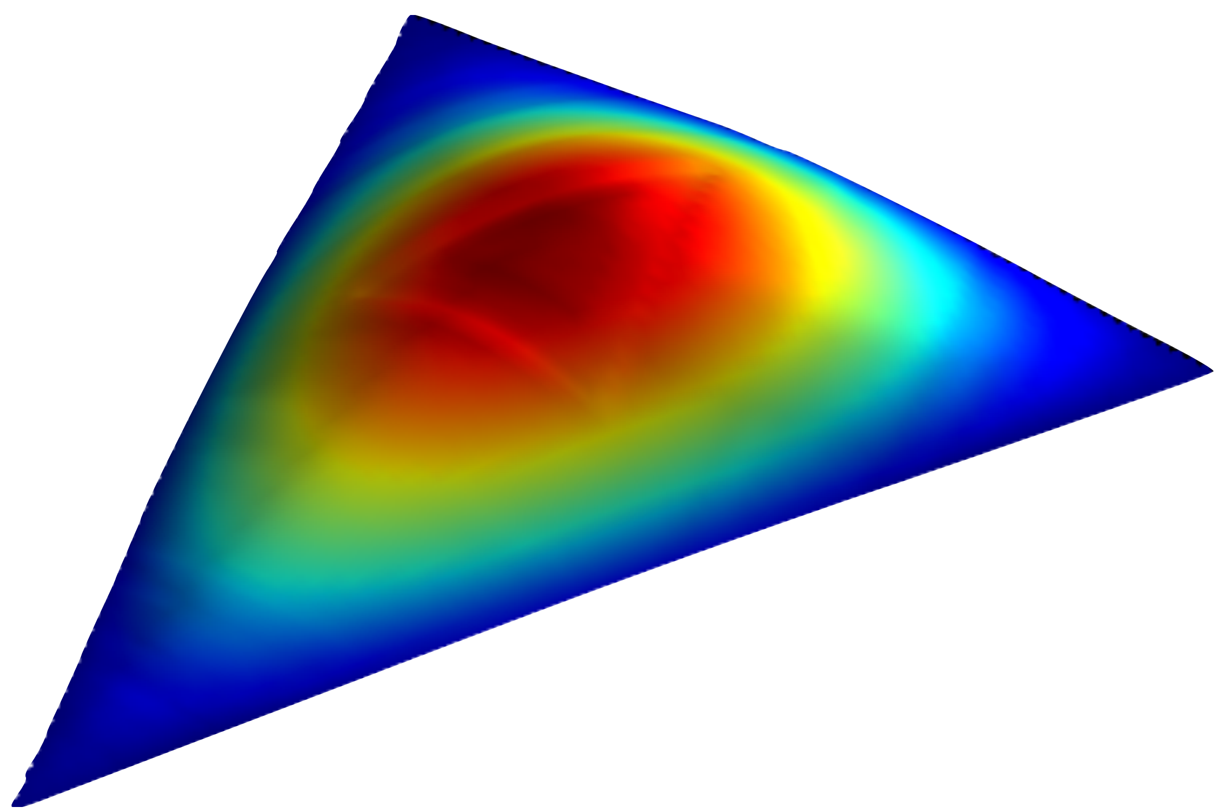
Numerically calculating observables from inflation and reheating: PyTransport and beyond



David Mulryne
Queen Mary University of London

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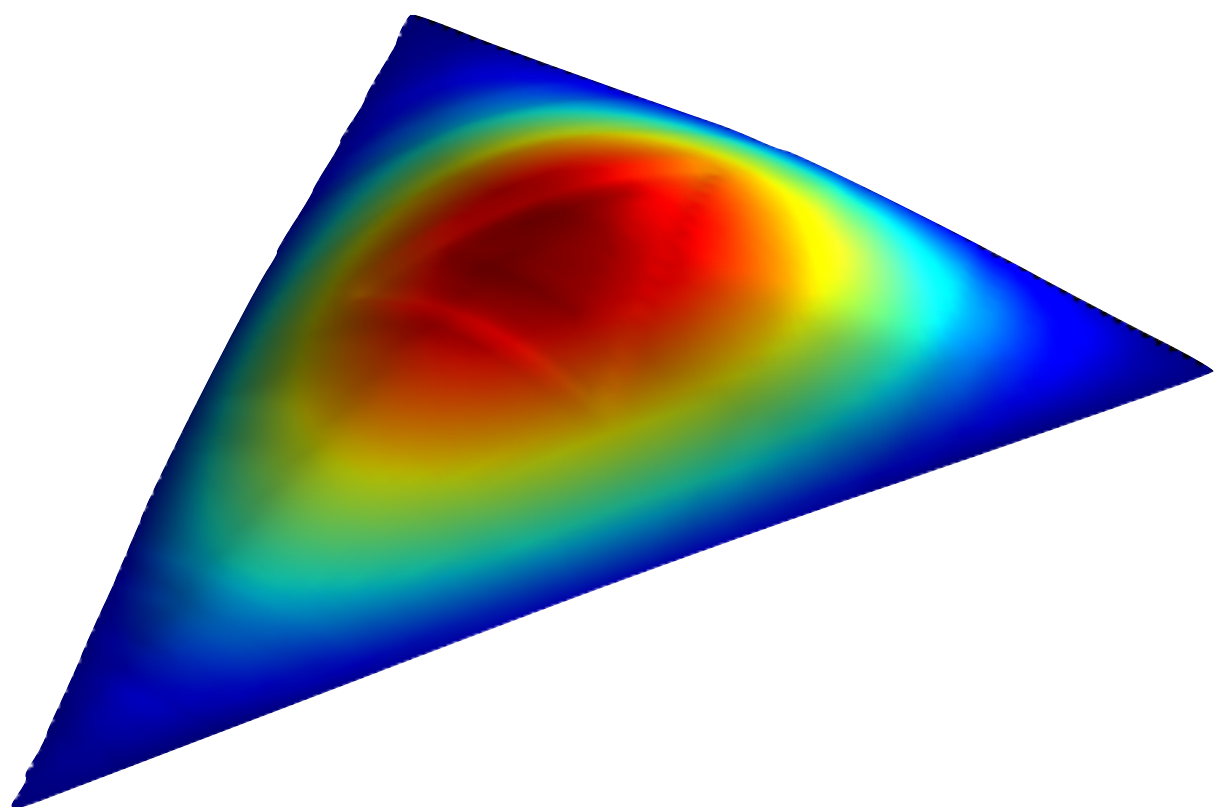

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Collaborators

Transport collaborators:
D Seery, M Dias, J Frazer, J Ronayne
arXiv:1609.00379; arXiv:1708.07130 + ongoing

Visit TransportMethod.com for more information

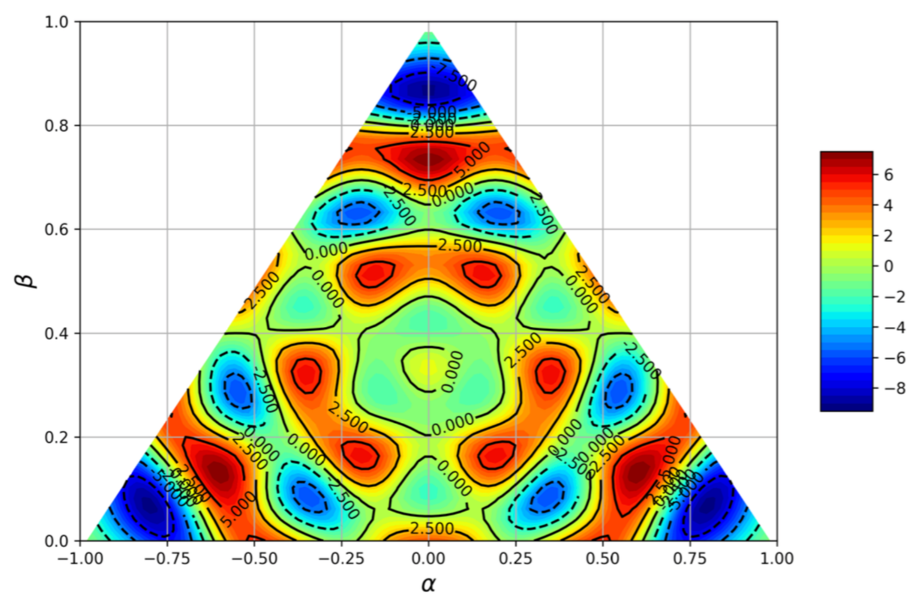
Non-perturbative reheating collaborations:
S Imrith, A Rajantie
arxiv:1801.02600; arXiv:1903.07487

Motivation

- Many many models of inflation
- New effects in (higher order) correlation functions could potentially allow us to detect new fields
- Models can be complicated, for example with curved field space metric
- In many systems the large N limit has interesting properties. To probe this limit for inflation, however, numerics are essential
- Without numerics, theory error even for simple models can be greater than observational uncertainty
- At very least we should be able to take any model of inflation and confront with (improving) observations

PyTransport

- PyTransport and sibling code CppTransport (developed by David Seery) solves transport equations for inflationary perturbations to produce full power spectrum and bispectrum
- Deals with models with arbitrary numbers of scalar fields, a curved field space metric, perturbative reheating (unreleased)
- Includes all tree-level effects on sub and super-horizon scales
- Publicly available and automated in sense user need only provide potential (and field space metric) — users welcome!



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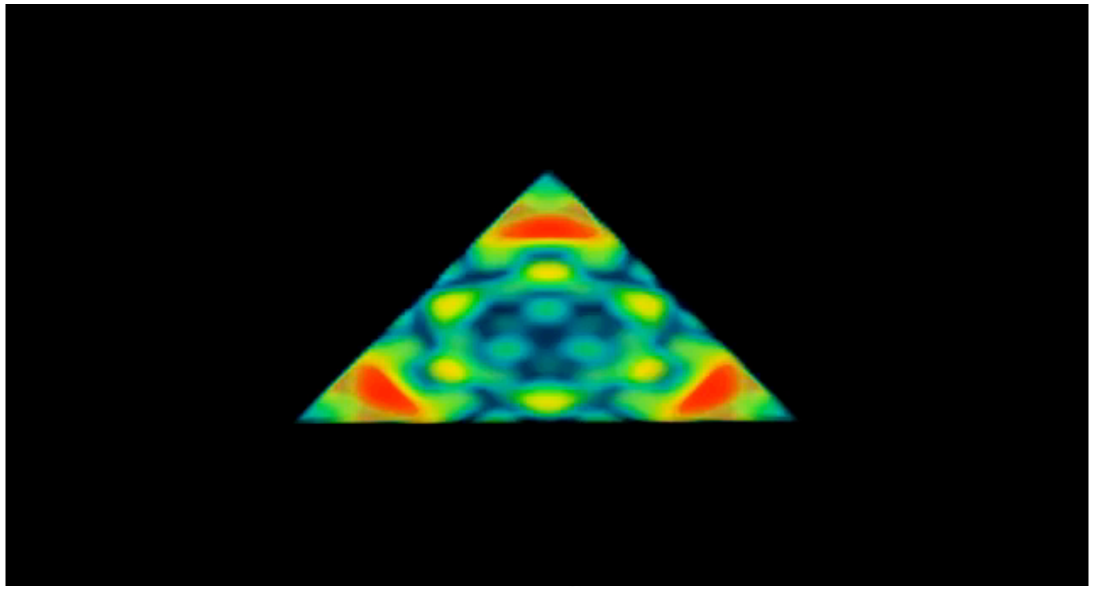

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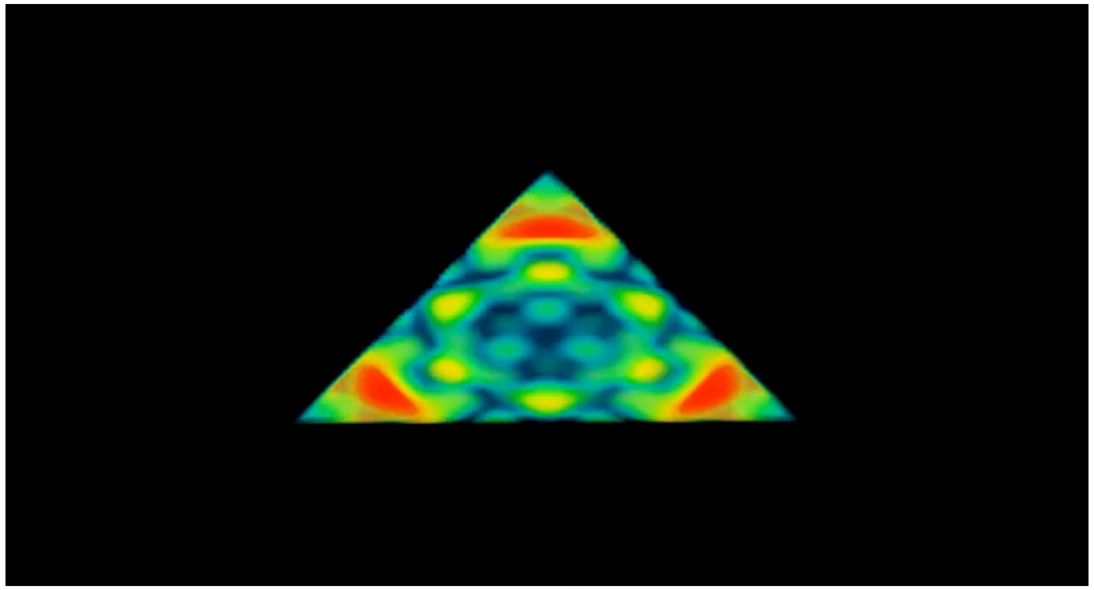

Observational quantities

- Statistical quantities we want to evaluate

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)P(k)$$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

$$f_{\rm NL} = \frac{5}{6}\frac{B(k_1,k_2,k_3)}{P(k_1)P(k_2)+P(k_1)P(k_3)+P(k_2)P(k_3)}$$

- Basic predictions

$$P(k)\sim Ak^{-3}$$

$$f_{\rm NL}\sim \text{slow roll (for canonical single field)}$$

Calculating statistics

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_p^2 R + \mathcal{L}_m\right]$$

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$$\mathcal{L}_m = -G_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V$$

$$\phi^I + \delta\phi^I$$

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$$\phi^I + \delta\phi^I$$

action expanded order by order in fluctuations Q^I
and gravitational waves (tensor) h_{ij}

Calculating statistics

$$S = S_{(2)} + S_{(3)}$$

$$\downarrow \qquad\qquad \searrow$$

$$\mathcal{O}(2) \text{ in } Q^I \qquad \mathcal{O}(3) \text{ in } Q^I$$

Maldacena 2003; Seery and Lidsey 2006; Chen *et al*. 2007; Elliston *et al.* 2012; many others

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Lagranian or Hamiltonian equations of motion for Q^I

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Lagranian or Hamiltonian equations of motion for Q^I

$$\langle Q^I(k_1)Q^J(k_2)\rangle = (2\pi)^3\delta(\mathbf{k_1}+\mathbf{k_2})\Sigma^{IJ}(k_1)$$

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$$Q^I \to \zeta$$

Calculating the statistics — transport method

- Our approach (schematically)

$$\frac{\mathrm{d}Q^I}{\mathrm{d}t} = u^I{}_J Q^J + \frac{1}{2} u^I{}_{JK} Q^J Q^k$$

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$$\frac{\mathrm{d}}{\mathrm{d}t} B^{IJK} = u^I{}_L B^{LJK} + u^I{}_{LM} \Sigma^{JL} \Sigma^{KM} + \text{cyclic perms}$$

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Background and k dependent quantities

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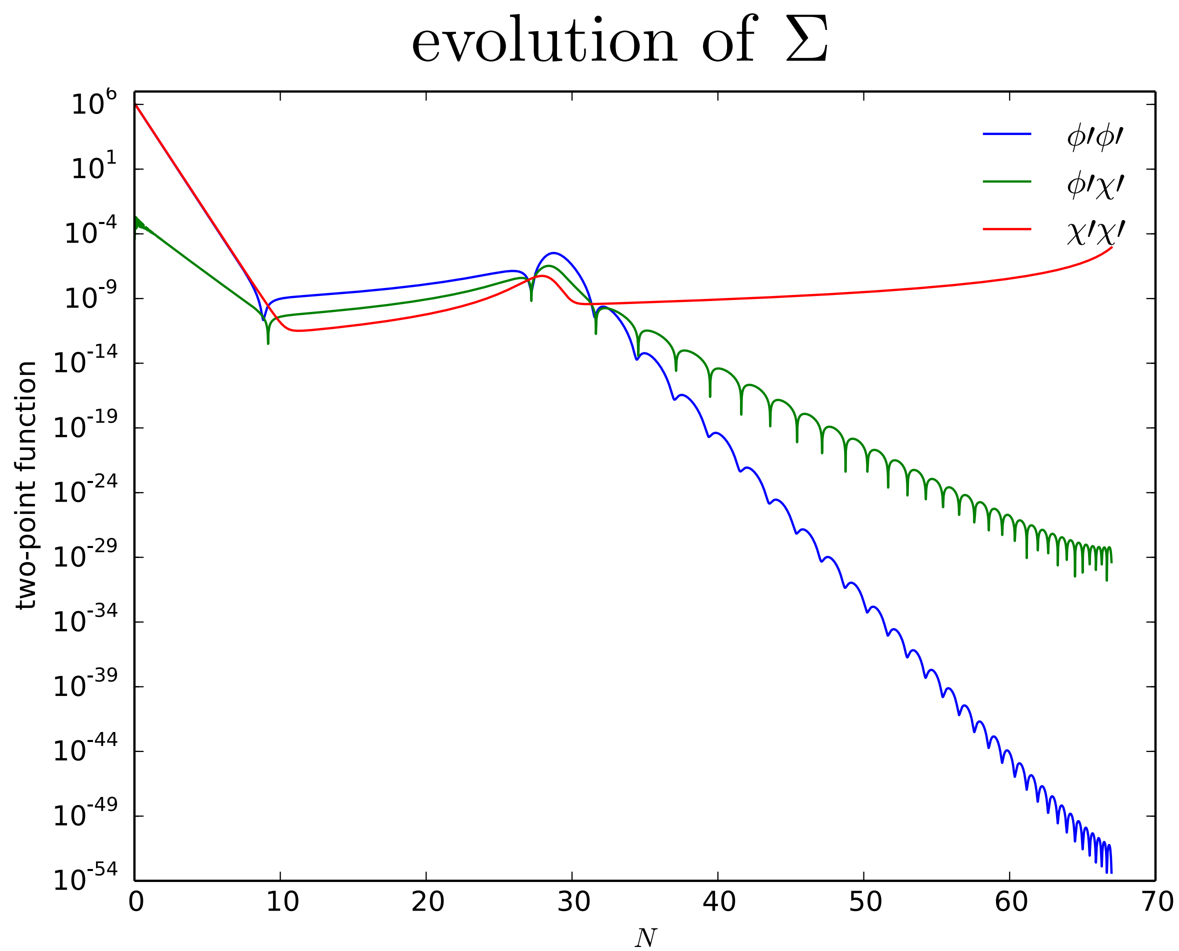
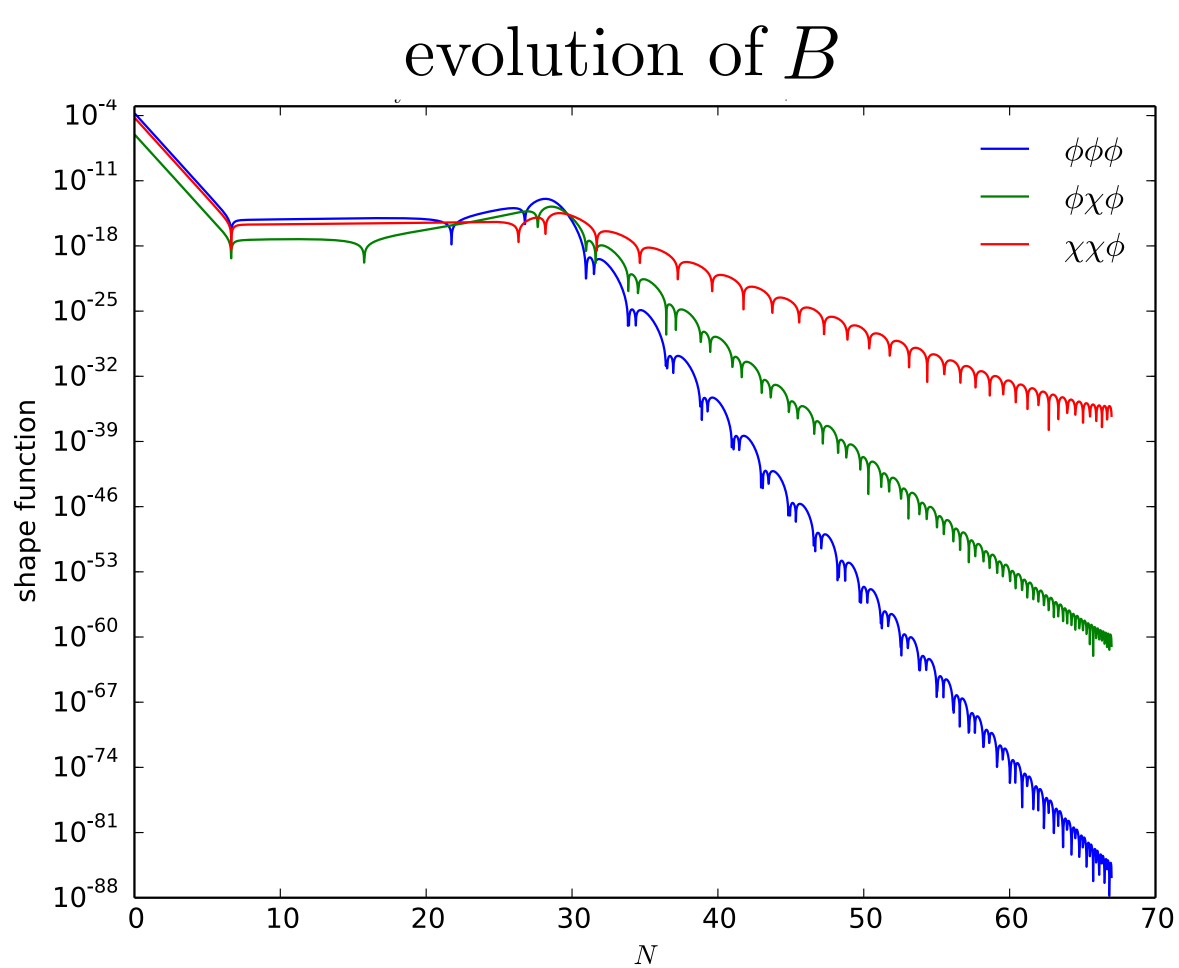
Ideal for a numerical implementation — solve from Bunch Davis vacuum


evolution of Σ
two-point function
N
$\phi\prime\phi\prime$
$\phi\prime\chi\prime$
$\chi\prime\chi\prime$
10^{6} 10^{1} 10^{-4} 10^{-9} 10^{-14} 10^{-19} 10^{-24} 10^{-29} 10^{-34} 10^{-39} 10^{-44} 10^{-49} 10^{-54}
0 10 20 30 40 50 60 70


evolution of B
shape function
N
$\phi\phi\phi$
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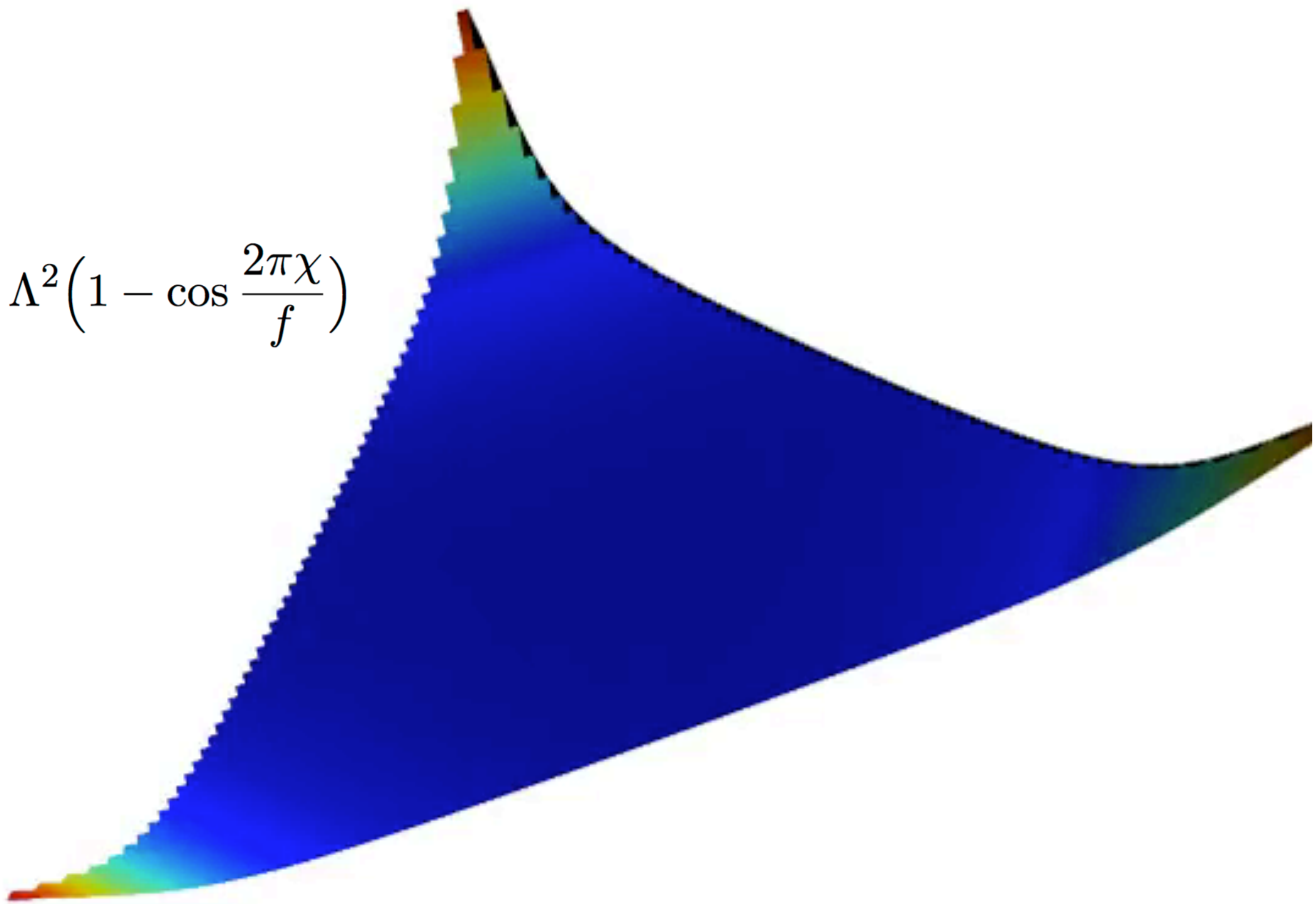
$\chi\chi\phi$
10^{-4} 10^{-11} 10^{-18} 10^{-25} 10^{-32} 10^{-39} 10^{-46} 10^{-53} 10^{-60} 10^{-67} 10^{-74} 10^{-81} 10^{-88}
0 10 20 30 40 50 60 70

$$V = \frac{1}{4} g\phi^4 + \Lambda^2 \left(1 - \cos \frac{2\pi\chi}{f}\right)$$



Slice through reduced bispectrum with $k_1 + k_2 + k_3$ fixed

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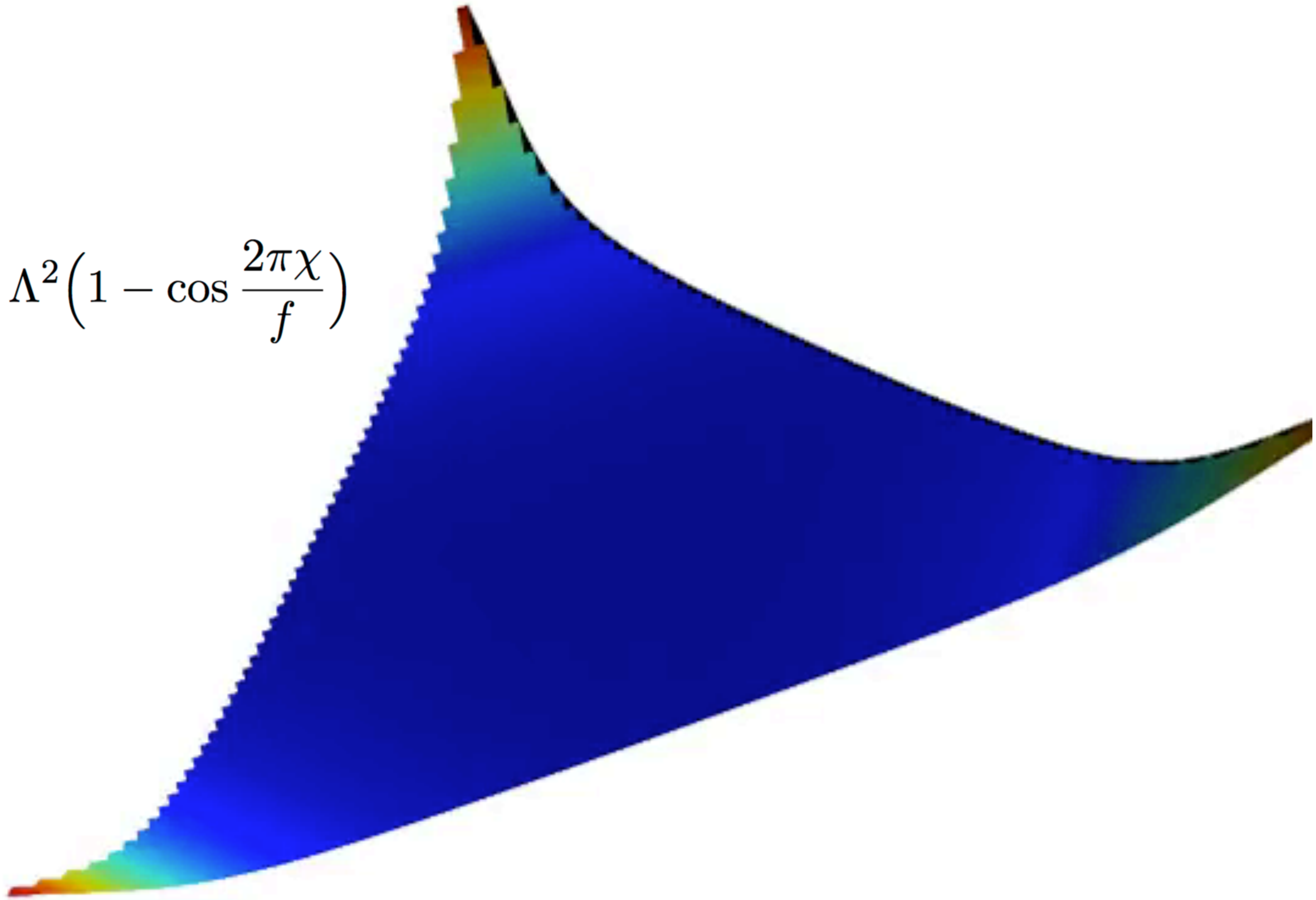

Slice through reduced bispectrum with $k_1 + k_2 + k_3$ fixed

Demonstration interlude

$$V = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$

Models

- **Model driven** - string theory, supergravity, MSSM, Standard Model. At a minimum we should be able to test all models
    - Either concrete models, or random potentials e.g. Dias, Frazer and Marsh (2017), Bjorkmo and Marsh (2017)
- **Phenomenological** - how do multi-field dynamics differ from single field dynamics? - the great hope is that we could detect new fields!
- **New effects** - extra light/heavy fields, curved field space metric -> curved trajectories, isocurvature modes -> Non-Gaussianity Byrnes et al. 2008; Hall and Choi Chen & Wang 2009; Tolley and M. Wyman 2010; Achúcarro et al. 2011 PBH production e.g. Germani, Prokopec (2017,1018), Tomberg, Räsänen (2018) , Byrnes, Cole, Patil (2018)
- **Probabilistic** for many fields a probabilistic interpretation may be needed for many fields e.g. Frazer 2014

Heavy field with turn (c.f. non-geodesic motion)

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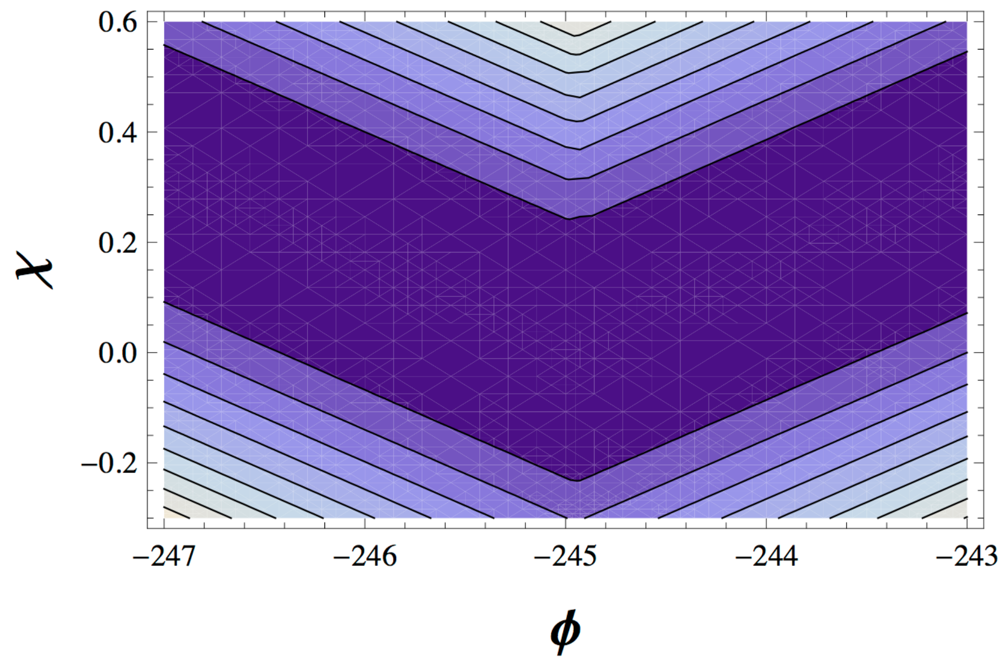
Goa, Langlois and Mizuno (2014)



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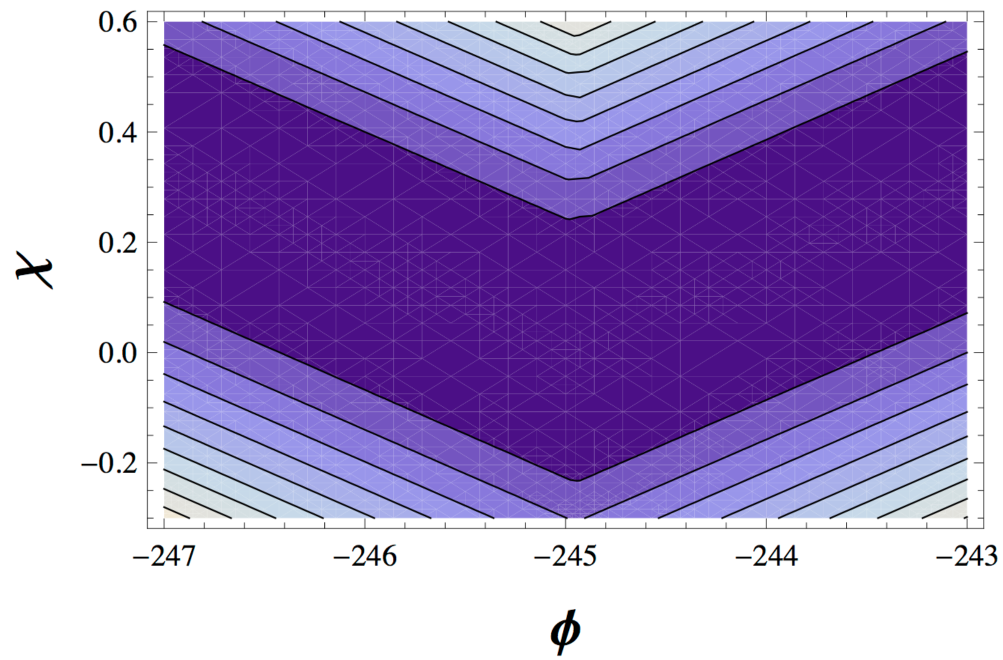

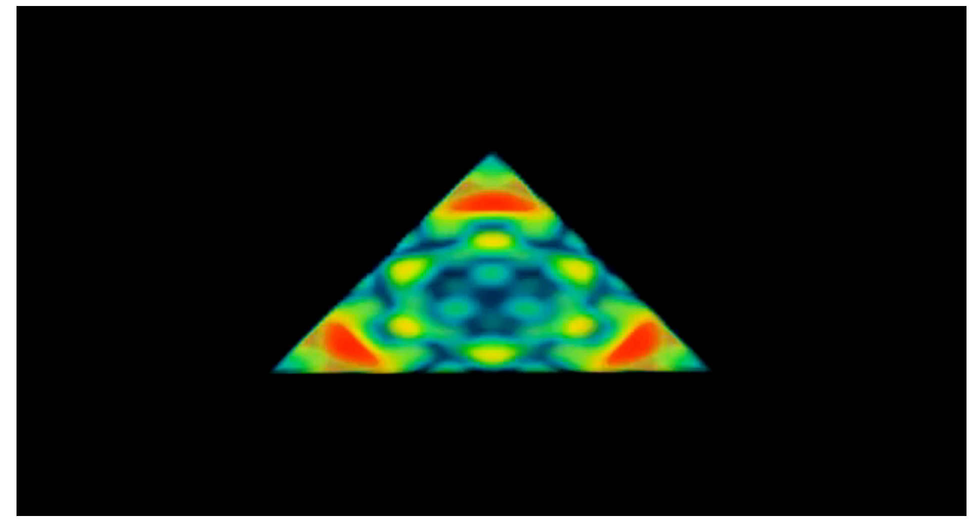

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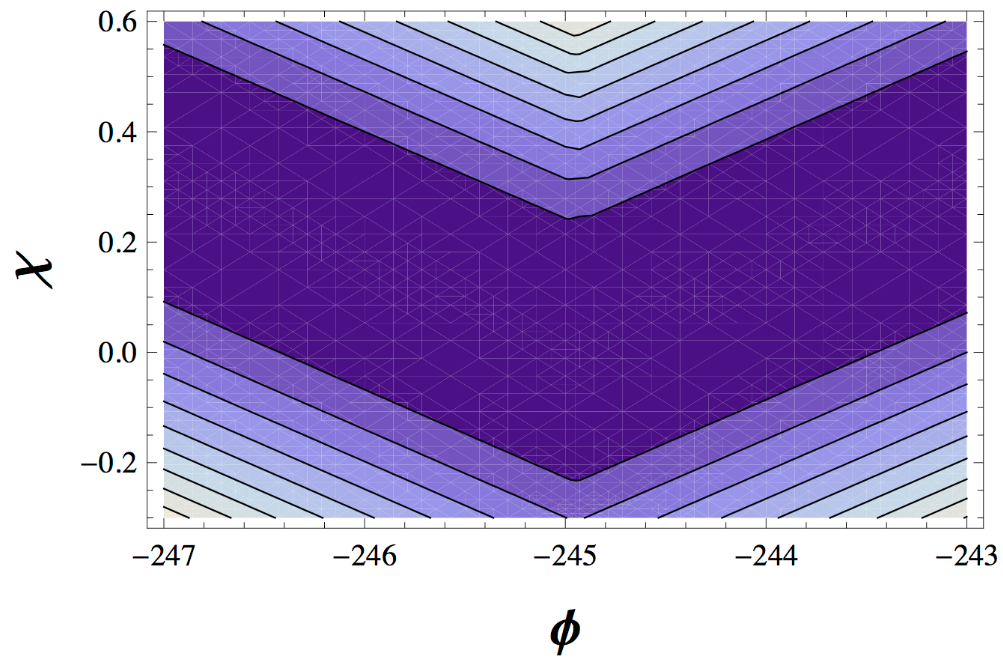

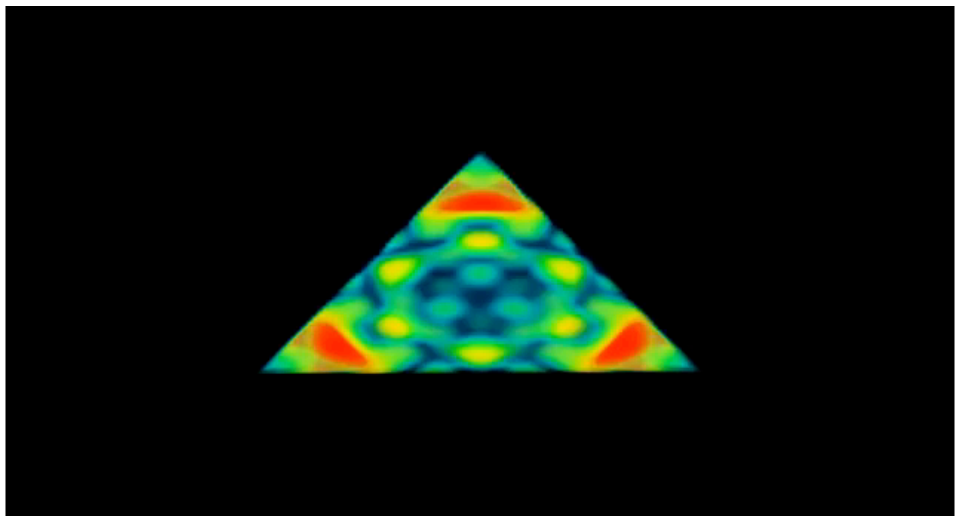

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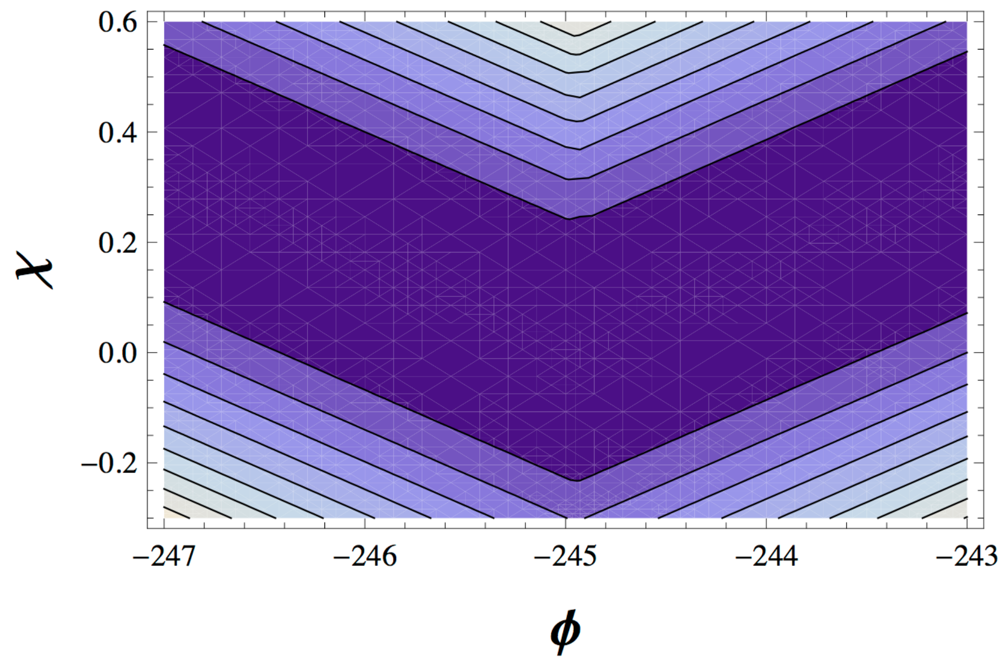

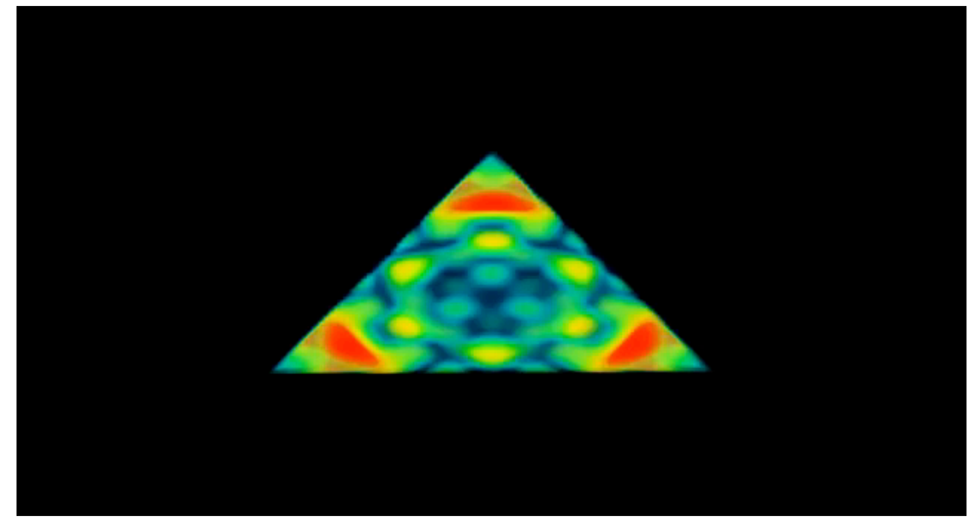

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$$\Gamma(\phi_1) = \frac{\Gamma_0}{\cosh^2\left(2\left(\frac{\phi_1 - \phi_{1(0)}}{\Delta\phi_1}\right)\right)}$$

$$G_{IJ} = \begin{pmatrix} 1 & \Gamma(\phi_1) & 0 \\ \Gamma(\phi_1) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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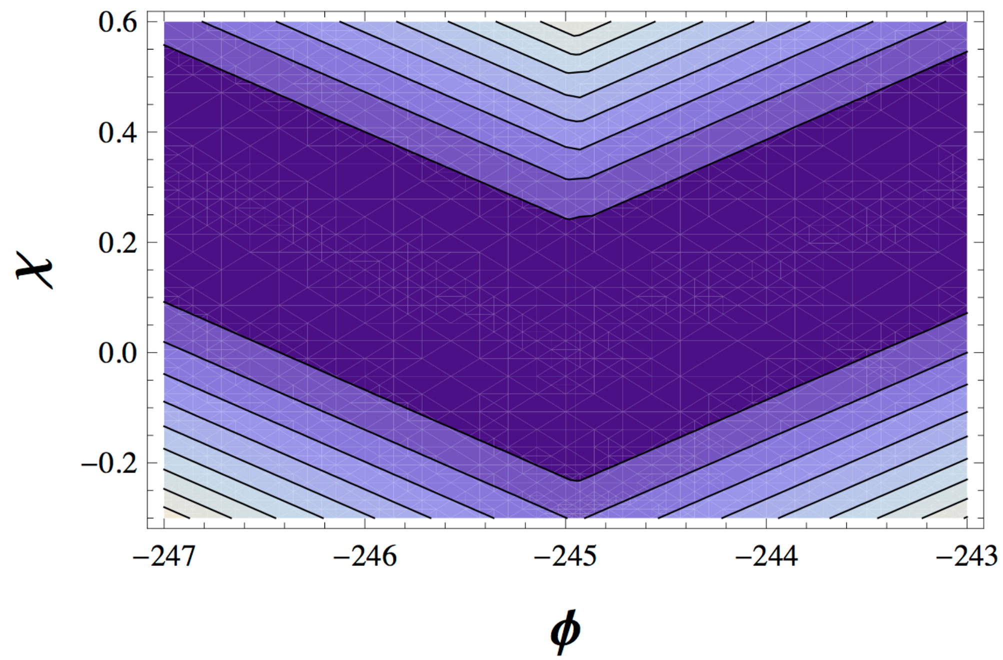

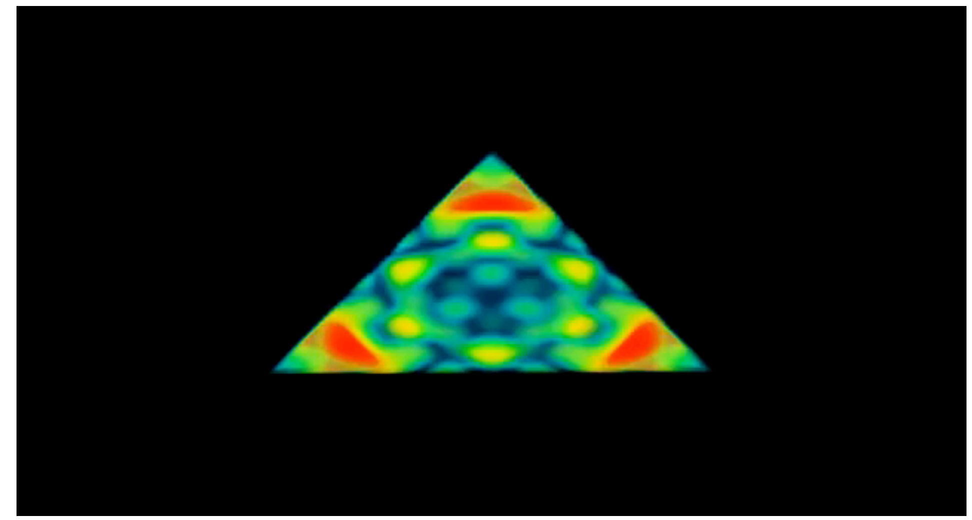

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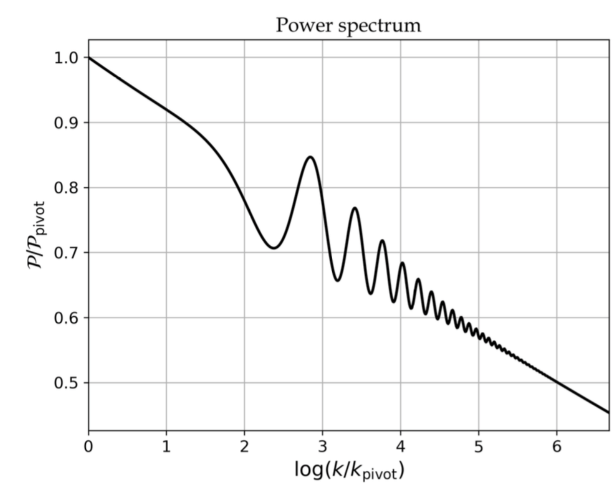

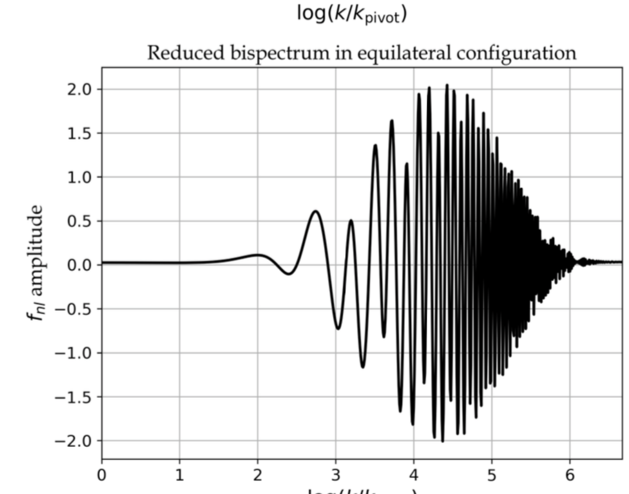

Non-Minimal coupling to gravity i.e. for multifield alpha attractors

Ronayne, Carrilho, Mulryne and Tenkanen (2018)

$$S_J = \int d^4x\sqrt{-g}\left(\frac{1}{2}\delta_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - \frac{M_{\rm P}^2}{2}\left(1+f(\phi^I)\right)g^{\mu\nu}R_{\mu\nu}(\Gamma) - V(\phi^I)\right)$$

$$f(\phi^I) = \sum_I \xi_I^{(n)}\left(\frac{\phi^I}{M_{\rm P}}\right)^n$$

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$$\Downarrow$$

$$S_{\rm E} = \int d^4x\sqrt{-g}\left(\frac{1}{2}G_{IJ}(\phi^I)\partial_\mu\phi^I\partial^\mu\phi^J - \frac{1}{2}M_{\rm P}^2R - V(\phi^I)\Omega^{-2}(\phi^I)\right)$$

$$G_{IJ} = \Omega^{-1}\delta_{IJ} + \frac{3}{2}\upsilon M_{\rm P}^2\Omega^{-2}\frac{\partial\Omega}{\partial\phi^I}\frac{\partial\Omega}{\partial\phi^J}$$

↑ (υ): 0 for metric, 1 for Palatini

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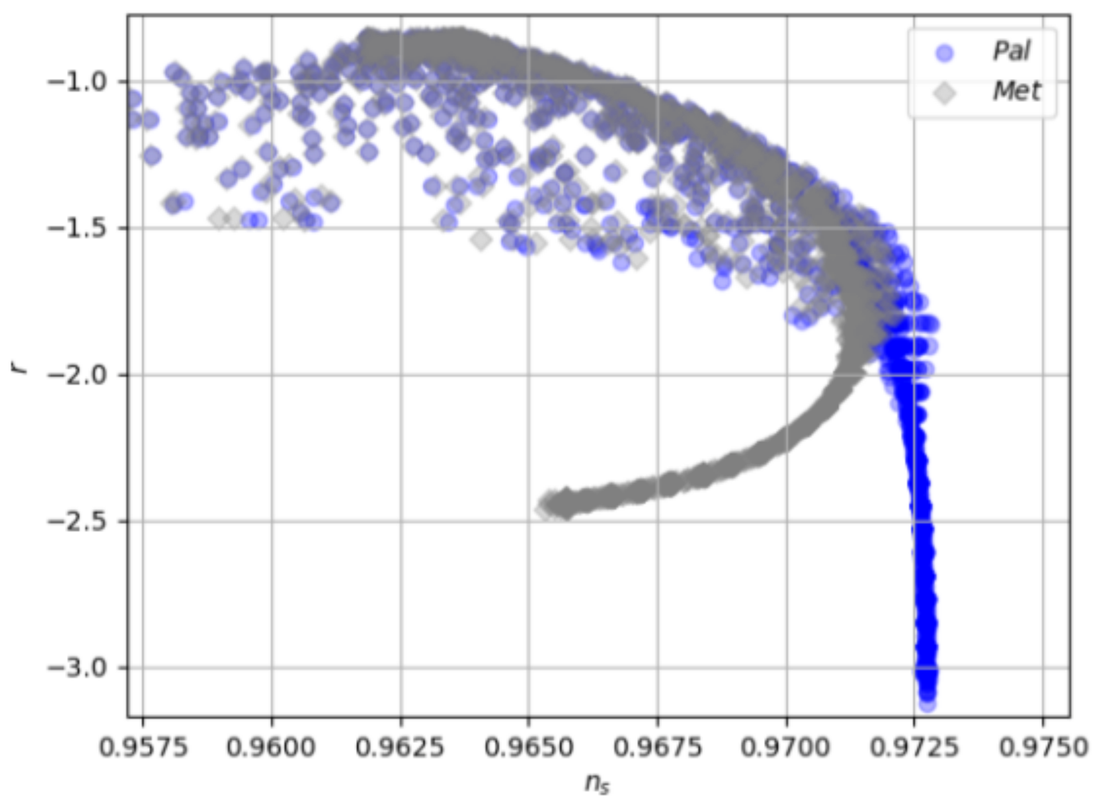
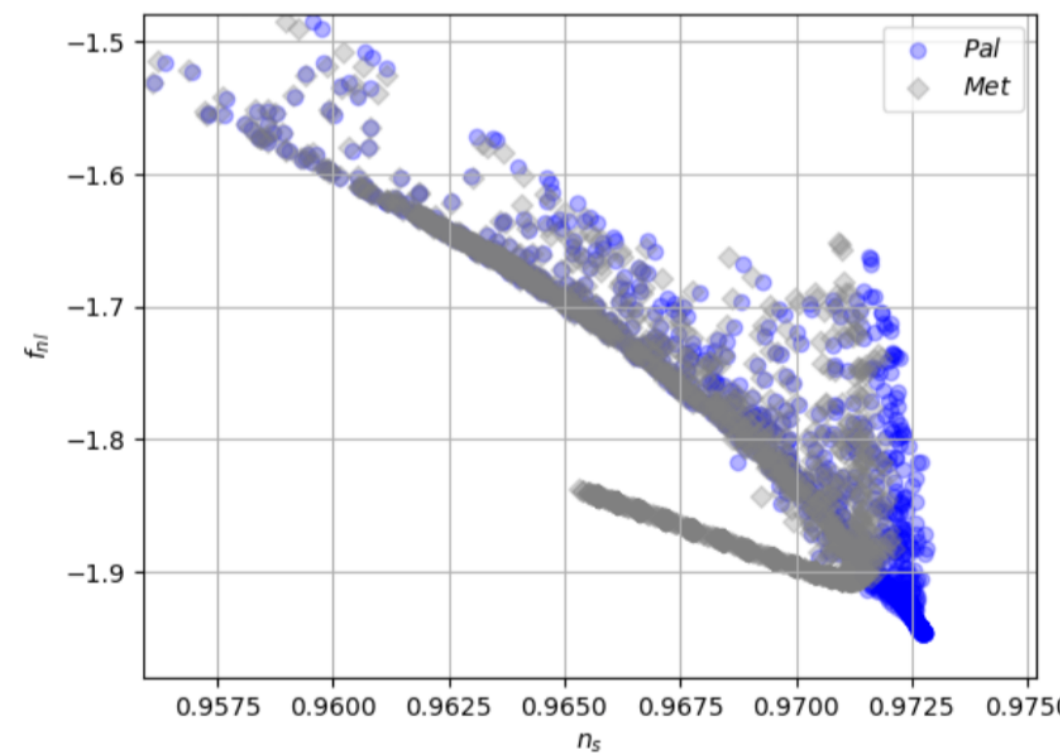
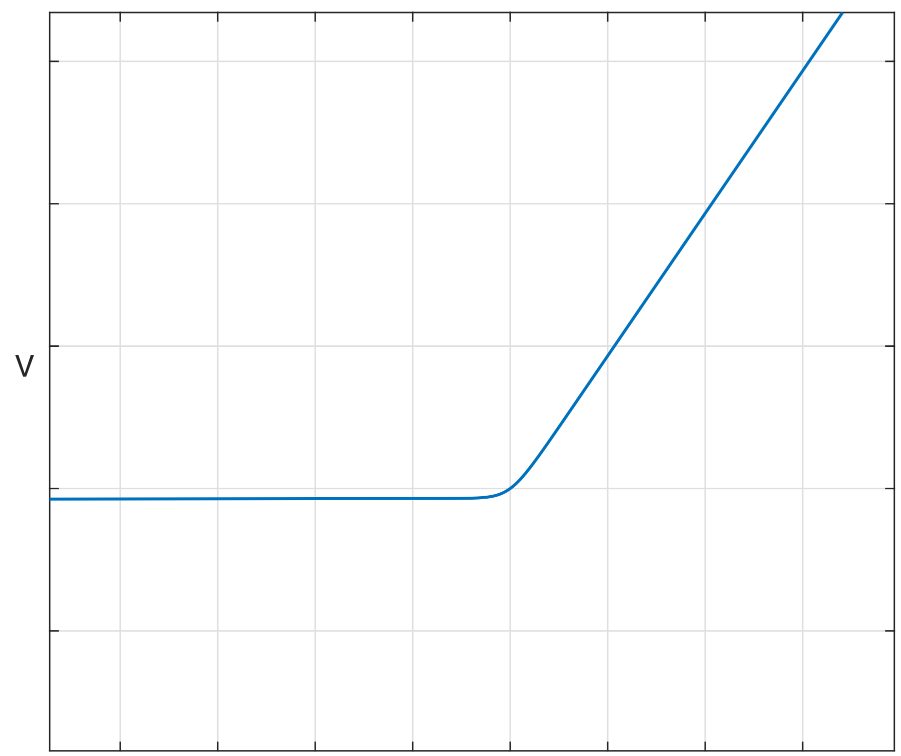
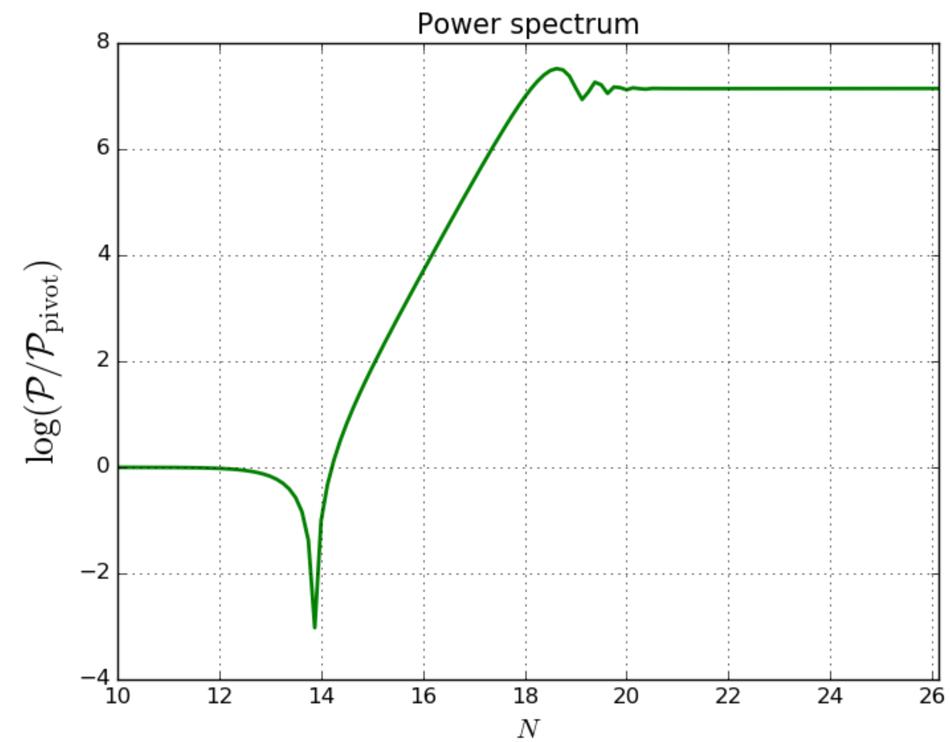


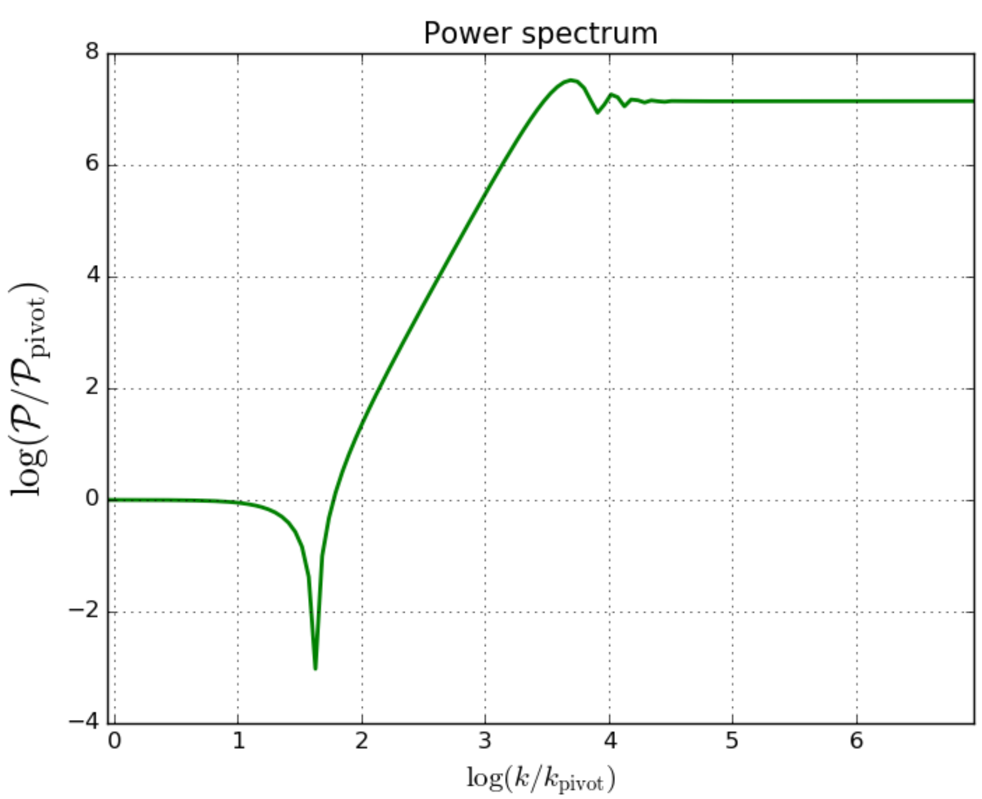


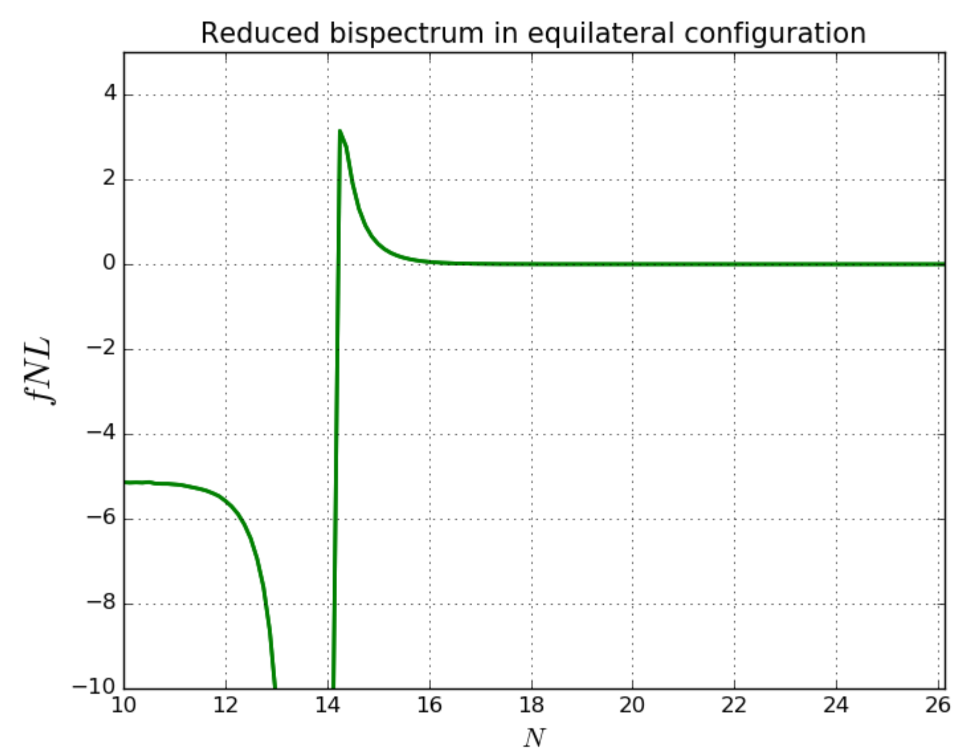
Primordial black holes

e.g. Germani, Prokopec (2017,1018), Tomberg Räsänen (2018) , Byrnes, Cole, Patil (2018)









Reheating

PyTransport with perturbative reheating (in progress)

- Often isocurvaure modes left at end of inflation and so zeta evolves
- Phenomenological way forward is to introduce decay to other radiation and other fluids, gives (with associated perturbed equations to second order)

$$D_t\dot{\phi}^I + 3H\dot{\phi}^I - \Gamma_a^{IJ}\dot{\phi}_J + G^{IJ}V_{,J} = 0$$

$$\dot{\rho}_a + 3H\gamma_a\rho_a + \Gamma_a^{IJ}\dot{\phi}_I\dot{\phi}_J = 0$$

PyTransport with perturbative reheating (in progress)

- e.g. N-axion (c.f. Kim, Liddle, Seery (2009), uncoupled case leads to observable non-Gaussianity)

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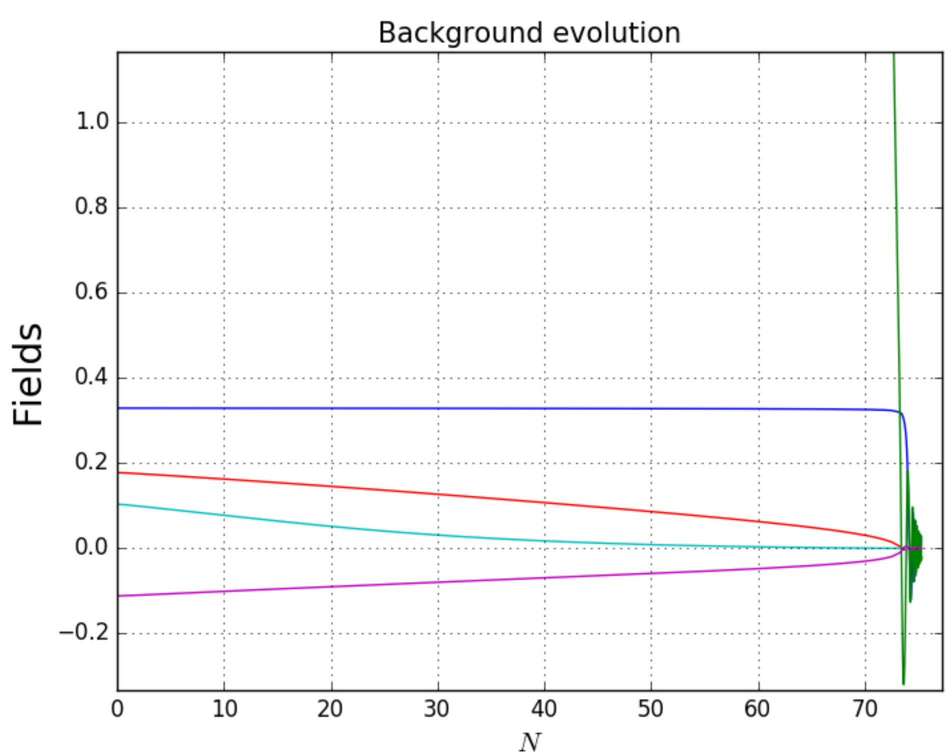

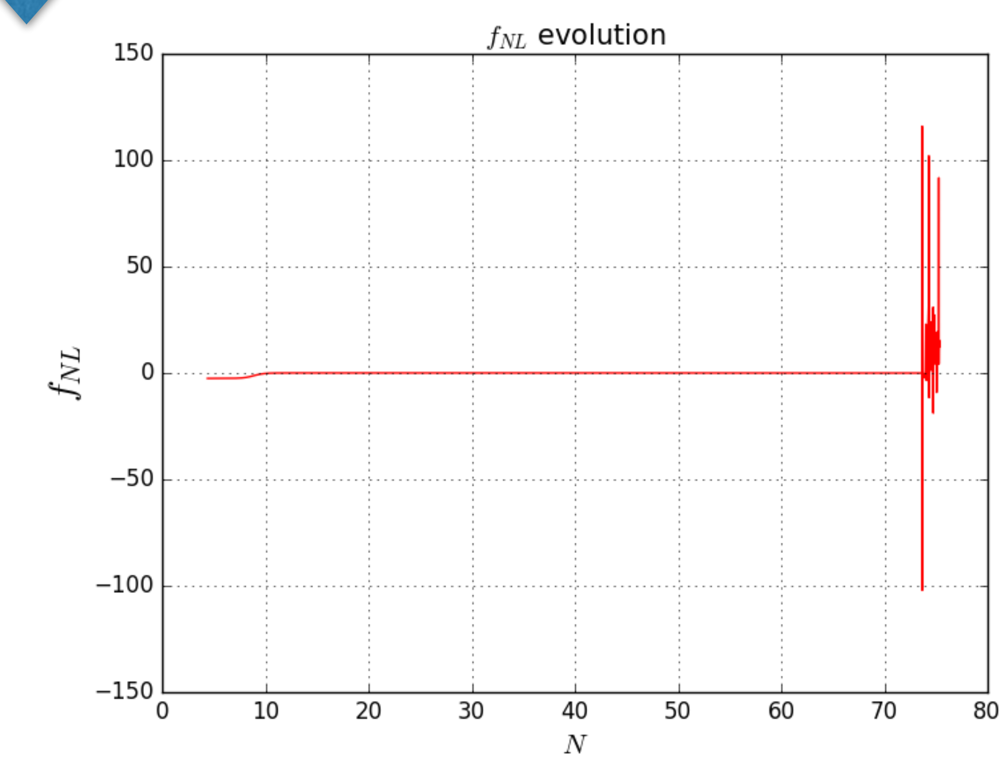

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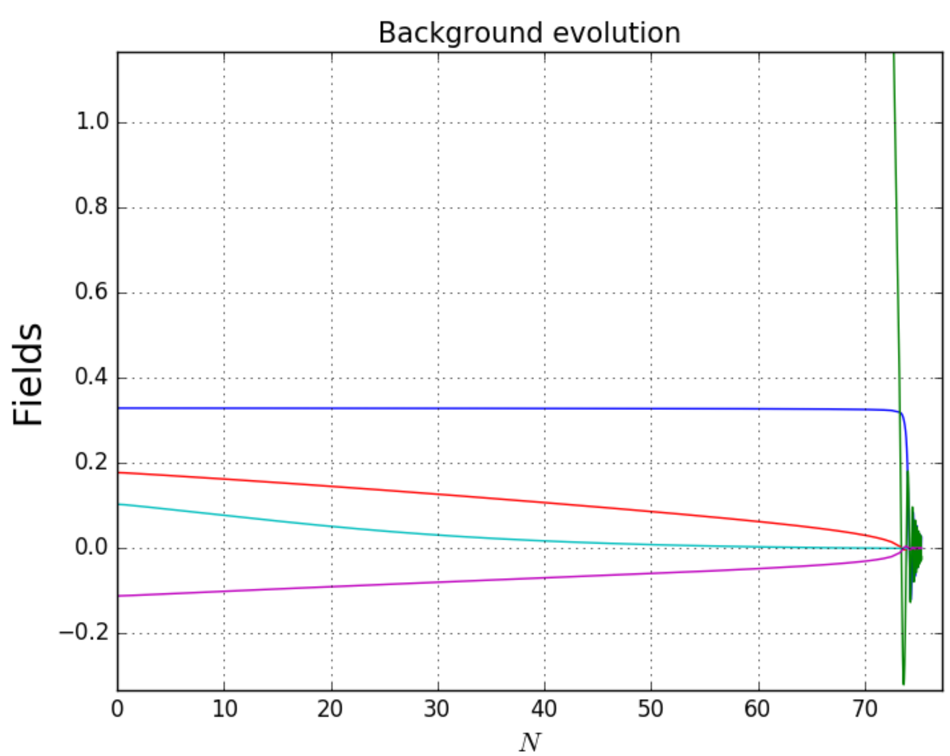

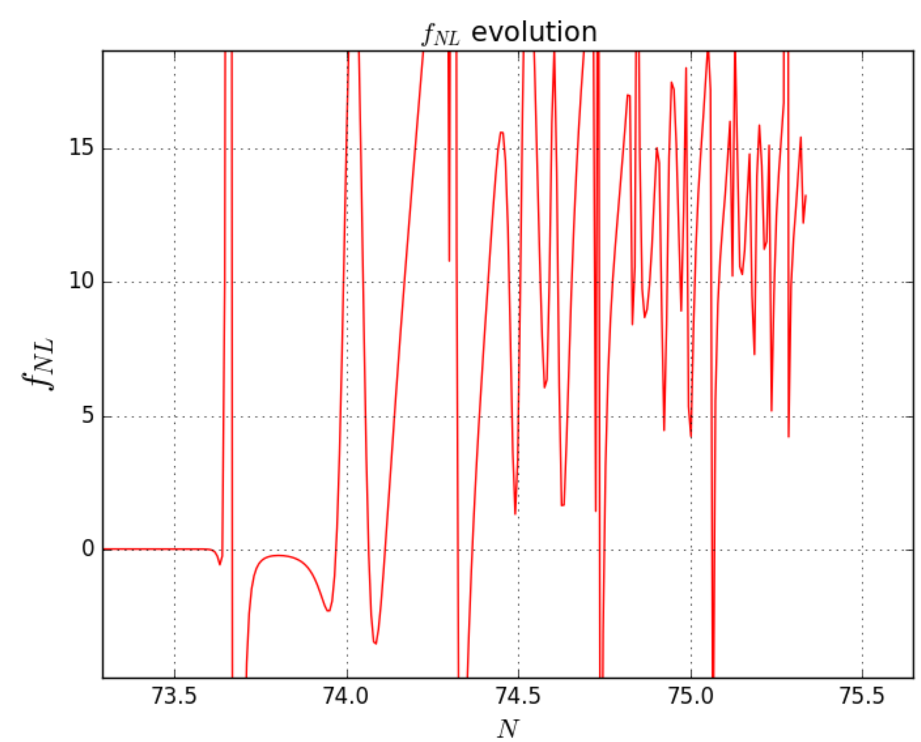

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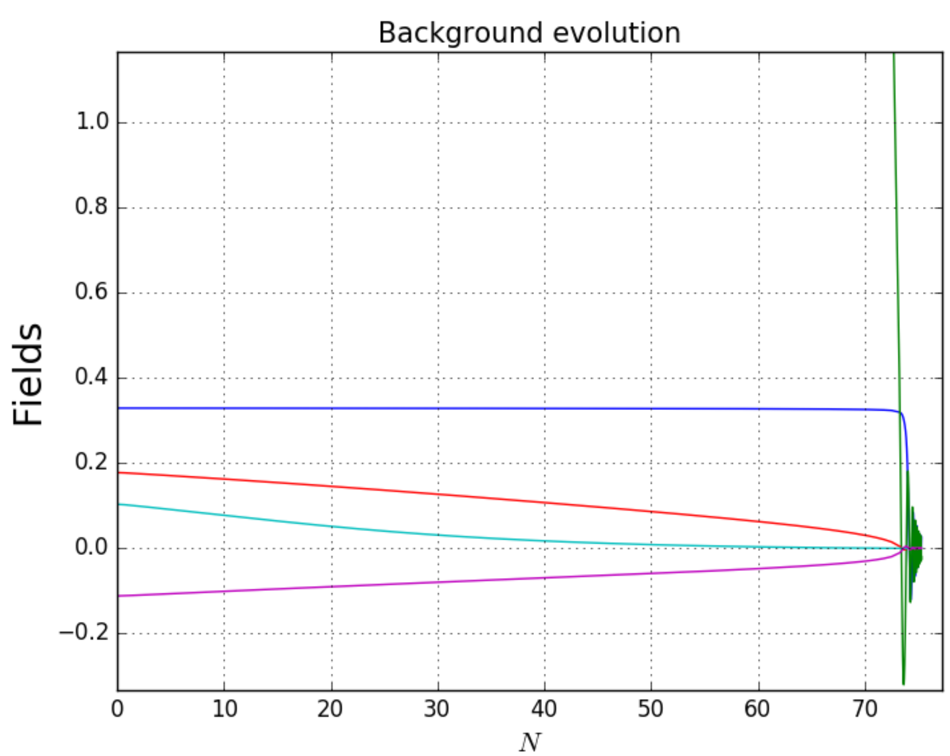

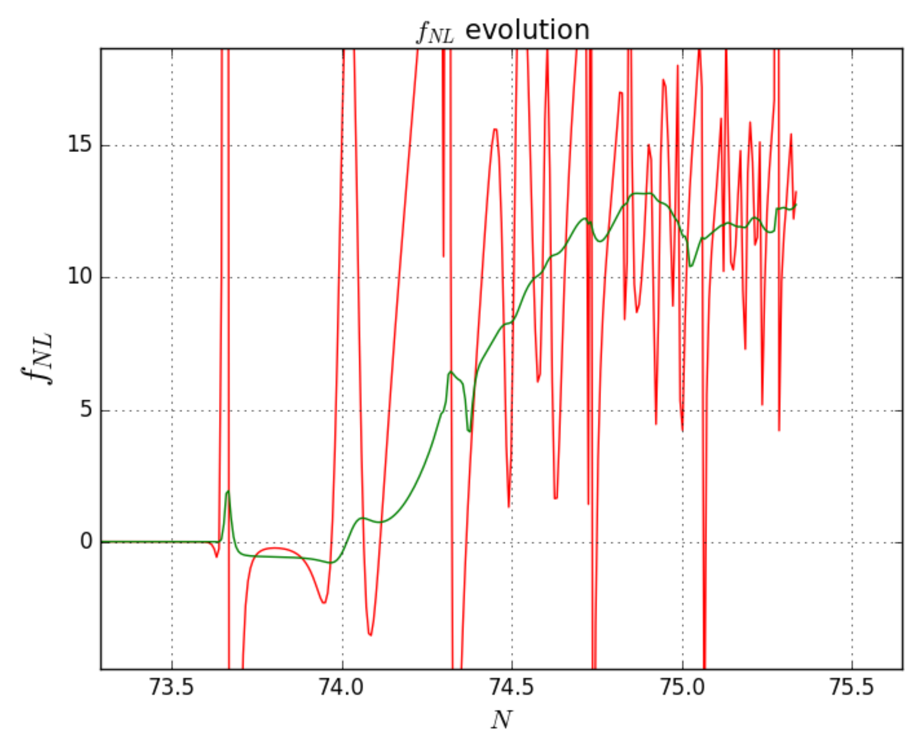

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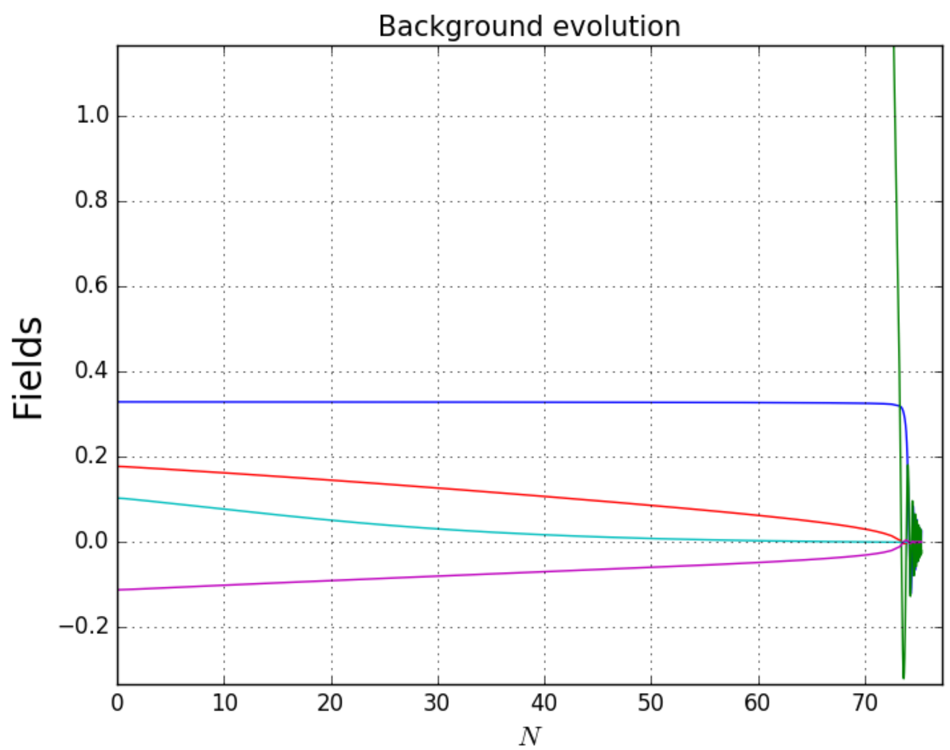
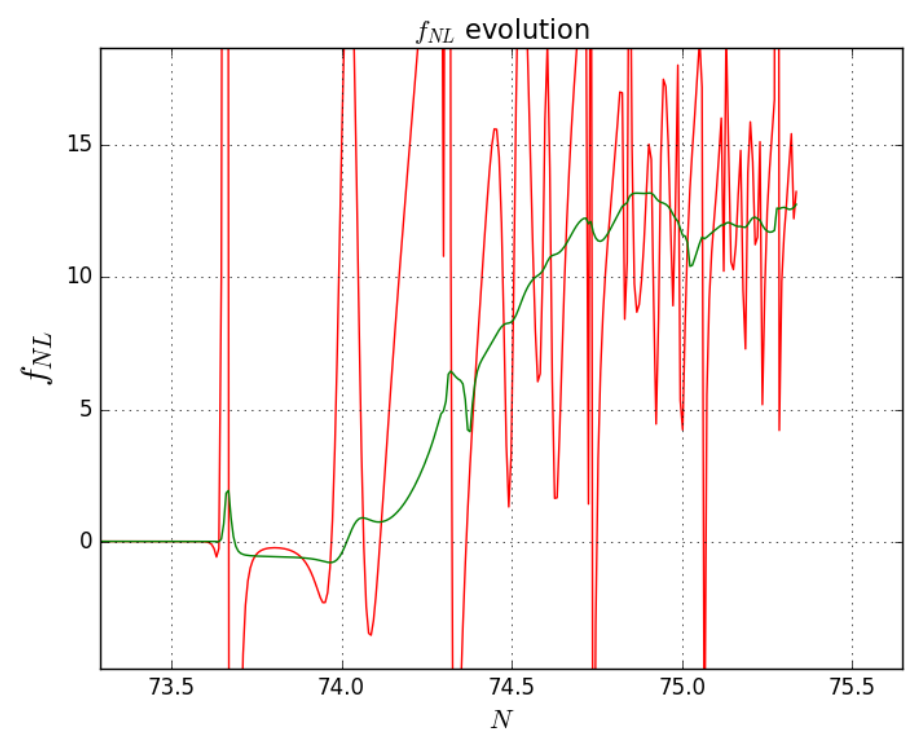


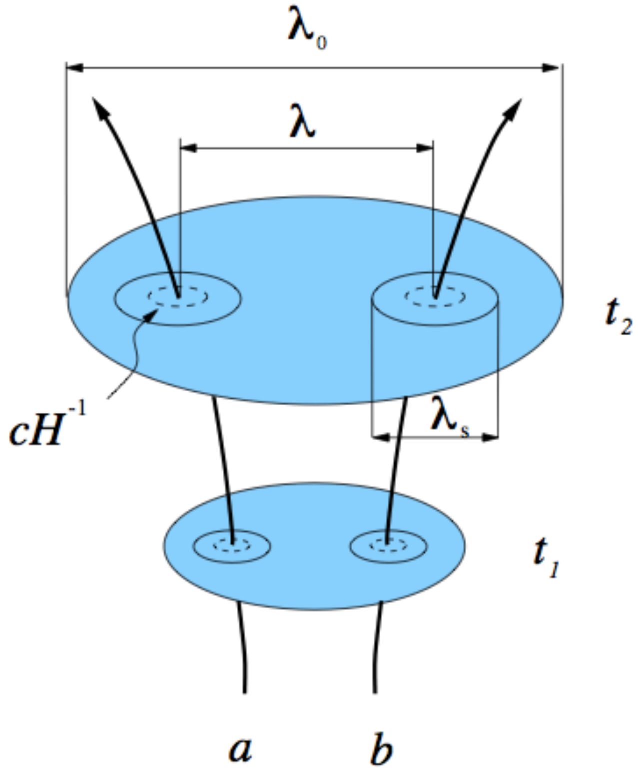


- More generally, Gaussian random landscape around a minimum (c.f. Bjorkmo and Marsh (2017)

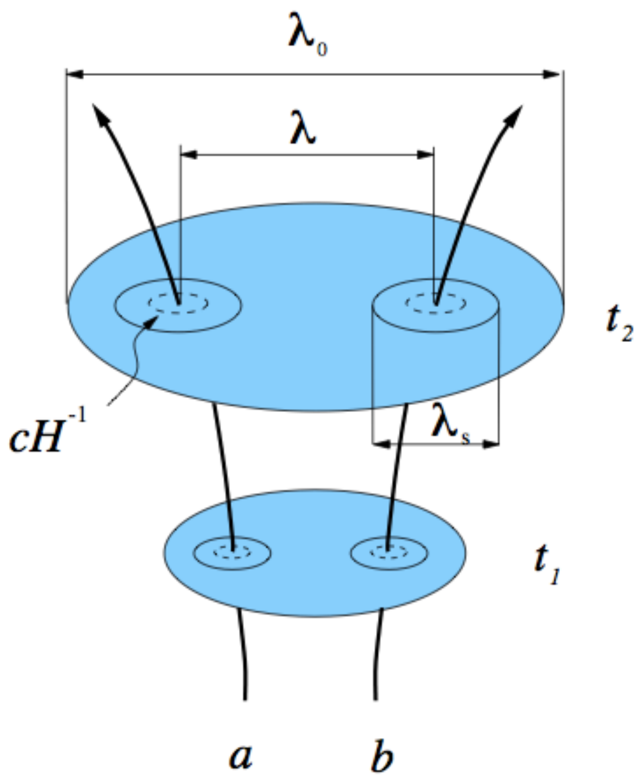
Perturbations through non-perturbative reheating

- What happens if isocurvature present and reheating is non-perturbative (i.e. some form of preheating)
- Dynamics must be tracked using lattice simulations
- Perturbations can be tracked using δN
- However usual expansion can't be used
- Archetypal example is massless preheating

$$V = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

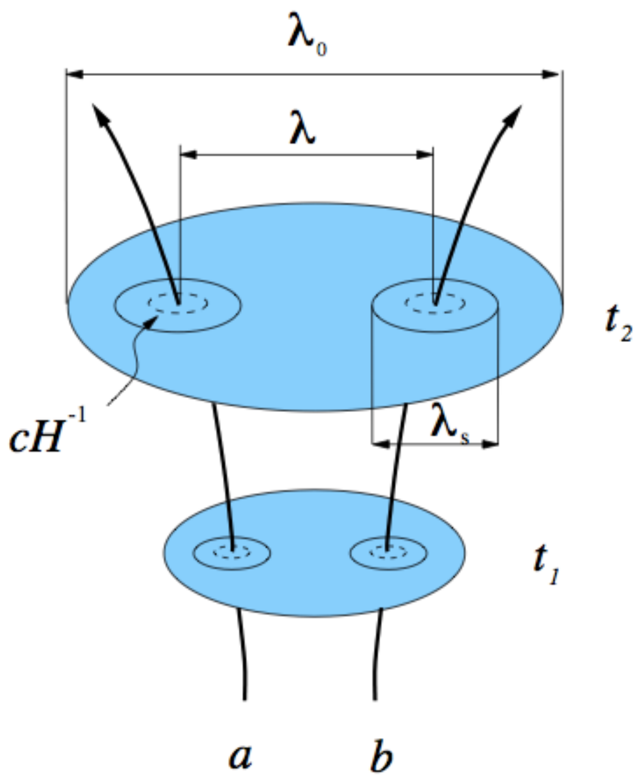


Wands et al., 2000



Wands et al., 2000

$$\zeta(\mathbf{x}) = \delta N(\mathbf{x}) = N(\vec{\chi}(\mathbf{x})) - \bar{N}$$



Wands et al., 2000

$$\zeta(\mathbf{x}) = \delta N(\mathbf{x}) = N(\vec{\chi}(\mathbf{x})) - \bar{N}$$

$$\delta N(\mathbf{x}) = N_{,I}\delta\chi^I(\mathbf{x}) + \frac{1}{2}N_{,IJ}\left(\delta\chi^I(\mathbf{x})\delta\chi^J(\mathbf{x}) - \overline{\delta\chi^I\delta\chi^J}\right)$$

- But….

$$V = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

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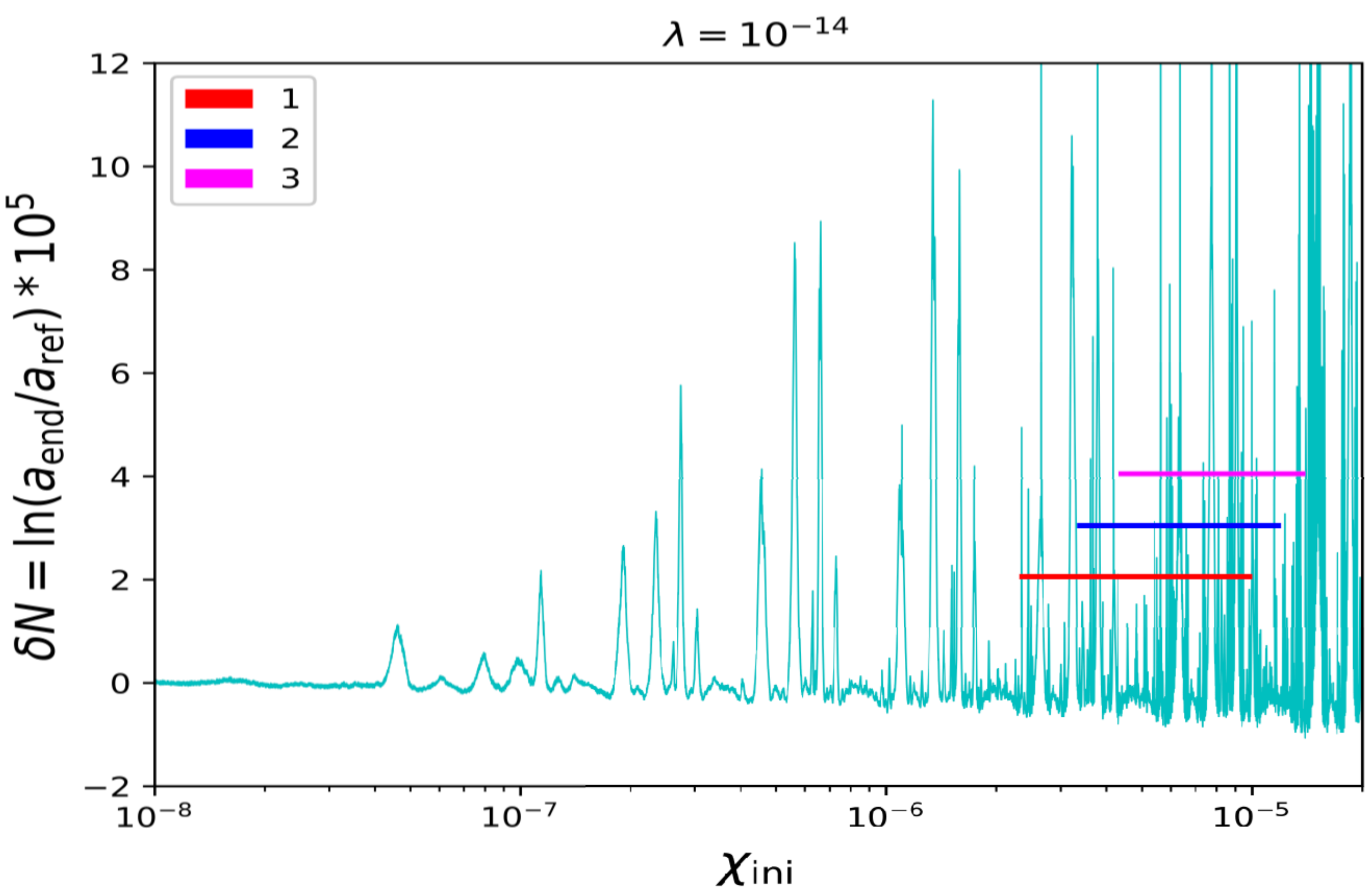

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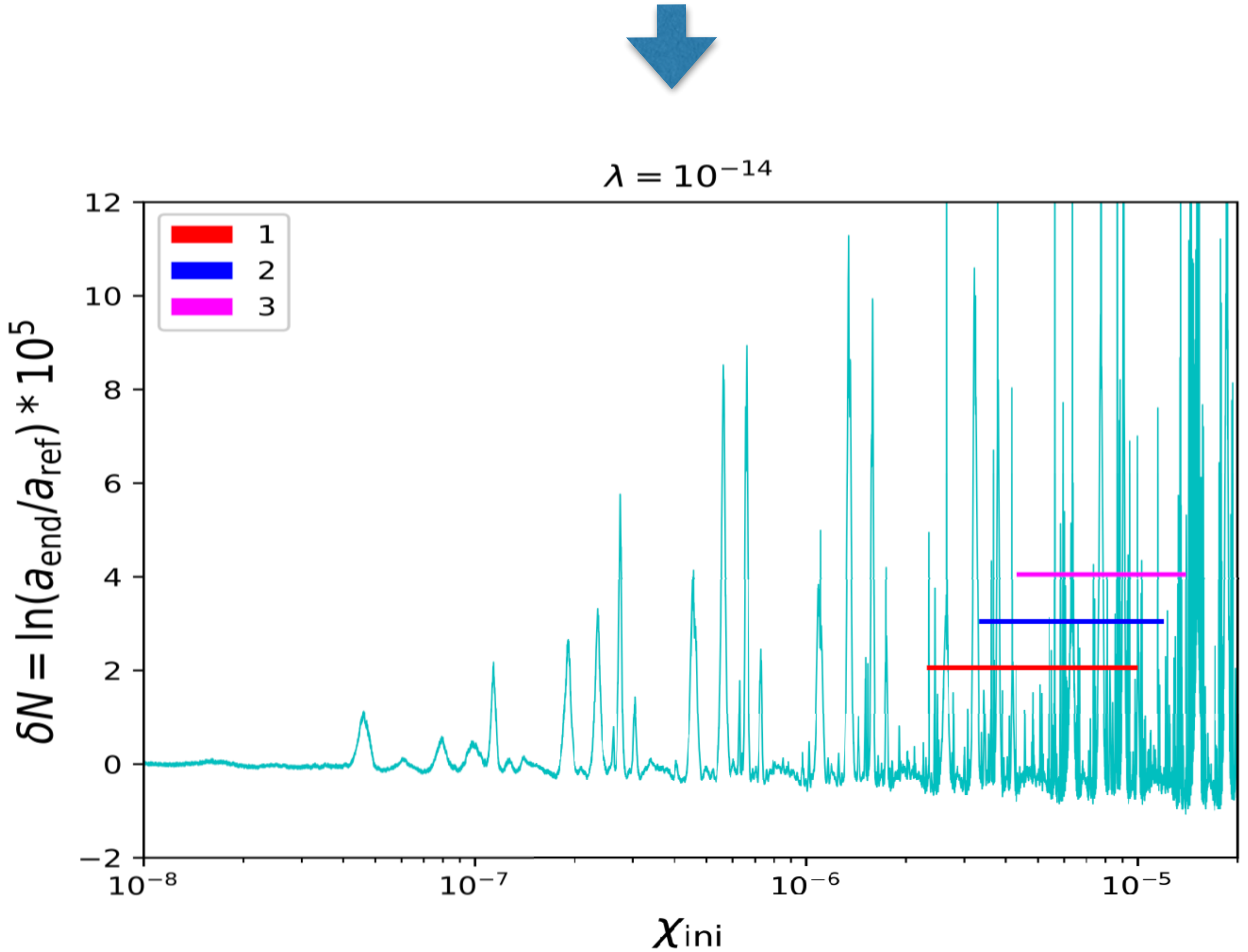

- A lot of work on this e.g. Chambers, Rajantie (2008), Bond, Frolov, Huang, Kofman (2009); Chambers, Nurmi, Rajantie (2010); Suyama and Yokoyama (2013); Bethke, Figueroa, Rajantie (2013)

- If one wants to know correlates, use full expression:

$$
\begin{aligned}
\langle\zeta_1 ... \zeta_m\rangle &= \langle(N_1-\bar{N})...(N_m-\bar{N})\rangle \\
&= \int \mathrm{d}\vec{\chi}_1 ... \int \mathrm{d}\vec{\chi}_m (N_1-\bar{N})...(N_m-\bar{N}) \\
&\qquad \times \mathcal{P}(\vec{\chi}_1, ..., \vec{\chi}_m)
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- But often don't know distribution, just the moments (from PyTransport for example)

- Try a different expansions see Suyama and S. Yokoyama (2013); Bethke, Figueroa, Rajantie (2013)

- Assume field space perturbations are close to Gaussian, and

$$\langle\delta\phi^I(\mathbf{x}_1)\delta\phi^J(\mathbf{x}_2)\rangle < \langle\delta\phi^I(\mathbf{x})\delta\phi^J(\mathbf{x})\rangle$$

- Leads to:

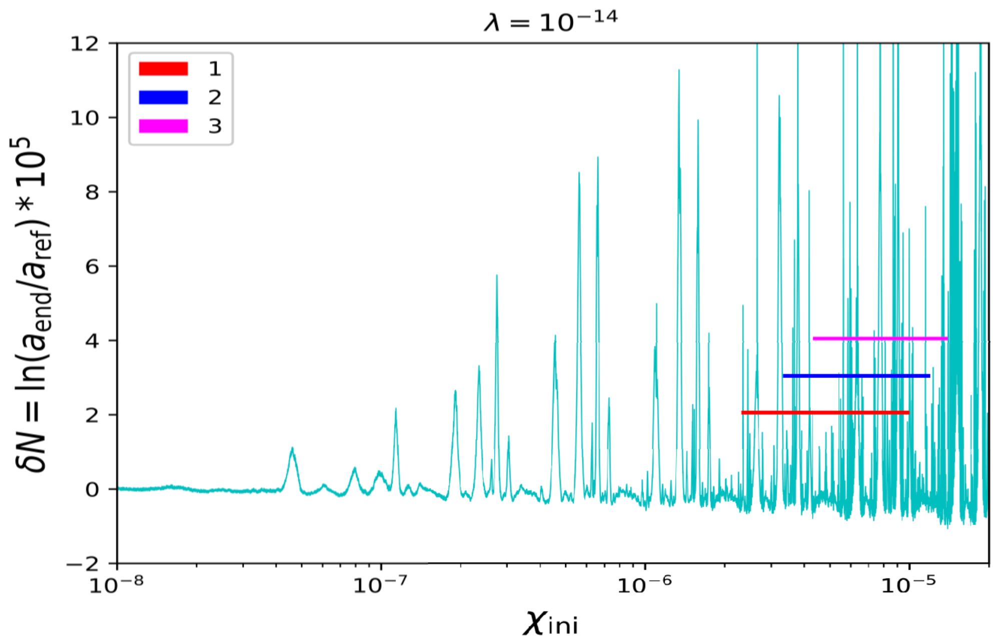
$$
\begin{aligned}
P_\zeta(k) &\approx \tilde{N}_I \tilde{N}_J \Sigma^{IJ}(k) \\
B_\zeta(k_1,k_2,k_3) &\approx \tilde{N}_I \tilde{N}_J \tilde{N}_K \alpha^{IJK}(k_1,k_2,k_3) \\
&\quad + \Big(\tilde{N}_I \tilde{N}_J \tilde{N}_{KL} \Sigma^{IK}(k_1) \Sigma^{JL}(k_2) \\
&\qquad\qquad + \text{cyclic} \Big).
\end{aligned}
$$

- With:

$$
\tilde{N}_I = \Sigma^{-1}_{IJ} \int \mathrm{d}\vec{\chi}_1 \, \mathcal{P}_{\mathrm{G}}(\vec{\chi}_1) N_1 \delta\chi_1^J
$$

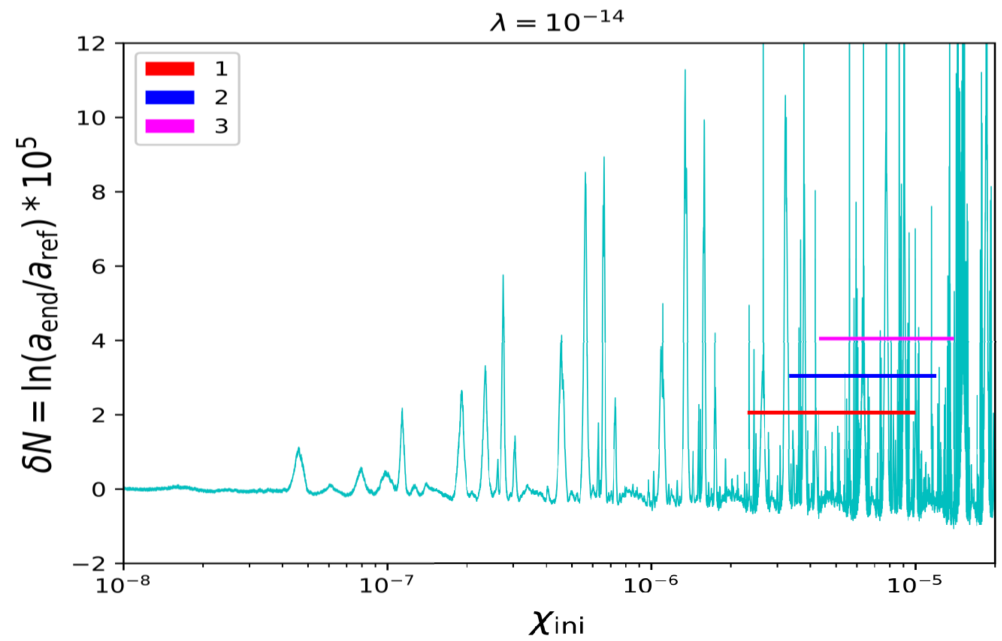
$$
\tilde{N}_{IJ} = \Sigma^{-1}_{IK} \Sigma^{-1}_{JL} \int \mathrm{d}\vec{\chi}_1 \, \mathcal{P}_{\mathrm{G}}(\vec{\chi}_1)(N_1 - \bar{N}) \delta\chi_1^K \delta\chi_1^L
$$

- Allows….



$\lambda = 10^{-14}$
1
2
3
$\delta N = \ln(a_{\rm end}/a_{\rm ref}) * 10^5$
$\chi_{\rm ini}$
12
10
8
6
4
2
0
−2
10^{-8}
10^{-7}
10^{-6}
10^{-5}

- Allows….



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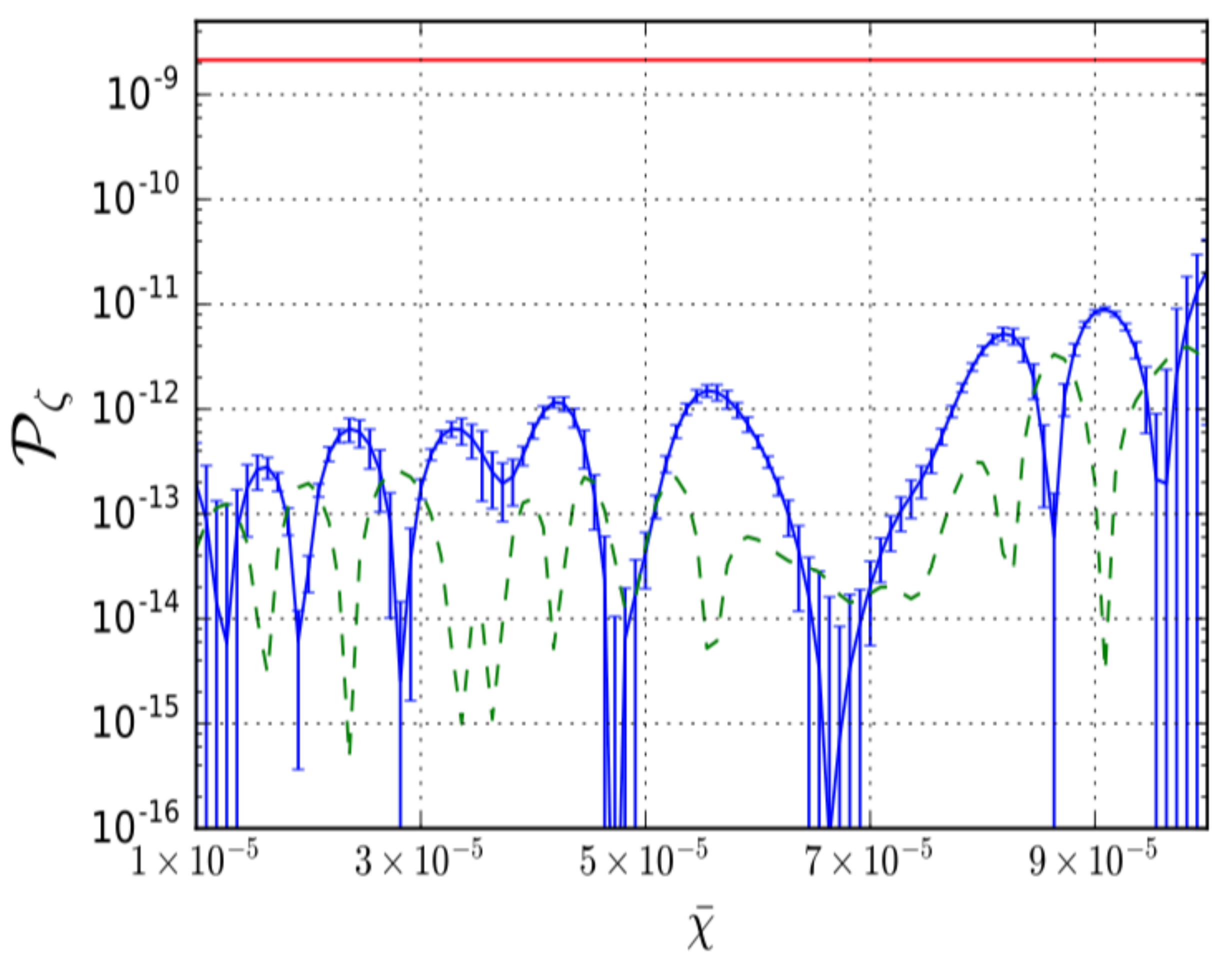
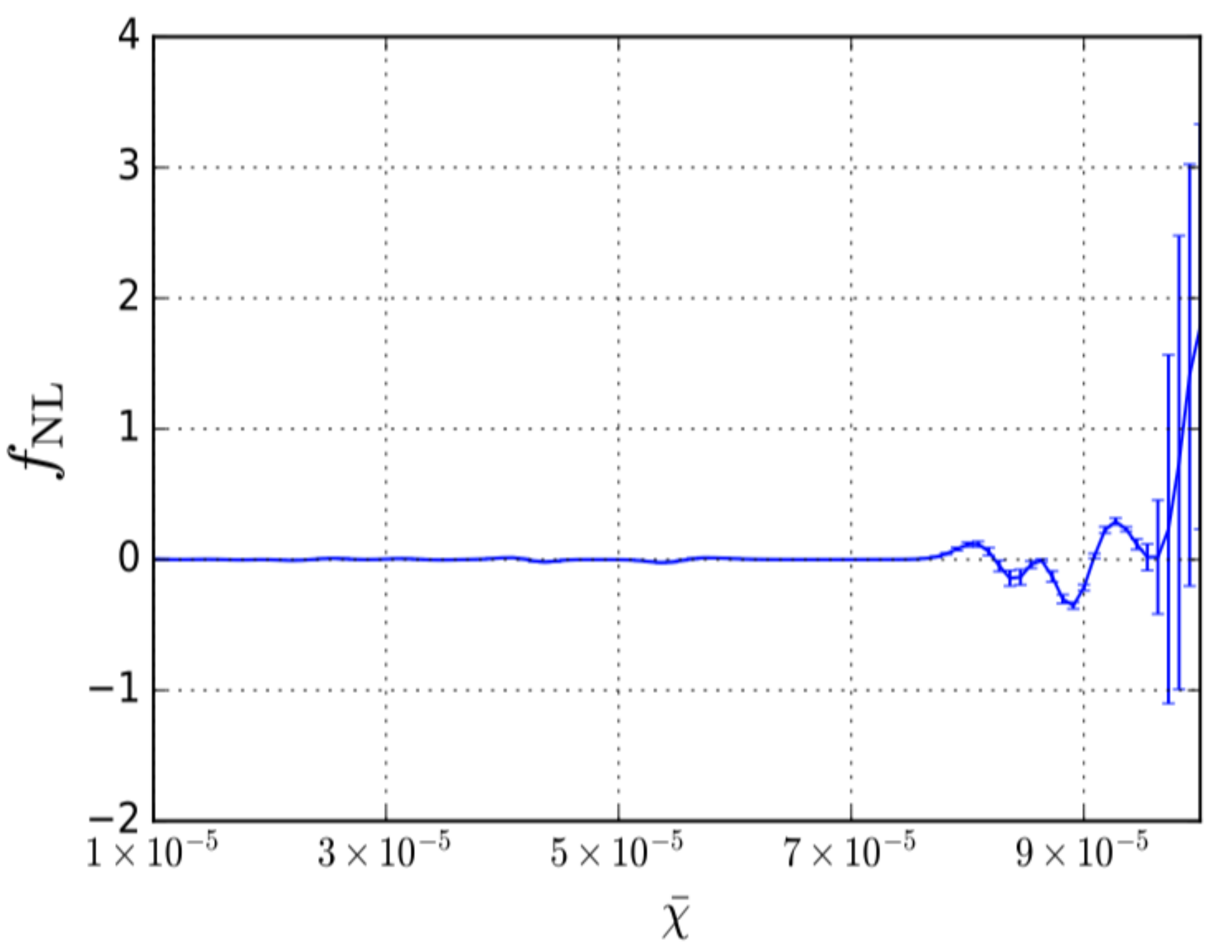


$\mathcal{P}_\zeta$
$\bar{\chi}$



f_{NL}
$\bar{\chi}$

Calculating the statistics — usual method

- Usual method Salopek and bond (1985):

$$Q^I(k) = \Psi^I{}_L(t,k)a^L(k) + \Psi^{*I}{}_L(t,k)a^{\dagger L}(-k)$$

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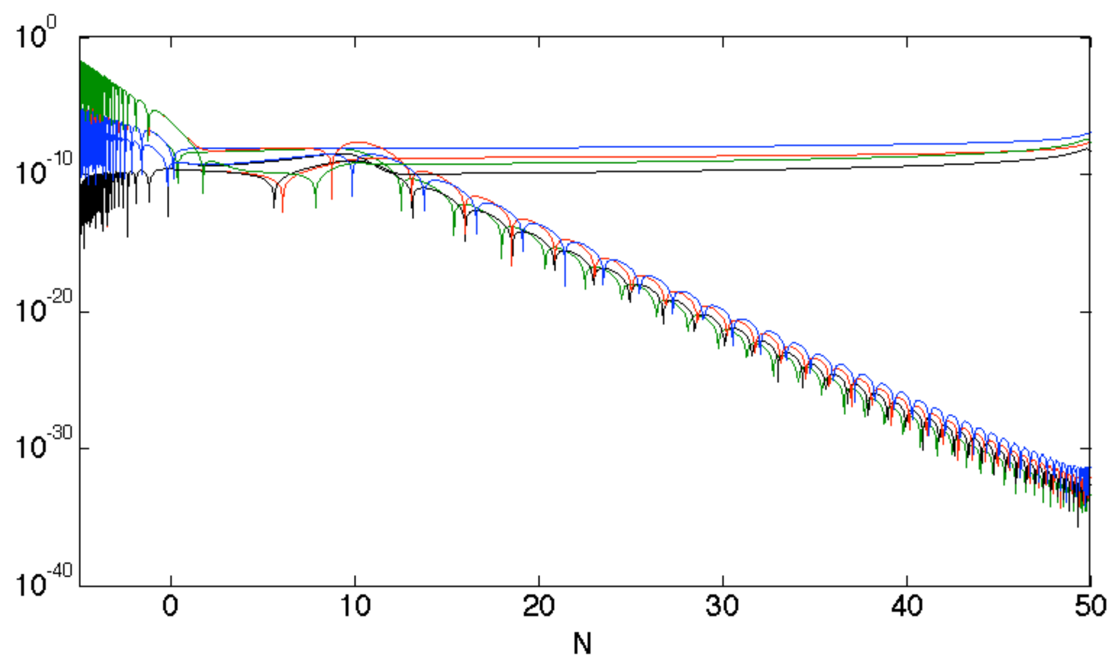

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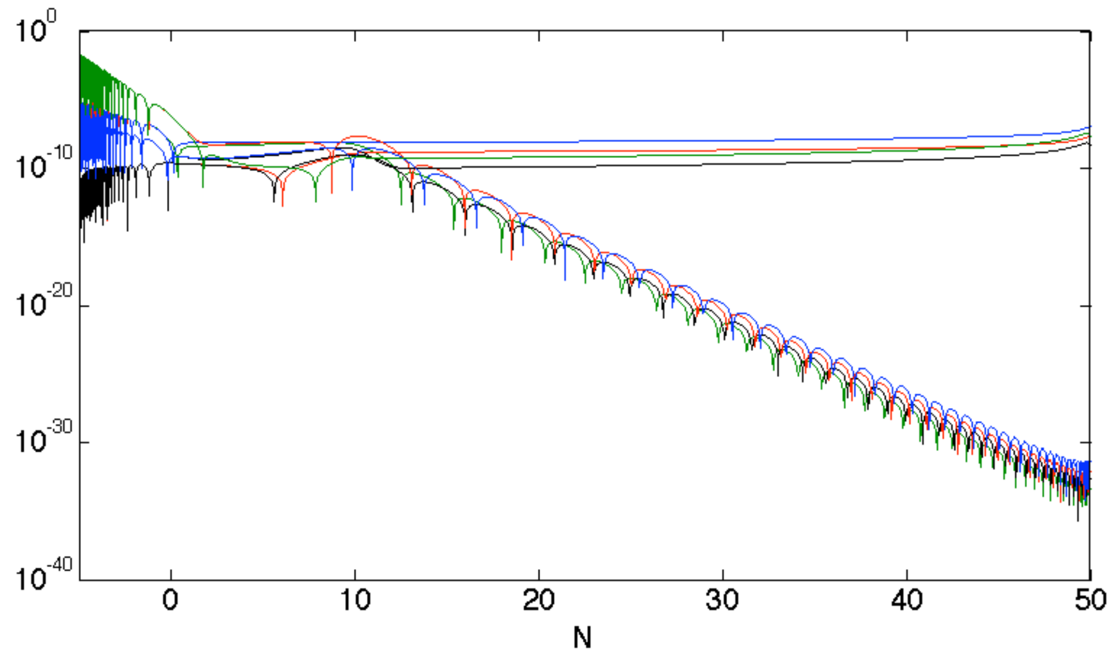

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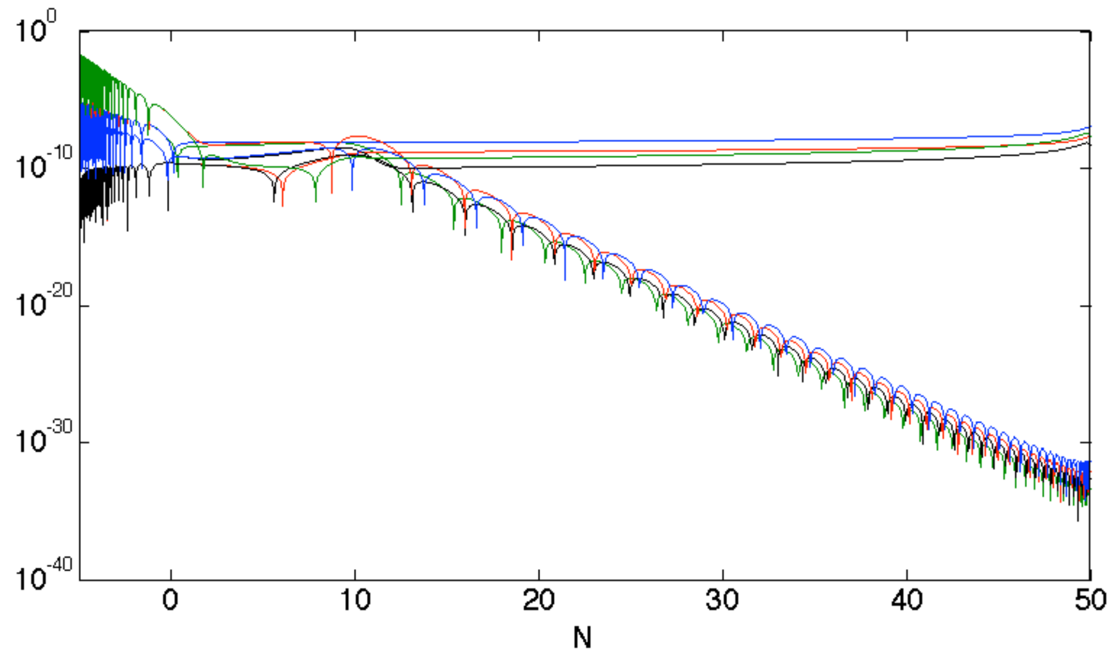

$$\Sigma^{IJ} = \Psi^{IL}\Psi^{*J}{}_L$$

$$\langle Q^I Q^J Q^L \rangle = -i \int_{-\infty}^{t} \mathrm{d}t' \langle [Q^I Q^J Q^L, H_{\rm int}(t')] \rangle$$

Maldacena (2003)