Simple flavon stabilization without domain wall problem

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Ref: [1] Phys.Lett. B788 (2019) 494-499 [arXiv:1810.05791 [hep-ph]].
[2] arXiv:1905.11517 [hep-ph]].

Inflation and the dark sector - Current challenges and future perspectives, June 3-7, 2019, Jyväskylä, Finland

What is the cosmological domain problem?

- Theory has discrete symmetry.
- Symmetry experiences spontaneous break down.

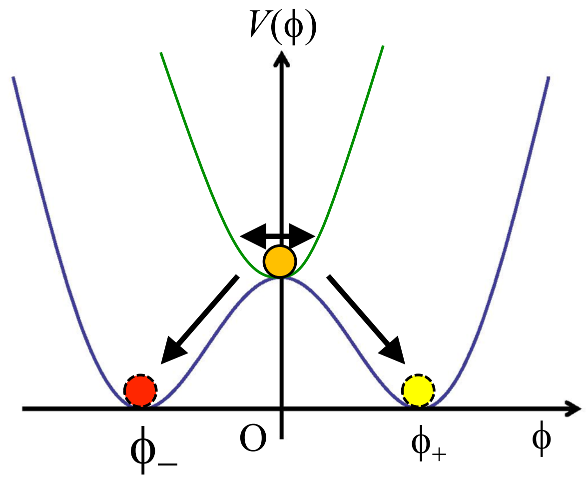
⇩

ϕ chooses either + or − within each coherent patch.
Domain walls form at the boundaries between + and − regions.

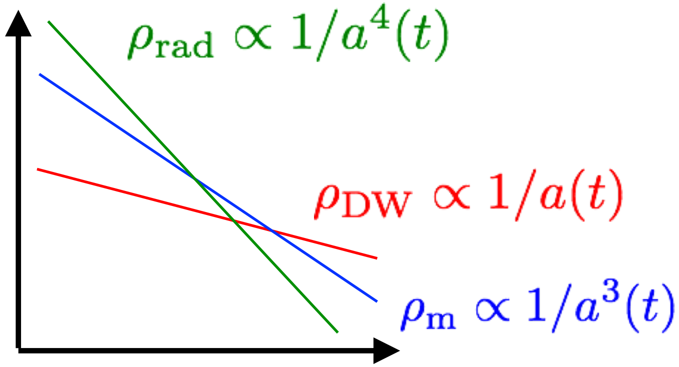
⇩

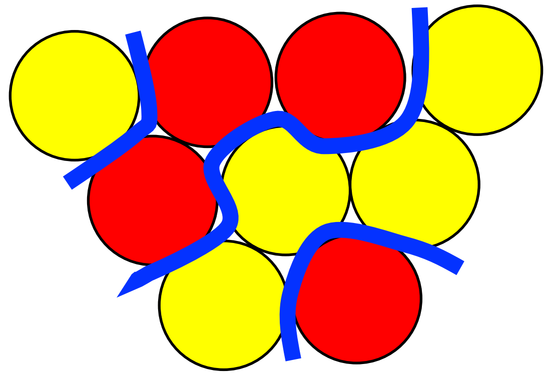
Energy density of DW will dominate soon.

Cosmological Domain wall Problem



Z_2 symmetry





Discrete Flavor symmetry

Flavor physics is to understand the lepton mass and mixing patterns.

It may be explained by some discrete symmetry.

In particular vacuum, the scalar fields acquire particular VEVs. flavons

These VEVs determine the lepton mass and mixing pattern.

VEV alignment

Usually very complicated to achieve.

Many vacua exist in the scalar potential.

↓

Domain walls would be produced in general.

Not many papers care about domain wall problem.

What we did:

We construct the SUSY flavor symmetry model base on discrete symmetry.

It has novel & simple mechanism for flavon stabilization.

It naturally avoids domain wall problem.

Plan of my talk

1. Introduction
    1.1. Brief history of model building using flavor symmetry in SUSY
    1.2. Brief review of AF model with A_4 symmetry
    1.3. Non-zero θ_{13} and S_4 flavor models
    1.4. Drawbacks of S_4 (& A_4) models in the literatures
2. Our S_4 flavor model
    2.1. Chaged lepton sector
    2.2. neutrino sector
    2.3. Mixing matrix
    2.4. Comparing with observations
3. Novel flavon stabilization
    3.1. General argument
    3.2. Concrete example
4. Evading the domain wall problem
5. Summary

1. Introduction

1.1. Brief history of model building using flavor symmetry in SUSY

One of the mainstream:

To explain the neutrino mass and mixing patterns using discrete flavor symmetry.

Before 2012: Tri-bimaximal (TB) neutrino mixing matrix.

(e.g.) using A_4 (Altarelli-Feruglio 2006)

In 2012: Non-zero reactor angle $\theta_{13} \neq 0$. Daya Bay and RENO experiments.

After 2012: TB mixing + corr. (e.g.) using A_4 (Kang et al. 2018)

Trimaximal (TM) mixing matrix. (e.g.) using S_4 (Luhn 2013, Ding et al. 2013)

1.2. Brief review of AF model with A_4 symmetry

"Standard model" of TB mixing   Altarelli-Feruglio 2006

Lepton sector

$$W_\ell = \frac{y_e}{\Lambda} e^c H_d (\varphi_T \ell) + \frac{y_\mu}{\Lambda} \mu^c H_d (\varphi_T \ell)' + \frac{y_\tau}{\Lambda} \tau^c H_d (\varphi_T \ell)'' + \frac{x_a}{\Lambda^2} H_u H_u \xi (\ell\ell) + \frac{x_b}{\Lambda^2} H_u H_u (\varphi_S \ell\ell) + \cdots$$

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ
A_4	3	1	$1'$	$1''$	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0
	LH leptons	RH leptons			Higgs	flavons		

1.2. Brief review of AF model with A_4 symmetry

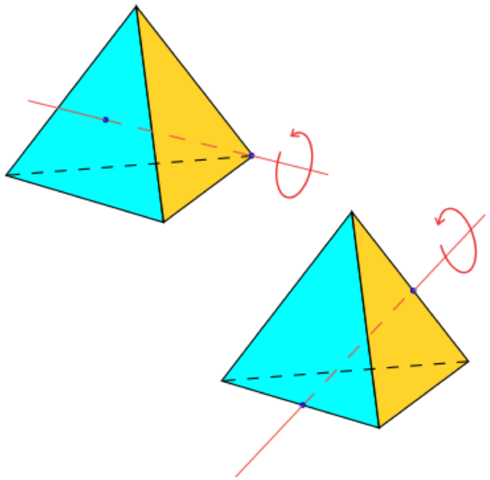
Altarelli-Feruglio 2006

A_4 symmetry

A_4: alternating group (even permutation of 4 objects)
≈ tetrahedral group (rotation symmetry of regular tetrahedron).

12 elements, generated by S and T, with $S^2=T^3=(ST)^3=1$.

$(1, S, T^2ST, TST^2, T, ST, TS, STS, T^2, ST^2, T^2S, ST^2S)$



Irreducible representations: **1**, **1'**, **1"**, **3**
The minimum group containing triplet without doublet.

For triplet, $T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$ $\quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$

$\omega \equiv e^{2\pi i/3}$

1.2. Brief review of AF model with A_4 symmetry

Altarelli-Feruglio 2006

A_4 symmetry

$$W_\ell = \frac{y_e}{\Lambda} e^c H_d(\varphi_T \ell) + \frac{y_\mu}{\Lambda} \mu^c H_d(\varphi_T \ell)' + \frac{y_\tau}{\Lambda} \tau^c H_d(\varphi_T \ell)'' + \frac{x_a}{\Lambda^2} H_u H_u \xi(\ell\ell) + \frac{x_b}{\Lambda^2} H_u H_u (\varphi_S \ell\ell) + \cdots$$

Field	1	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ
A_4	3	1	$1'$	$1''$	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0

The product of two triplets: **3x3=1+1'+1"+3$_S$+3$_A$.**

Contraction rules:

$$\mathbf{1} \sim a_1 b_1 + a_2 b_3 + a_3 b_2 \equiv (ab)$$

$$\mathbf{1}' \sim a_3 b_3 + a_1 b_2 + a_2 b_1 \equiv (ab)'$$

$$\mathbf{1}'' \sim a_2 b_2 + a_3 b_1 + a_1 b_3 \equiv (ab)''$$

$$\mathbf{3}_A \sim \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{pmatrix} \equiv (ab)_{\mathbf{3_A}}$$

$$\mathbf{3}_S \sim \begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_3 b_1 - a_1 b_3 \end{pmatrix} \equiv (ab)_{\mathbf{3_S}}$$

for $a = (a_1, a_2, a_3)$ & $b = (b_1, b_2, b_3)$

1.2. Brief review of AF model with A_4 symmetry

Altarelli-Feruglio 2006

A_4 symmetry

$$W_\ell = \frac{y_e}{\Lambda} e^c H_d(\varphi_T \ell) + \frac{y_\mu}{\Lambda} \mu^c H_d(\varphi_T \ell)' + \frac{y_\tau}{\Lambda} \tau^c H_d(\varphi_T \ell)''$$
$$+ \frac{x_a}{\Lambda^2} H_u H_u \xi(\ell\ell) + \frac{x_b}{\Lambda^2} H_u H_u (\varphi_S \ell\ell) + \cdots$$

Take the following VEV alignments:

$$\langle \varphi_T \rangle = (v_T, 0, 0)^T, \quad \langle \varphi_S \rangle = (v_S, v_S, v_S)^T, \quad \langle \xi \rangle = v_\xi$$

$$\Rightarrow \quad \mathcal{M}_\ell = \frac{v_T v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{M}_\nu = \frac{v_u^2}{\Lambda^2} \left[a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + b \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \right],$$

$$(a = 2x_a v_\xi, \ b = 2x_b v_S)$$

Using TB mixing matrix

$$U_{\rm TB} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix} \quad \Rightarrow \quad U_{\rm TB}^T \mathcal{M}_\nu U_{\rm TB} = \frac{v_u^2}{\Lambda^2} \begin{pmatrix} a+3b & & \\ & a & \\ & & -a+3b \end{pmatrix}$$

1.2. Brief review of AF model with A_4 symmetry

Altarelli-Feruglio 2006

How to obtain the VEV alignments: $\langle\varphi_T\rangle=(v_T,0,0),\ \langle\varphi_S\rangle=(v_S,v_S,v_S),\ \langle\xi\rangle=v_\xi$

Field	1	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	$1'$	$1''$	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2

Driving fields are just introduced to stabilize flavons.

$$w_d=M(\varphi_0^T\varphi_T)+g(\varphi_0^T\varphi_T\varphi_T)+g_1(\varphi_0^S\varphi_S\varphi_S)+g_2\tilde{\xi}(\varphi_0^S\varphi_S)+g_3\xi_0(\varphi_S\varphi_S)+g_4\xi_0\xi^2+g_5\xi_0\xi\tilde{\xi}+g_6\xi_0\tilde{\xi}^2.$$

$$\Rightarrow\begin{cases}\varphi_T=(v_T,0,0)^T, & v_T=-\dfrac{3M}{2g}.\\ \varphi_S=(v_S,v_S,v_S)^T, & v_S^2=-\dfrac{g_4}{3g_3}u^2\\ (\tilde{\xi}=0,) & \\ \xi=u, & \end{cases}$$

(Flat direction will be stabilized by one-loop radiative corrections.) → Domain wall can be produced.

More degenerate vacua derived by acting A_4-elements. ↓ Domain wall can be produced.

1.3. Non-zero θ_{13} and S_4 flavor models

In 2012, Daya Bay and RENO experiments reported non-zero θ_{13}.

Present best fit value: $\theta_{13}/^\circ = 8.61^{+0.12}_{-0.13}$ Esteban et al. 2019

⇨ TB mixing model is ruled out.

One good way is to use S_4 flavor symmetry. e.g., Luhn 2013, Ding et al. 2013

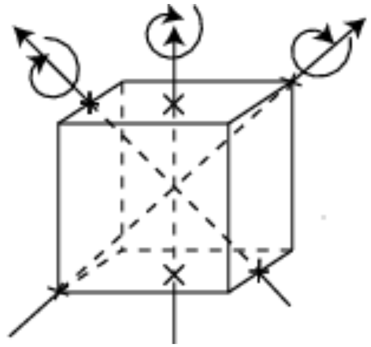
It leads to one of trimaximal mixing, TM_1.

$$U_{\mathrm{TM}_1} = \begin{pmatrix} 2/\sqrt{6} & * & * \\ -1/\sqrt{6} & * & * \\ -1/\sqrt{6} & * & * \end{pmatrix}$$

Preserves 1^{st} column of TB mixing matrix.

Maybe most favored by experiments. e.g., King 2019

1.3. Non-zero θ_{13} and S_4 flavor models



S_4 symmetry

S_4: symmetry group (permutation of 4 objects)
≈ hexahedral group (rotation symmetry of cube).

24 elements, generated by S, T and U,
with $S^2=T^3=U^2=(ST)^3=(SU)^2=(TU)^2=(STU)^4=1$.

$(1, S, T^2ST, TST^2, T, ST, TS, STS, T^2, ST^2, T^2S, ST^2S,$
$U, TU, SU, T^2U, STSU, ST^2SU, STU, TSU, T^2SU, ST^2U, TST^2U, T^2STU)$

Irreducible representations: **1**, **1'**, **2**, **3**, **3'**

For doublet **2**, $T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$ $S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

For triplet **3** and **3'**, $T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$ $S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$ $U = \mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$\omega \equiv e^{2\pi i/3}$

1.4. Drawbacks of S_4 (& A_4) models in the literatures

(- Non-zero θ_{13} cannot be explained (for A_4 only).)

- Driving fields exist only for obtaining desired VEV alignments.

- Dynamics is not explained.

- Flat directions exist.

- Domain walls are likely to form.

⇔ Our model

- Has simple stabilization mechanism. → - No driving field.
  - No flat direction.

- Evades domain wall formation.

Dynamically realized.

2. Our S_4 flavor model

$$W = W_\ell + W_\nu + W_{\rm f}$$

2.1. Chaged lepton sector

$$W_\ell = \frac{y_\tau}{\Lambda}\tau^c H_d(\phi_\ell \ell)_{\mathbf{1}} + \frac{y_\mu}{\Lambda^2}\mu^c H_d(\phi_\ell\phi_\ell)_{\mathbf{3}'}\ell - \frac{y_e}{\Lambda^3}e^c H_d\left[\phi_\ell(\phi_\ell\phi_\ell)_{\mathbf{3}'}\right]_{\mathbf{3}}\ell$$

Take the VEV alignments: $\langle\phi_\ell\rangle = (0, v_\ell, 0)^T$ $\langle H_d\rangle = v_d$

$$\Rightarrow \quad \mathcal{M}_\ell = \frac{v_\ell v_d}{\Lambda}\begin{pmatrix} 2y_e v_\ell^2/\Lambda^2 & 0 & 0 \\ 0 & 2y_\mu v_\ell/\Lambda & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$\frac{v_\ell}{\Lambda} \sim O(0.1)$$

	ℓ	e^c	μ^c	τ^c	H_u	H_d	ϕ_ℓ	$\phi_{\mathbf{1}}$	$\phi_{\mathbf{2}}$	$\phi_{\mathbf{3}'}$	$\psi_{\mathbf{3}'}$
S_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{3}'$
$\mathrm{U}(1)_R$	5/6	1/6	1/2	5/6	0	0	1/3	1/3	1/3	1/3	1/3
Z_6^ℓ	0	−3	−2	−1	0	0	1	0	0	0	0

LH leptons — RH leptons — Higgs — flavons

2.2. neutrino sector

$$W_\nu = \frac{H_u^2}{\Lambda^2}\left[c_1\phi_{\mathbf{1}}(\ell\ell)_{\mathbf{1}} + c_2\phi_{\mathbf{2}}(\ell\ell)_{\mathbf{2}} + c_{3'}\phi_{\mathbf{3'}}(\ell\ell)_{\mathbf{3'}} + c_\psi\psi_{\mathbf{3'}}(\ell\ell)_{\mathbf{3'}}\right]$$

Take the VEV alignments:

$$\langle\phi_{\mathbf{1}}\rangle = v_1,\ \ \langle\phi_{\mathbf{2}}\rangle = (v_2, v_2)^T,\ \ \langle\phi_{\mathbf{3'}}\rangle = (v_{3'}, v_{3'}, v_{3'})^T,\ \ \langle\psi_{\mathbf{3'}}\rangle = (0, v_\psi, -v_\psi)^T$$

$$\Rightarrow \mathcal{M}_\nu = \frac{v_u^2}{\Lambda^2}\left[w_1\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix} + w_2\begin{pmatrix}0 & 1 & 1\\ 1 & 1 & 0\\ 1 & 0 & 1\end{pmatrix} + w_{3'}\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix} + w_\psi\begin{pmatrix}0 & 1 & -1\\ 1 & 2 & 0\\ -1 & 0 & -2\end{pmatrix}\right]$$

$$w_1 \equiv c_1 v_1,\ w_2 \equiv c_2 v_2,\ w_{3'} \equiv c_{3'} v_{3'},\ w_\psi \equiv c_\psi v_\psi.\ \ \langle H_u\rangle = v_u$$

If w_ψ=0, it becomes the mass matrix that can be diagonalized with U_{TB}.

$$U_{\rm TB}^T\mathcal{M}_\nu U_{\rm TB} = \frac{v_u^2}{\Lambda^2}\begin{pmatrix}w_1 - w_2 & 0 & 0\\ 0 & w_1 + 2w_2 & 0\\ 0 & 0 & -w_1 + w_2\end{pmatrix}$$

Non-zero w_ψ breaks TB symmetry, but Z_2 (=SU of S_4) remains, leads to TM_1.

2.3. Mixing matrix

Non-zero w_ψ breaks TB symmetry, but Z_2 (=SU of S_4) remains, leads to TM_1.

$$U^\nu = U_{\rm TB} U_{23}$$

$$U_{23} \equiv \begin{pmatrix} 1 & 0 \\ 0 & u_{23} \end{pmatrix}$$

⇨

$$U^{\nu T} \mathcal{M}_\nu U^\nu = \frac{v_u^2}{\Lambda^2} \begin{pmatrix} w_1 - w_2 + 3w_{3'} & 0 \\ 0 & u_{23}^T \begin{pmatrix} w_1 + 2w_2 & \sqrt{6} w_\psi \\ \sqrt{6} w_\psi & -w_1 + w_2 + 3w_{3'} \end{pmatrix} u_{23} \end{pmatrix}$$

Mass matrix can be diagonalized for

$$u_{23} = \begin{pmatrix} \cos\theta & e^{i\eta}\sin\theta \\ -e^{-i\eta}\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\beta} \end{pmatrix}$$

where $\tan 2\theta = \frac{2|C^* D + CB^*|}{|D|^2 - |B|^2}, \; e^{i\eta} = \frac{C^* D + CB^*}{|C^* D + CB^*|}$

$B = w_1 + 2w_2, \; C = \sqrt{6} w_\psi, \; D = -w_1 + w_2 + 3w_{3'}$

⇨

$$U^\nu = \begin{pmatrix} 2/\sqrt{6} & \cos\theta/\sqrt{3} & e^{i\eta}\sin\theta/\sqrt{3} \\ -1/\sqrt{6} & \cos\theta/\sqrt{3} - e^{-i\eta}\sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} + e^{i\eta}\sin\theta/\sqrt{3} \\ -1/\sqrt{6} & \cos\theta/\sqrt{3} + e^{-i\eta}\sin\theta/\sqrt{2} & -\cos\theta/\sqrt{2} + e^{i\eta}\sin\theta/\sqrt{3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

TM_1 mixing matrix

2.4. Comparing with observations

$$U^{\nu} = \begin{pmatrix} 2/\sqrt{6} & \cos\theta/\sqrt{3} & e^{i\eta}\sin\theta/\sqrt{3} \\ -1/\sqrt{6} & \cos\theta/\sqrt{3} - e^{-i\eta}\sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} + e^{i\eta}\sin\theta/\sqrt{3} \\ -1/\sqrt{6} & \cos\theta/\sqrt{3} + e^{-i\eta}\sin\theta/\sqrt{2} & -\cos\theta/\sqrt{2} + e^{i\eta}\sin\theta/\sqrt{3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

$$U^{\rm MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix},$$

Sum rules:

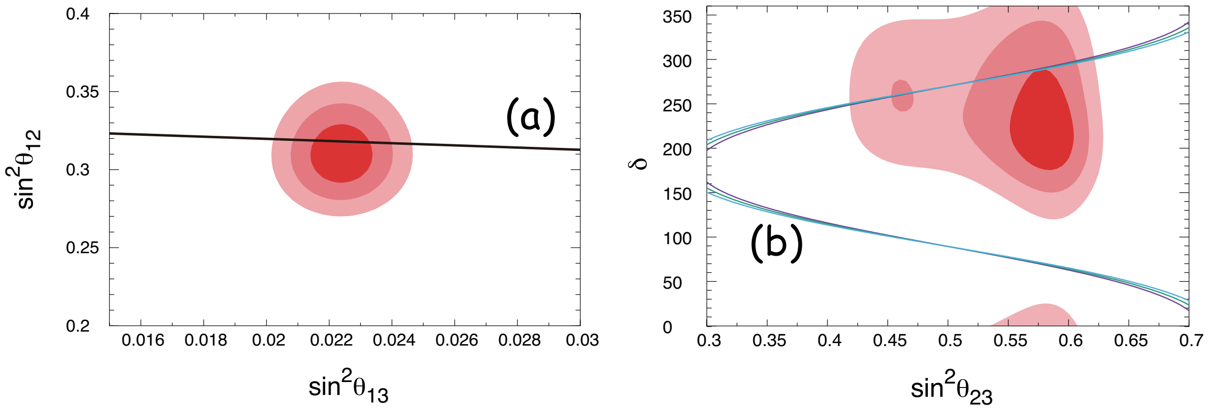
$$|U^{\nu}_{e1}|^2 = (c_{12}c_{13})^2$$

$$|U^{\nu}_{\mu 1}|^2 - |U^{\nu}_{\tau 1}|^2 = 0$$

⇒

(a) $\cos\theta_{12}\cos\theta_{13} = \dfrac{2}{\sqrt{6}}$

(b) $\cos(2\theta_{23})\left(\dfrac{2}{3} - \cos(2\theta_{12})\right) + \sin(2\theta_{12})\sin(2\theta_{23})\sin\theta_{13}\cos\delta = 0$



Allowed region taken from Esteban et al. 2018,2019

4 Model para. $w_1, w_2, w_{3'}, w_{\psi}$

9 obsevables:

$m_1, m_2, m_3, \theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$

2 sum rules

9=2+(4x2-1)

3. Novel flavon stabilization

3.1. General argument

	ℓ	e^c	μ^c	τ^c	H_u	H_d	ϕ_ℓ	ϕ_1	ϕ_2	$\phi_{3'}$	$\psi_{3'}$
S_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{3}'$
$\mathrm{U}(1)_R$	5/6	1/6	1/2	5/6	0	0	1/3	1/3	1/3	1/3	1/3
Z_6^ℓ	0	−3	−2	−1	0	0	1	0	0	0	0

Basic Idea:

The flavons are stabilized by the balance between the negative soft SUSY breaking mass and non-renormalizable terms in the potential.

Flavon sector: $W_{\mathrm{f}} \sim \frac{\phi^6}{\Lambda^3} \longrightarrow V \sim -m^2|\phi|^2 + \frac{|\phi|^{10}}{\Lambda^6} \longrightarrow \langle|\phi|\rangle \sim (m\Lambda^3)^{1/4}$

Desired VEV alignment $\langle\phi_\ell\rangle = (0, v_\ell, 0)^T,\ \langle\phi_{\mathbf{1}}\rangle = v_1,\ \langle\phi_{\mathbf{2}}\rangle = (v_2, v_2)^T$

$\langle\phi_{\mathbf{3}'}\rangle = (v_{3'}, v_{3'}, v_{3'})^T,\ \langle\psi_{\mathbf{3}'}\rangle = (0, v_\psi, -v_\psi)$

This is always an extremum of the potential independent of its form.

The desired VEV alignment is always an extremum of the potential.

For $\phi_\ell = (\phi_{\ell,1}, \phi_{\ell,2}, \phi_{\ell,3})^T$ (Z_6^ℓ symmety ensures no mixing with other flavons.)

$$W_{\mathrm{f},\ell} = \frac{1}{\Lambda^3}\left(\phi_{\ell,2}^6 + \phi_{\ell,2}^5\phi_{\ell,1} + \phi_{\ell,2}^5\phi_{\ell,3} + O(\phi_{\ell,1}^2, \phi_{\ell,3}^2, \phi_{\ell,1}\phi_{\ell,3})\right)$$

Substituting $\phi_\ell = (0, \phi_{\ell,2}, 0)^T$, minimizing V along $\phi_{\ell,2}$ $\longrightarrow$ $\langle|\phi_{\ell,2}|\rangle = v_\ell$

No linear term for $\phi_{\ell,1}$, $\phi_{\ell,3}$ $\Longleftrightarrow$ $\langle\phi_\ell\rangle = (0, v_\ell, 0)^T$ is extremum.

Forbidden by symmetry.

$W_{\mathrm{f},\ell}$ must be invariant under S_4 and Z_6^ℓ (where $\phi_\ell \to \Omega\phi_\ell$ $(\Omega \equiv e^{2\pi i/6})$).

Then, e.g., $\phi_\ell \to \Omega^2 T\phi_\ell = \begin{pmatrix} \Omega^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \Omega^4 \end{pmatrix}\phi_\ell$ $\Rightarrow$ $\phi_{\ell,2}$ is invariant. $\phi_{\ell,1}$, $\phi_{\ell,3}$ are not.

(Note that $\Omega^2 T$ is the generator of remaining $Z_3 \subset S_4 \times Z_6^\ell$ after sym. breaking.)

3.2. Concrete example

The extremum is the minimum? ⇨ Curvature of the potential should be positive.

$$V=\sum_{i=1}^{3}\left|\frac{\partial W_{\mathrm{f},\ell}}{\partial\phi_{\ell},i}\right|^2+\left|\frac{\partial W_{\mathrm{f},\mathbf{1}}}{\partial\phi_{\mathbf{1}}}\right|^2+\sum_{i=1}^{2}\left|\frac{\partial W_{\mathrm{f},\mathbf{2}}}{\partial\phi_{\mathbf{2},\mathbf{i}}}\right|^2+\sum_{i=1}^{3}\left|\frac{\partial W^{\phi}_{\mathrm{f},\mathbf{3'}}}{\partial\phi_{\mathbf{3'},\mathbf{i}}}\right|^2+\left|\frac{\partial W_{\mathrm{f},\mathbf{1}}}{\partial\xi_{\mathbf{1}}}\right|^2+\sum_{i=1}^{3}\left|\frac{\partial W^{\psi}_{\mathrm{f},\mathbf{3'}}}{\partial\psi_{\mathbf{3'},\mathbf{i}}}\right|^2+\left|\frac{\partial W_{\mathrm{f},\mathbf{1}}}{\partial\xi'_{\mathbf{1}}}\right|^2+V_{\mathrm{SB}}\;(+V_A)$$

$$W_{\mathrm{f},\ell}=\frac{1}{\Lambda^3}\left[h_1\left(\phi_\ell^3\right)^2_{\mathbf{1'}}+h_2\left(\phi_\ell^2\right)_{\mathbf{1}}\left(\left(\phi_\ell^2\right)^2_{\mathbf{3'}}\right)_{\mathbf{1}}\right],\quad W_{\mathrm{f},\mathbf{1}}=\frac{g_0}{\Lambda^3}\phi_{\mathbf{1}}^6,\quad W_{\mathrm{f},\mathbf{2}}=\frac{1}{\Lambda^3}\left[g_1\left(\phi_{\mathbf{2}}^2\right)^3_{\mathbf{1}}+g_2\left(\phi_{\mathbf{2}}^3\right)^2_{\mathbf{1}}\right],\quad W_{\mathrm{f},\xi}=\frac{g_\xi}{\Lambda^3}\xi_{\mathbf{1}}^6,$$

$$W^{\phi}_{\mathrm{f},\mathbf{3'}}=\frac{1}{\Lambda^3}\left[g_3\left(\phi_{\mathbf{3'}}^2\right)^3_{\mathbf{1}}+g_4\left(\left[\phi_{\mathbf{3'}}^5\right]\phi_{\mathbf{3'}}\right)_{\mathbf{1}}\right],\quad W^{\psi}_{\mathrm{f},\mathbf{3'}}=\frac{1}{\Lambda^3}\left[g_5\left(\psi_{\mathbf{3'}}^2\right)^3_{\mathbf{1}}+g_6\left(\left[\psi_{\mathbf{3'}}^5\right]\psi_{\mathbf{3'}}\right)_{\mathbf{1}}\right],\quad W_{\mathrm{f},\xi'}=\frac{g_{\xi'}}{\Lambda^3}\xi_{\mathbf{1}}'^6,$$

$$V_{\mathrm{SB}}=-m_\ell^2\sum_{i=1}^{3}|\phi_{\ell,i}|^2-m_{\mathbf{1}}^2|\phi_{\mathbf{1}}|^2-m_{\mathbf{2}}^2\sum_{i=1}^{2}|\phi_{\mathbf{2},i}|^2-m_{\mathbf{3'}}^2\sum_{i=1}^{3}|\phi_{\mathbf{3'},i}|^2-m_\xi^2|\xi_{\mathbf{1}}|^2-m_\psi^2\sum_{i=1}^{3}|\psi_{\mathbf{3'},i}|^2-m_{\xi'}^2|\xi'_{\mathbf{1}}|^2$$

The A terms are added if massless without them. ($m_{3/2} \ll m_{flavon}$)

$$V_A=\frac{3am_{3/2}}{\Lambda^3}\left[h_1\left(\phi_\ell^3\right)^2_{\mathbf{1'}}+h_2\left(\phi_\ell^2\right)_{\mathbf{1}}\left(\left(\phi_\ell^2\right)^2_{\mathbf{3'}}\right)_{\mathbf{1}}+g_0\phi_{\mathbf{1}}^6+g_1\left(\phi_{\mathbf{2}}^2\right)^3_{\mathbf{1}}+g_2\left(\phi_{\mathbf{2}}^3\right)^2_{\mathbf{1}}\right.$$
$$\left.+g_3\left(\phi_{\mathbf{3'}}^2\right)^3_{\mathbf{1}}+g_4\left(\left[\phi_{\mathbf{3'}}^5\right]\phi_{\mathbf{3'}}\right)_{\mathbf{1}}+g_\xi\xi_{\mathbf{1}}^6+g_5\left(\psi_{\mathbf{3'}}^2\right)^3_{\mathbf{1}}+g_6\left(\left[\psi_{\mathbf{3'}}^5\right]\psi_{\mathbf{3'}}\right)_{\mathbf{1}}+g_{\xi'}\xi_{\mathbf{1}}'^6\right]+\mathrm{h.c.}$$

(e.g.) potential of $\phi_{\mathbf{3'}}$

1) Find an extremum at $\langle\phi_{\mathbf{3'}}\rangle = (v_{3'}, v_{3'}, v_{3'})^T$. ⟶ $v_{\mathbf{3'}} = \left(\frac{1}{14580}\right)^{1/8}\left(\frac{m_{\mathbf{3'}}\Lambda^3}{|g_3|}\right)^{1/4}$

2) Expand $\phi_{3'}$ around the extremum as $\phi_{\mathbf{3'},j} = v_{3'} + \frac{1}{\sqrt{2}}(\phi^R_{\mathbf{3'},j} + i\phi^I_{\mathbf{3'},j})$ for j=1,2,3.

3) Find the mass matrix for $\phi^A_{\mathbf{3'},j}$ for j=1,2,3 and A=R,I.

$$\mathcal{M}^2_{\phi_{\mathbf{3'}}} = \begin{pmatrix} \mathcal{M}^{(R)2}_{\phi_{\mathbf{3'}}} & 0 \\ 0 & \mathcal{M}^{(I)2}_{\phi_{\mathbf{3'}}} \end{pmatrix},$$

$$\mathcal{M}^{(A)2}_{\phi_{\mathbf{3'}}} = \begin{pmatrix} D_A & E_A & E_A \\ E_A & F_A & G_A \\ E_A & G_A & F_A \end{pmatrix},$$

$$D_R = \frac{8(5g_3^2 + 18g_3g_4 + 36g_4^2)}{15g_3^2} m^2_{\mathbf{3'}},$$

$$E_R = \frac{8(g_3 - 3g_4)(5g_3 + 6g_4)}{15g_3^2} m^2_{\mathbf{3'}},$$

$$F_R = \frac{4(7g_3^2 + 72g_4^2)}{15g_3^2} m^2_{\mathbf{3'}},$$

$$G_R = \frac{4(13g_3^2 + 18g_3g_4 - 36g_4^2)}{15g_3^2} m^2_{\mathbf{3'}}.$$

$$D_I = -\frac{16(g_3 - 3g_4)(g_3 + 6g_4)}{15g_3^2} m^2_{\mathbf{3'}},$$

$$E_I = -D_I/2,$$

$$F_I = -\frac{4(g_3^2 - 24g_3g_4 - 72g_4^2)}{15g_3^2} m^2_{\mathbf{3'}},$$

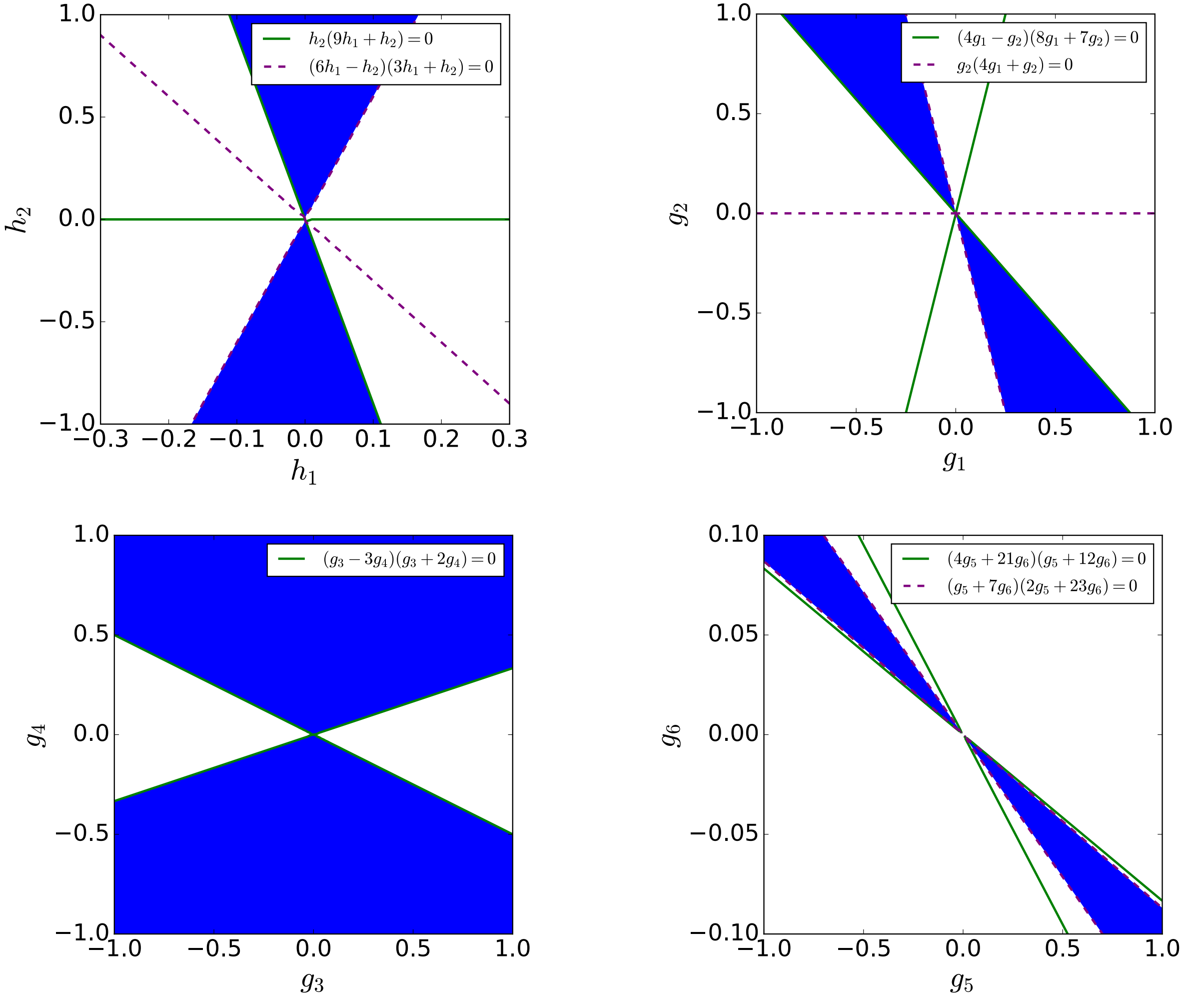
$$G_I = -\frac{4(g_3^2 + 30g_3g_4 + 36g_4^2)}{15g_3^2} m^2_{\mathbf{3'}}.$$

4) Find the conditions for positive curvatures.

3 R & 2 I are positive of O($m^2_{\mathbf{3'}}$) if $(g_3 - 3g_4)(g_3 + 2g_4) < 0$, one I is massless.

5) Include V_A to find O($|a| m_{3/2} m_{\mathbf{3'}}$) for last one.

Conditions for positive curvature. ⇨ Flavons are stabilized.



$$W_{\mathrm{f},\ell} = \frac{1}{\Lambda^3}\left[h_1\left(\phi_\ell^3\right)^2_{\mathbf{1}'} + h_2\left(\phi_\ell^2\right)_{\mathbf{1}}\left(\left(\phi_\ell^2\right)^2_{\mathbf{3}'}\right)_{\mathbf{1}}\right],$$

$$W_{\mathrm{f},\mathbf{2}} = \frac{1}{\Lambda^3}\left[g_1\left(\phi_{\mathbf{2}}^2\right)^3_{\mathbf{1}} + g_2\left(\phi_{\mathbf{2}}^3\right)^2_{\mathbf{1}}\right],$$

$$W^{\phi}_{\mathrm{f},\mathbf{3}'} = \frac{1}{\Lambda^3}\left[g_3\left(\phi_{\mathbf{3}'}^2\right)^3_{\mathbf{1}} + g_4\left(\left[\phi_{\mathbf{3}'}^5\right]\phi_{\mathbf{3}'}\right)_{\mathbf{1}}\right],$$

$$W^{\psi}_{\mathrm{f},\mathbf{3}'} = \frac{1}{\Lambda^3}\left[g_5\left(\psi_{\mathbf{3}'}^2\right)^3_{\mathbf{1}} + g_6\left(\left[\psi_{\mathbf{3}'}^5\right]\psi_{\mathbf{3}'}\right)_{\mathbf{1}}\right],$$

4. Evading the domain wall problem

Chigusa, SK, Nakayama 2019

If m_{SB} > H during inflation, flavons fall into the minimum dynamically, and such a region expands that covers the whole observable universe.

If m_{SB} < H during inflation, we need negative Hubble-induced mass terms.

$$V_{\rm SB}^{(H)} = -c_\ell H^2 \sum_{i=1}^{3} |\phi_{\ell,i}|^2 - c_{\mathbf{1}} H^2 |\phi_{\mathbf{1}}|^2 - c_{\mathbf{2}} H^2 \sum_{i=1}^{2} |\phi_{\mathbf{2},i}|^2 - c_{\mathbf{3'}} H^2 \sum_{i=1}^{3} |\phi_{\mathbf{3'},i}|^2 - c_\xi H^2 |\xi_{\mathbf{1}}|^2 - c_\psi H^2 \sum_{i=1}^{3} |\psi_{\mathbf{3'},i}|^2 - c_{\xi'} H^2 |\xi'_{\mathbf{1}}|^2$$

These terms have the same forms as V_{SB}, the structure of the minima is the same.

$$\langle \phi_\ell \rangle = (0, v_\ell^H, 0)^T, \quad \langle \phi_{\mathbf{1}} \rangle = v_1^H, \ \langle \phi_{\mathbf{2}} \rangle = (v_2^H, v_2^H)^T, \ \langle \xi_{\mathbf{1}} \rangle = v_\xi^H, \ \langle \xi'_{\mathbf{1}} \rangle = v_{\xi'}^H,$$

$$\langle \phi_{\mathbf{3'}} \rangle = (v_{3'}^H, v_{3'}^H, v_{3'}^H)^T, \ \langle \psi_{\mathbf{3'}} \rangle = (0, v_\psi^H, -v_\psi^H),$$

where $v^H \sim O(H\Lambda^3)^{1/4}$

Since flavons have masses of O(H) around this minimum, they fall quickly into the minimum dynamically during inflation.

Rough schetch of the scenario

Potential has negative mass and higher dimensional terms.

During inflation: $V = -H^2|\phi|^2 + \frac{|\phi|^{10}}{\Lambda^6}$

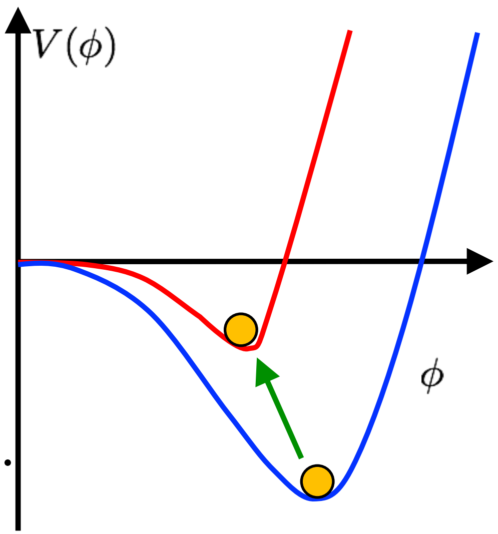
ϕ settles down in the minimum at $(H\Lambda^3)^{1/4}$ very quickly.

After inflation ($H \gtrsim m_{soft}$): $V = -H^2|\phi|^2 + \frac{|\phi|^{10}}{\Lambda^6}$

ϕ remains trapped in the moving minimum at $(H(t)\Lambda^3)^{1/4}$.

($H \lesssim m_{soft}$): $V = -m_{\rm soft}^2|\phi|^2 + \frac{|\phi|^{10}}{\Lambda^6}$

ϕ stays at rest in the present vacuum at $(m_{\rm soft}\Lambda^3)^{1/4}$.



⇨ Symmetry is never restored. ⇨ No domain wall in our Universe.

(c.f.) Reva 2010 mentioned the rough idea to use $-H^2 \phi^2$.

Minimum tracking

Ema, Nakayama, Takimoto 2016

Power n of higher dimentional terms must be large.

For $W_{\rm NR} = \phi^n$, $V_{\rm NR} = |\phi|^{2(n-1)}$ and the minimum is at $v^H \simeq H^{1/(n-2)}$. (n > 3)

Then, for $H = p/t$, $v^H \propto t^{-1/(n-2)}$.

Oscillation amplitude around the temporal minimum: $\Delta\phi$

Number density consevation: $H(\Delta\phi)^2 \propto a^{-3} \xrightarrow[a = t^p]{} \Delta\phi \propto t^{(1-3p)/2}$

Since $\frac{\Delta\phi}{v^H} \propto t^{\frac{(1-3p)(n-2)+2}{2(n-2)}}$, then $(1-3p)(n-2)+2 \leq 0$ for not to overshoot the origin.

$$\Rightarrow \quad n \geq \frac{6p}{3p-1} = \begin{cases} 6 & \text{(RD)} \\ 4 & \text{(MD)} \end{cases}$$ $\longrightarrow$ Our choice is n=6.

5. Summary

We construct the SUSY S_4 flavor symmetry model.

It has novel & simple mechanism for flavon stabilization.

- The VEV alignment is obtained by the balance of the tachyonic mass and higher dimensional terms in the potential.
- No driving field.

It naturally avoids domain wall problem.

- The same structure of the VEV alignment is realized by negative Hubble-induced mass terms in the potential.
- Flavons settle down in the minimum quickly during inflation, and keep trapped in the temporal minimum after inflation.
- This dynamical flavon stabilization avoids the domain wall problem.

This stabilization mechanism can be applied to any such models based on discrete flavor symmetry.