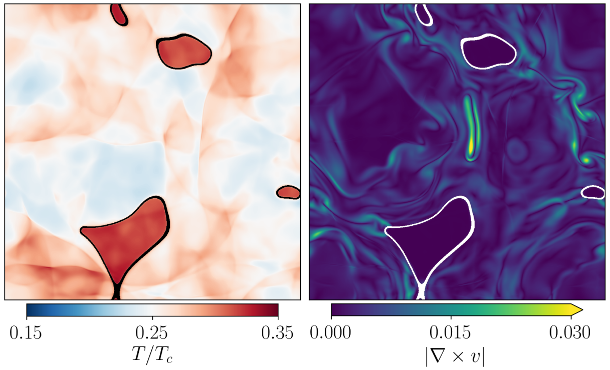
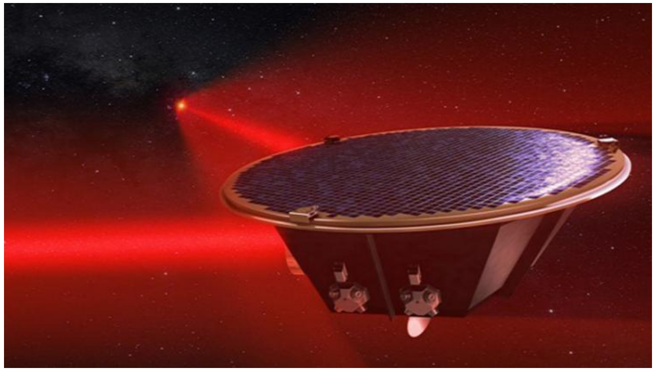


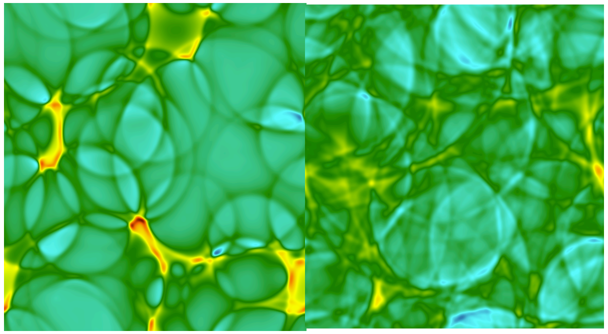


# Gravitational waves from phase transitions in the early Universe

**Mark Hindmarsh**

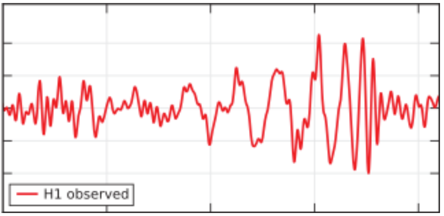
Helsinki Institute of Physics & Dept of Physics, University of Helsinki
and
Department of Physics & Astronomy, University of Sussex

IDS workshop
Jyväskylä
3. kesäkuuta 2019

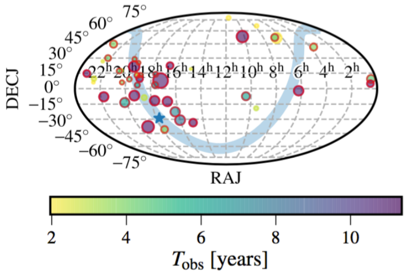


# Gravitational wave astronomy

Sources



Binary Supermassive Black Holes in galactic nuclei



Compact Binaries in our Galaxy & beyond

Compact objects captured by upermassive Black Holes

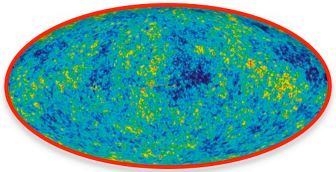
Rotating NS, Supernovae

*wave period* age of universe years hours sec ms

*log(frequency)* -16 -14 -12 -10 -8 -6 -4 -2 0 +2

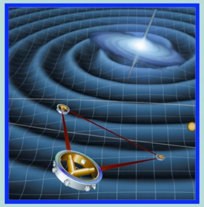
Detectors

Cosmic microwave background polarization



Pulsar Timing

Space Interferometers

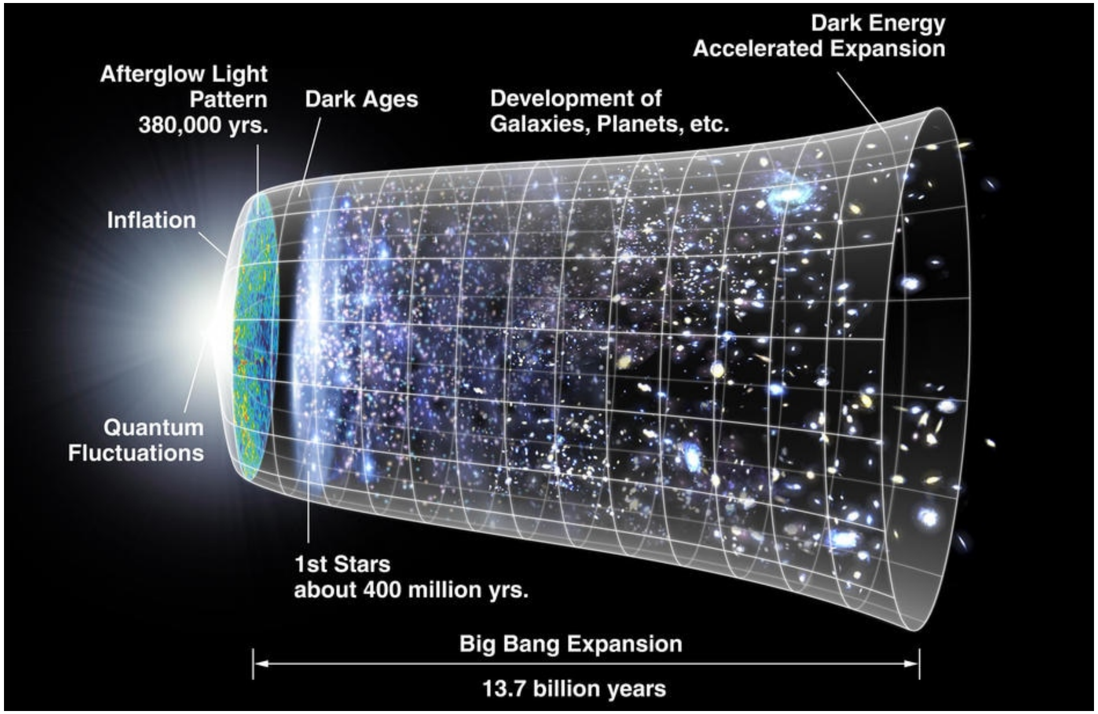


Terrestrial interferometers

NASA

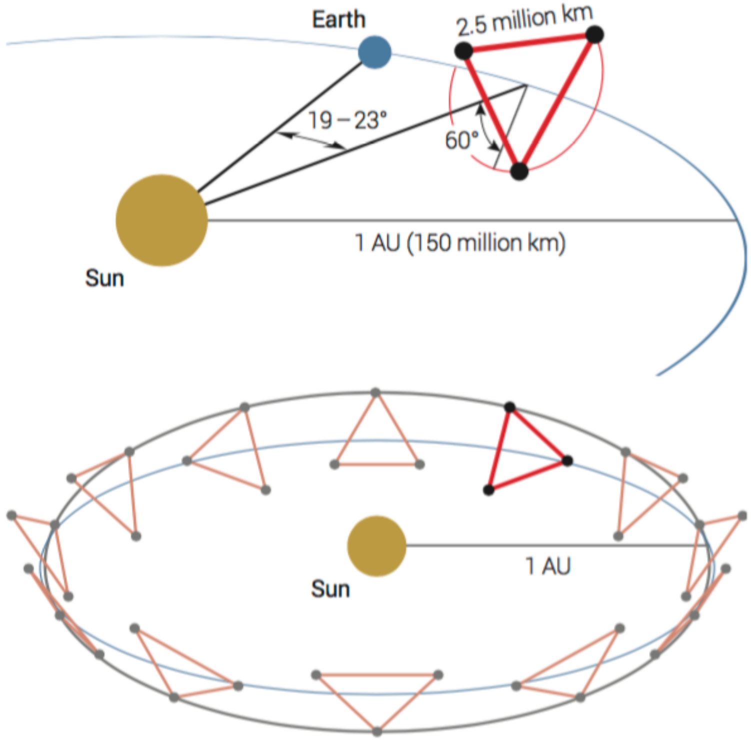
# Gravitational wave cosmology

- Gravitational waves are hard to observe
- Once made, not absorbed by intervening matter
- Complete history of the universe visible in GWs



# Space-based gravitational wave detectors

- Approved:
  - LISA (ESA L3 2034)
- LISA sensitivity
  - Peak: $10^{-3}$ - $10^{-2}$ Hz
  - Arm length $l \approx 10^9$ m
  - $\Delta l \approx 10^{-12}$ m
  - Characteristic strain $h \approx 10^{-21}$
- Proposed:
  - DECIGO (Japan, ?)
  - Taiji, Tianqin (China, ?)
  - Big Bang Observer (USA, ?)



LISA

# Gravitational waves from the early universe

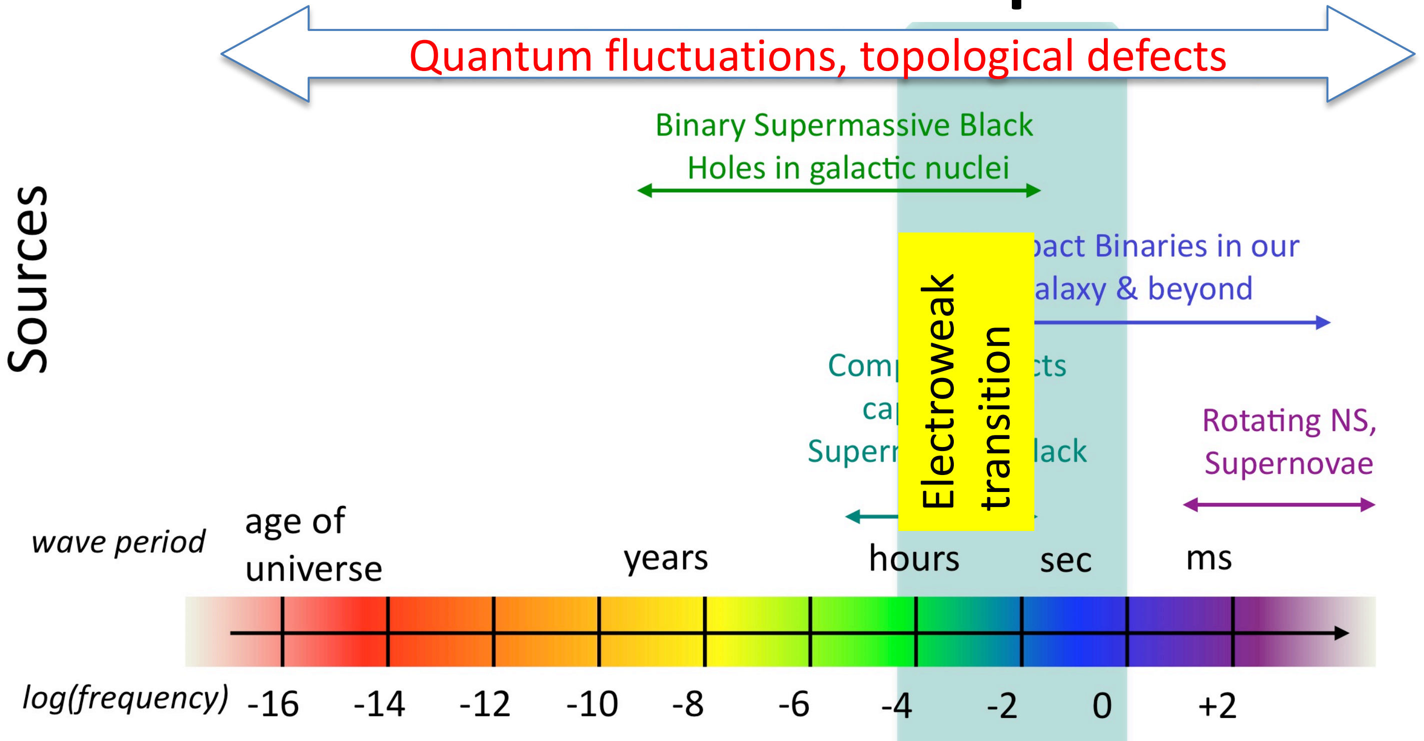
- Events at time *t* generate waves with minimum frequency $f \approx 1/t$ (Hubble rate)
- Redshifted to a frequency now: $f_0 = [a(t)/a(t_0)]\, f$
- Minimum frequencies (redshifted Hubble rates):

| Event | Time/s | Temp/GeV | $f_0$/Hz |
|---|---|---|---|
| QCD phase transition | $10^{-3}$ | 0.1 | $10^{-8}$ |
| EW phase transition | $10^{-11}$ | 100 | $10^{-5}$ |
| ? | $10^{-25}$ | $10^{9}$ | 100 LIGO |
| End of inflation | $\geq 10^{-36}$ | $\leq 10^{16}$ | $\leq 10^{8}$ |

- Inflation and topological defects: waves on all scales

Caprini, Figueroa (2018), Christensen (2019)

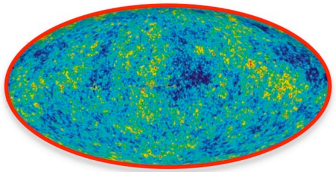
# Gravitational wave spectrum



Sources

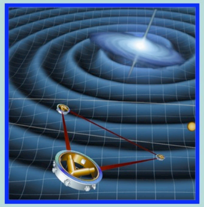
Detectors

Cosmic microwave background polarization



Pulsar Timing

Space Interferometers



Terrestrial interferometers

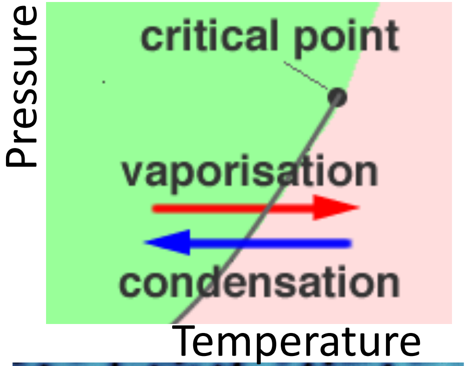
NASA

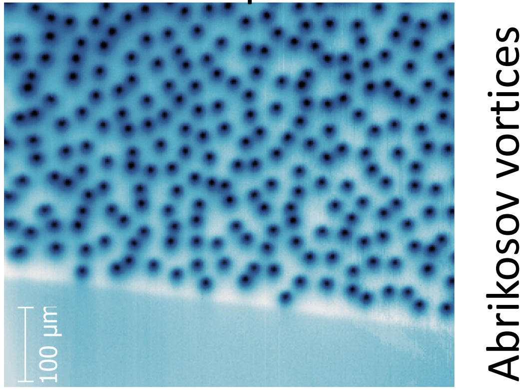
# Phase transitions in the early Universe

- At very high temperatures and pressures, the state of matter in the Universe changes
  - Tc ~ 100 MeV QCD (cross-over)
  - Tc ~ 100 GeV Higgs/electroweak
  - Tc >> 100 GeV ???
- Departures from equilibrium and homogeneity (shear stress)
  - First order phase transition: relativistic condensation or `fizz’

    Steinhardt (1982)
  - Formation of topological defects

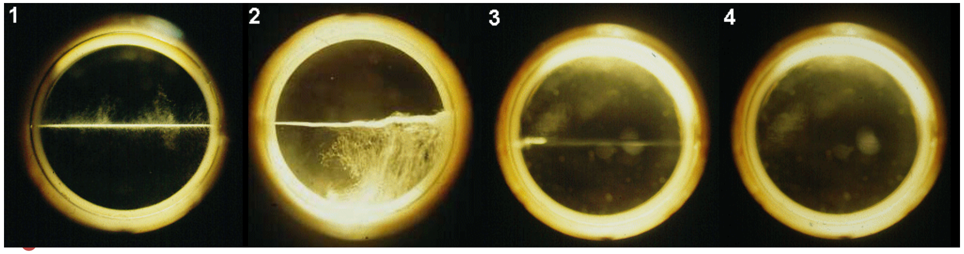
    Kibble (1976)



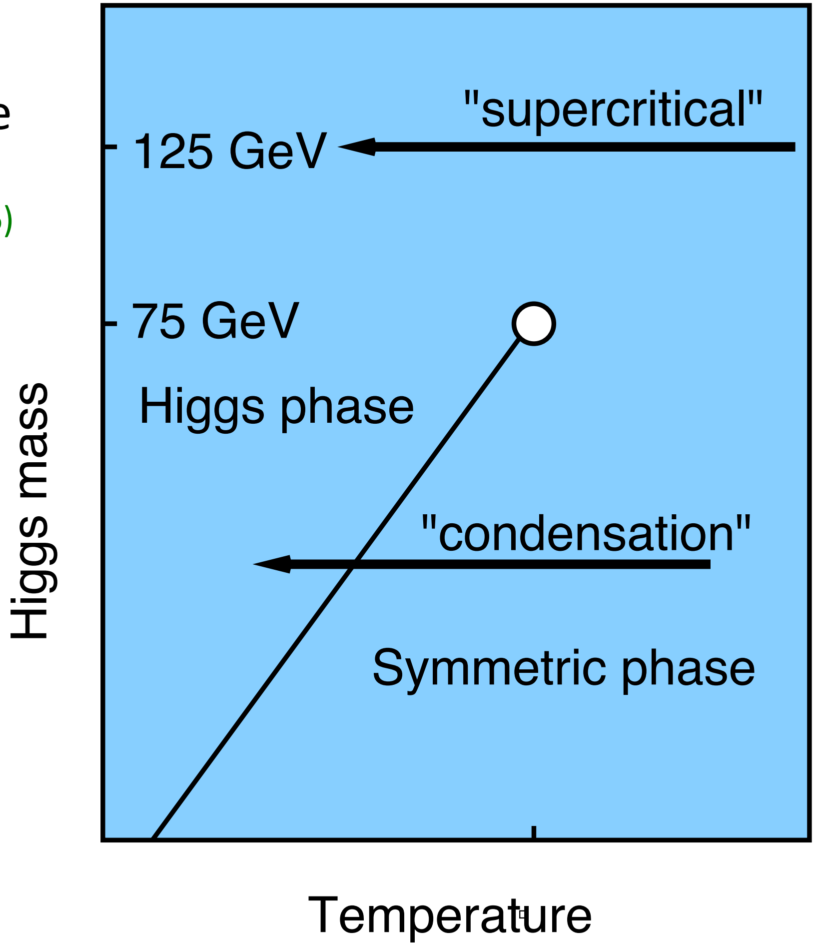


# Electroweak transition

- SM is not weakly coupled at high T
- Non-perturbative techniques:
  - Dimensional reduction to 3D effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - SU(2)-Higgs on 4D lattice
    Czikor, Fodor, Heitger (1998)
- SM transition at $m_h \approx 125$ GeV is a cross-over - a supercritical fluid

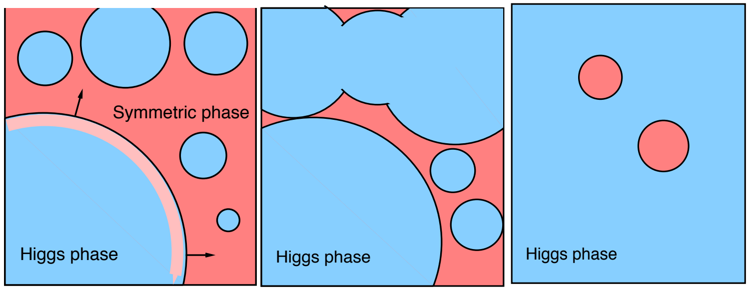


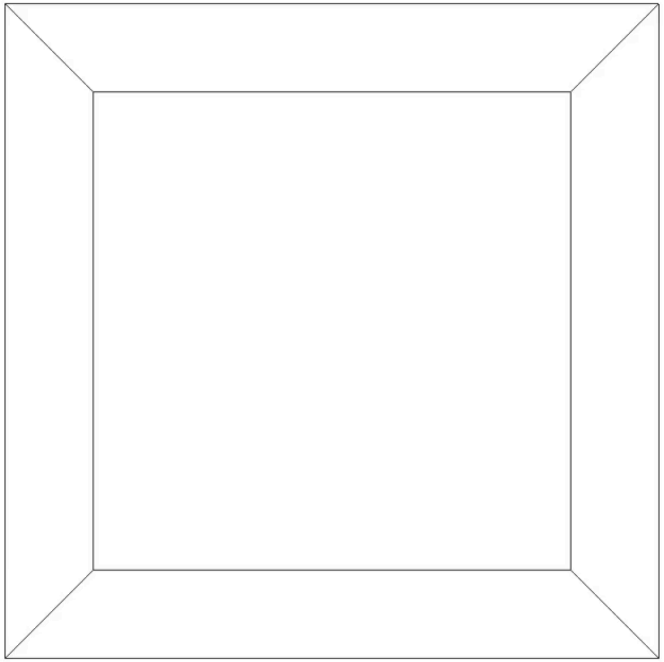
- Search for 1st order transition is a search for physics beyond SM



# First order phase transitions

- 1st order transition proceeds by nucleation of bubbles of Higgs phase
- Expanding bubbles generate pressure waves in hot fluid
- Shear stresses - detectable gravitational waves?





Kinetic energy density

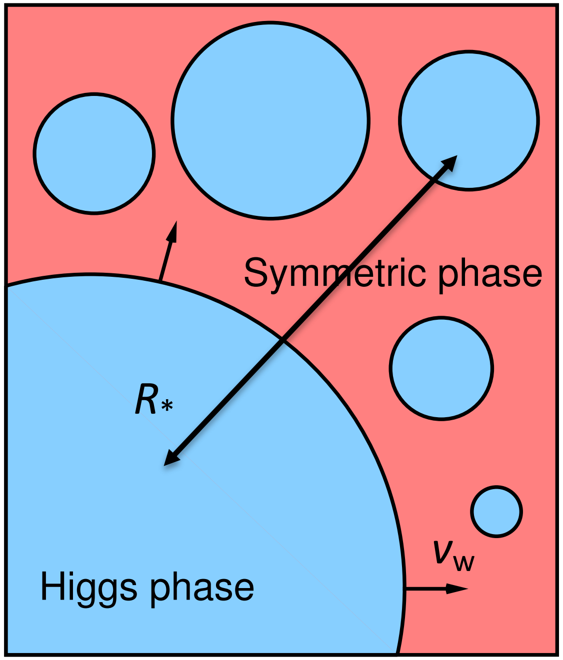
D Weir, University of Helsinki

Steinhardt (1982); Gyulassy et al (1984);
Witten (1984); Enqvist et al (1992);

# GWs from first order phase transitions

- Parametrise transition:
  - $T_n$ = nucleation temperature
  - $\alpha$ = (scalar potential)/(thermal energy)
  - $R_*$ = mean bubble centre separation
  - $v_\mathrm{w}$ = bubble wall speed
  - $g_\mathrm{eff}$ = effective d.o.f.
- Notes on calculating parameters:
  - Bubble nucleation rate/volume = $p(T)$
  - Transition rate parameter b = $d\ \ln(p)/dt$
  - $v_\mathrm{w}$ non-equilibrium
  - Calculate $T_n$ , $\alpha$, $R_*$ from $V_\mathrm{T}(\mathrm{f})$, b and $v_\mathrm{w}$
- Aim: GW power spectrum

$$\frac{d\Omega_\mathrm{gw}}{d\ln f}(T_\mathrm{n}, R_* H_\mathrm{n}, \alpha, v_\mathrm{w}, g_\mathrm{eff}^{\pm})$$



# GWs from phase transitions

- Gravitational waves generated by shear stress fluctuations

$$\Omega_{\mathrm{GW}} \sim \frac{1}{G\rho} \left\langle \left| \dot{h}_{ij}(t) \right|^2 \right\rangle$$

$$\dot{h}_{ij} \sim G \int dt' \cos[k(t-t')] T^{TT}_{ij}(k,t')$$

- Shear stress ~ kinetic energy
- Kinetic energy from scalar potential energy

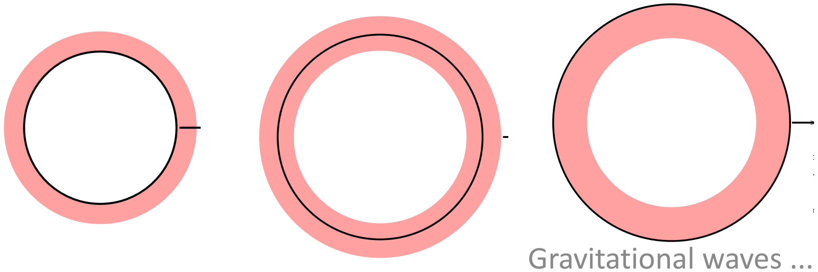
$$T^{TT}_{ij} \sim (\rho + p) U_i U_j \sim \rho K$$

  - $K(\alpha, v_w)$ = fluid kinetic energy fraction
- Timescales $\tau_v$ and $\tau_c$
  - $\tau_v$ duration of stresses from fluid velocity
  - $\tau_c$ coherence time of stress fluctuations

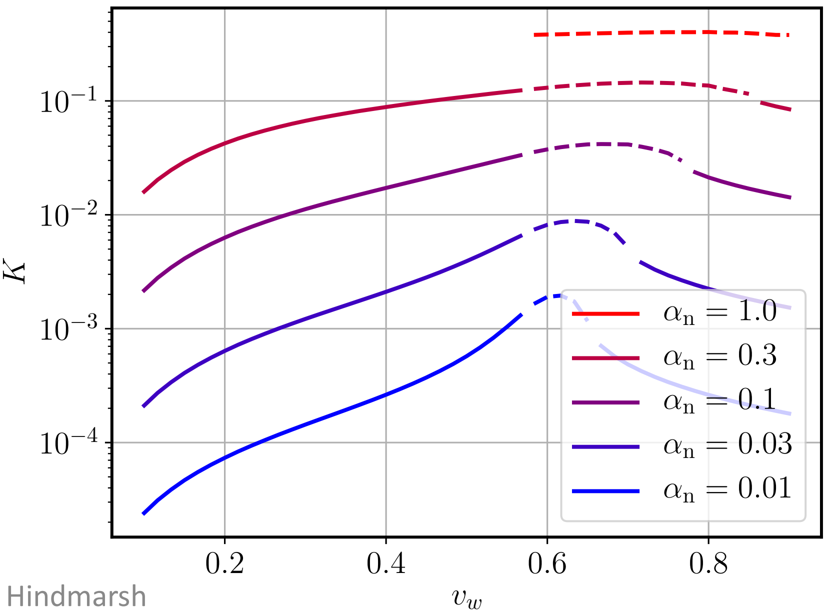
$$\Omega_{\mathrm{gw},0} \sim \Omega_{\mathrm{rad},0} (H_n \tau_v)(H_* \tau_c) K^2$$

- KE fraction estimate from single bubble

Kamionkowsky et al 1994, Espinosa et al 2010



Single-bubble KE fraction



# Simulations of thermal phase transitions

- Preparatory: 1M hrs CSC, Finland
- 2015/6: 17M CPU-hours
  Tier-0 (Hazel Hen, Stuttgart)
- $4200^3$ lattice on 24k cores
- GW density fraction power spectrum

$$\frac{d\Omega_{\rm gw}}{d\ln k} = \frac{1}{12H^2}\frac{k^3}{2\pi^2}\langle|\dot{h}_{ij}^{\rm tt}(\mathbf{k})|^2\rangle$$

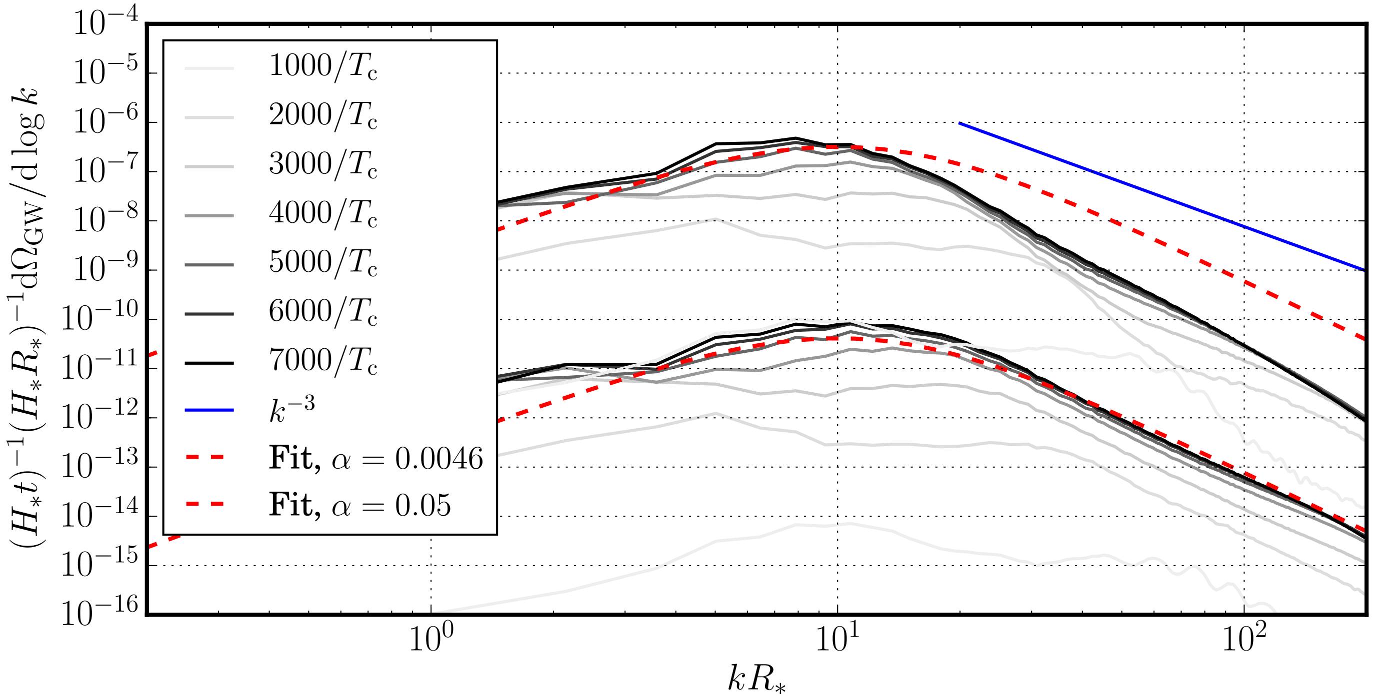
SUOMEN AKATEMIA
FINLANDS AKADEMI
ACADEMY OF FINLAND

Hindmarsh, Huber, Rummukainen, Weir 2017

Fluid kinetic energy in slice ($1200^3$)

Weir 2017

# GW power spectra: deflagration

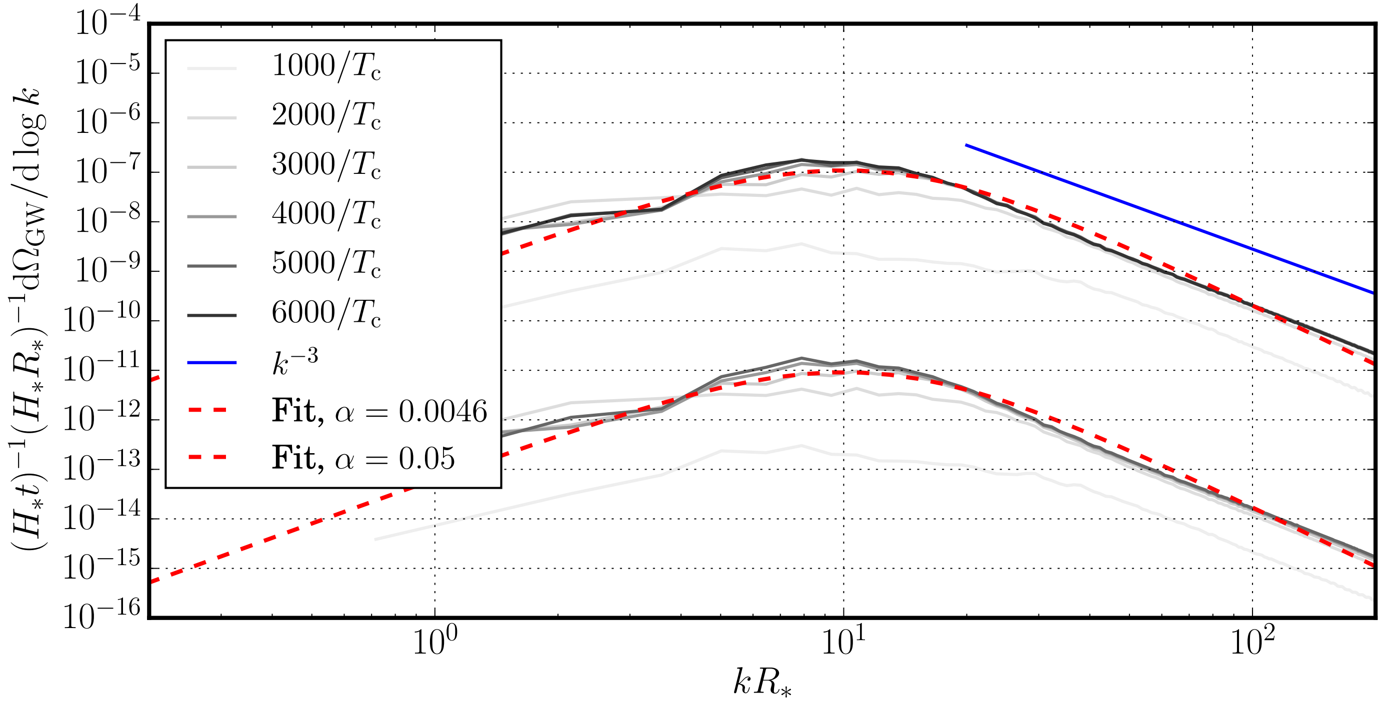


- Transition strength:
  $\alpha = 0.0046, 0.05$
- Wall speed:
  $v_w = 0.44$
- Mean bubble separation:
  $R_* = 1900/T_c$

- Domed peak at $kR_* \sim 10$
- Approx $k^{-4}$ spectrum at high $k$

Hindmarsh, Huber, Rummukainen, Weir (2017)

# GW power spectra: detonation

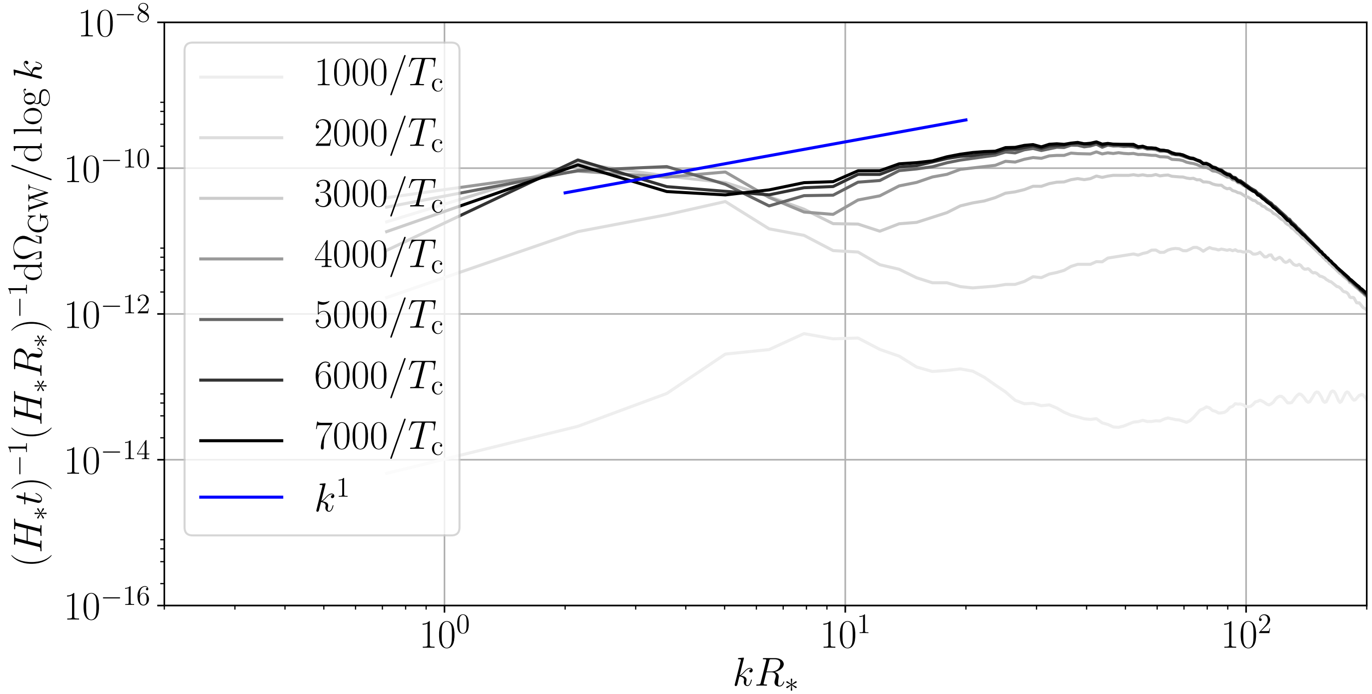


- Transition strength:
  $\alpha = 0.0046, 0.05$
- Wall speed:
  $v_w = 0.92$
- Mean bubble separation:
  $R_* = 1900/T_c$

- Domed peak at $kR_* \sim 10$
- Approx $k^{-3}$ spectrum at high $k$

Hindmarsh, Huber, Rummukainen, Weir (2017)

# GW power spectra: near $c_s$



- Transition strength: $\alpha = 0.0046$
- Wall speed: $v_w = 0.56$
- Mean bubble separation: $R_* = 1900/T_c$

- Peak at $kR_* \sim 40$
- Rising plateau at $kR_* \sim 10$ (approx $k$ spectrum)
- Peak length scale from sound shell thickness

Hindmarsh, Huber, Rummukainen, Weir (2017)

# Lifetime of sound waves

- Sound damped by viscosity

$$\left(\frac{4}{3}\eta_{\rm s} + \zeta\right) \nabla^2 V^i_\parallel$$

- Shear viscosity $\eta_{\rm s} \sim T^3/e^4 \ln(1/e)$
- Lifetime of scale *R*: $\tau_\eta(R) \sim e^4 \ln(1/e) R^2 T.$
- Longer than Hubble time for scales

$$R \gg \frac{v_{\rm w}}{H_*}\left(\frac{T_{\rm c}}{m_{\rm Pl} e^4}\right) \sim 10^{-11}\frac{v_{\rm w}}{H_*}\left(\frac{T_{\rm c}}{100\ {\rm GeV}}\right),$$

- Effective lifetime is Hubble time, unless …
- … non-linearities develop, at $\tau_{\rm nl} \sim R_*/\bar{U}_{\rm f}$
- Pure acoustic waves only if $\bar{U}_{\rm f} < R_* H_*$

# Acoustic GW power spectrum

Hindmarsh, Huber, Rummukainen, Weir (2017)

- Density fraction of GWs by acoustic production:

$$\Omega_{\rm gw,0}(f) = 0.68 F_{\rm gw,0}\tilde{\Omega}_{\rm gw}(H_* R_*) K^2\, C\left(\frac{f}{f_{p,0}}\right)$$

  - Fluid kinetic energy density fraction $K(\alpha, v_w)$
    (estimate from single-bubble fluid kinetic energy) Espinosa et al (2010)
  - Dimensionless constant (from simulations) $\tilde{\Omega}_{gw} \sim 0.01$
  - Phenomenological fit $C(s) = s^3\left(\frac{7}{4+3s^2}\right)^{\frac{7}{2}}$
  - Peak frequency

$$f_{\rm p,0} \simeq 26\left(\frac{1}{H_* R_*}\right)\left(\frac{T_{\rm n}}{10^2\,{\rm GeV}}\right)\left(\frac{g_{\rm eff}}{100}\right)^{\frac{1}{6}}\ \mu{\rm Hz},$$

  - Matter-era dilution:

$$F_{\rm gw,0} = (3.57 \pm 0.05)\times 10^{-5}\left(\frac{100}{g_{\rm eff}}\right)^{\frac{1}{3}}.$$

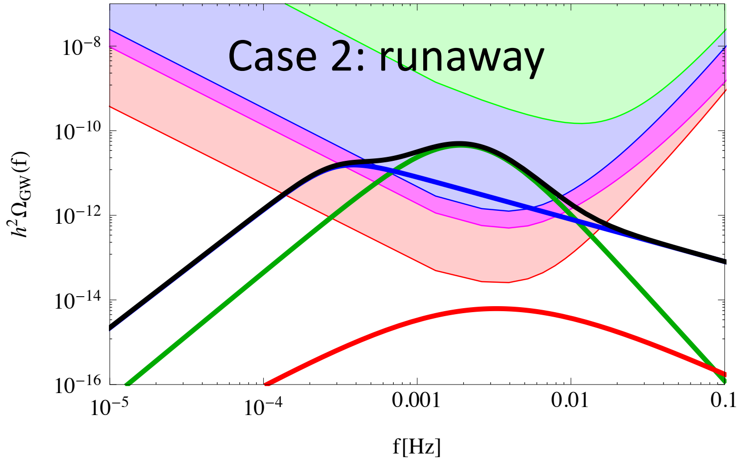
  - NB $\alpha$ from $\Delta(e-3p)/4$, not latent heat $\Delta(e+p)$
    - Latent heat 4x larger at $T_c$

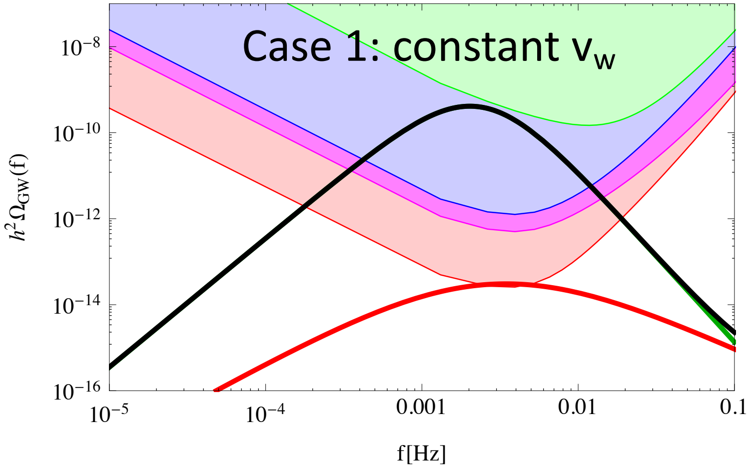
# LISA CWG party line 2016

- Three contributions to total power:
  - Scalar field   f
  - Acoustic   ac
  - Turbulent   tu

$$\Omega_{\rm gw} = \Omega^{\phi}_{\rm gw} + \Omega^{\rm ac}_{\rm gw} + \Omega^{\rm tu}_{\rm gw}$$

- Scalar field: bubble wall collisions
  - relevant only for runaway walls
  - “envelope approximation”
  - Kosowsky, Turner 1992
  - Huber, Konstandin 2008
- Acoustic production:
  - M.H. et al 2013, 2015, 2017
- Turbulent production:
  - Caprini, Durrer, Servant 2009

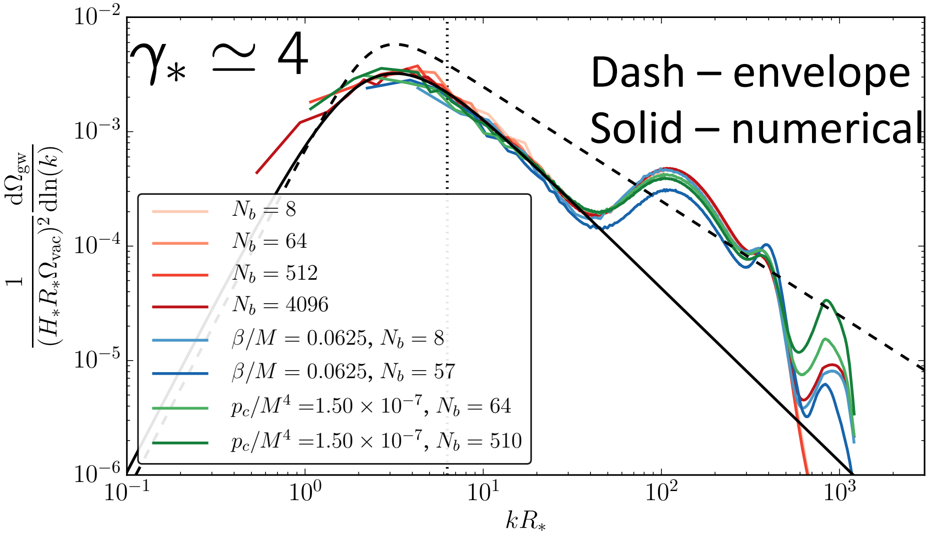




# Developments 1: scalar field

- Numerical simulations show differences from envelope approximation

Cutting, MH, Weir 2018

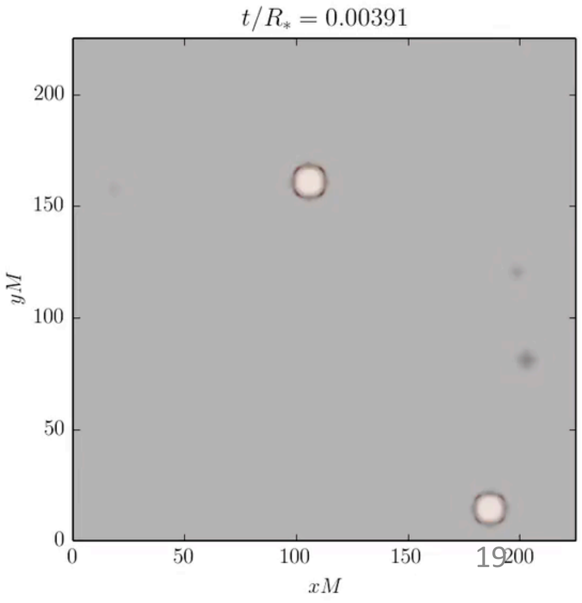


$$\frac{d\Omega_{\rm gw}^{\rm fit}}{d\ln k} = \Omega_{\rm p}^{\rm fit} \frac{(3+b)^c \tilde{k}^b k^3}{(b\tilde{k}^{(3+b)/c} + 3k^{(3+b)/c})^c}$$

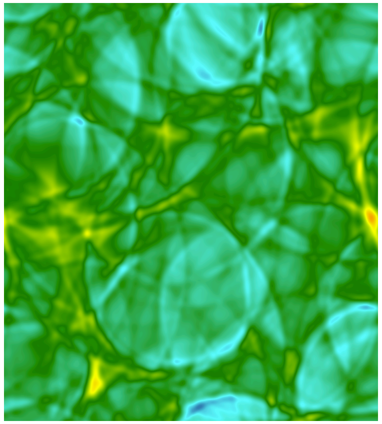
$$\Omega_{\rm p}^{\rm fit} = (3.22 \pm 0.04) \times 10^{-3} (H_{\rm n} R_*)^2 \Omega_\phi^2,$$

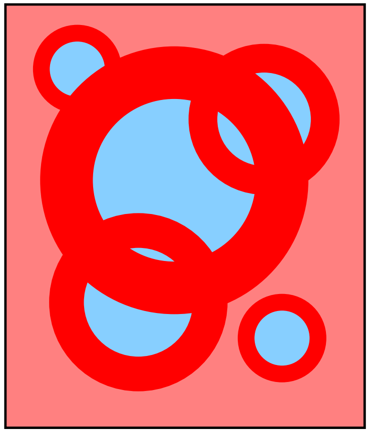
$$\tilde{k} R_* = 3.20 \pm 0.04,$$

$$b = 1.51 \pm 0.04, \quad c = 2.18 \pm 0.15$$



# Developments 2: sound shell model

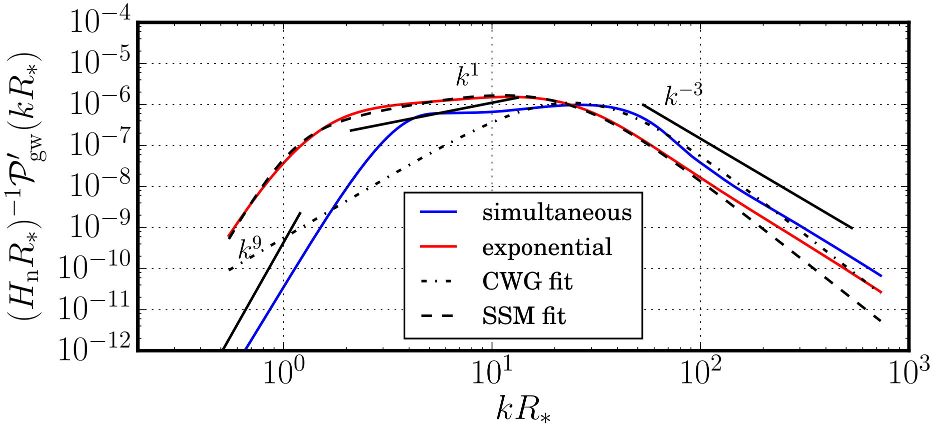




- Gaussian velocity field from weighted addition of sound shells $\mathbf{v}_\mathbf{q}(t_\mathrm{i})$

  MH 2017, MH, Hijazi (in prep 2019)

- Two length scales:
  - Bubble spacing $R_*$
  - Shell width $R_* \, |v_\mathrm{w} - c_\mathrm{s}|$
- Double broken power law
  - $P_{gw} \sim k^9, k^1, k^{-3}$
- Amplitude 10% - 20% agreement w/ simulations
- Systematically too large for deflagrations



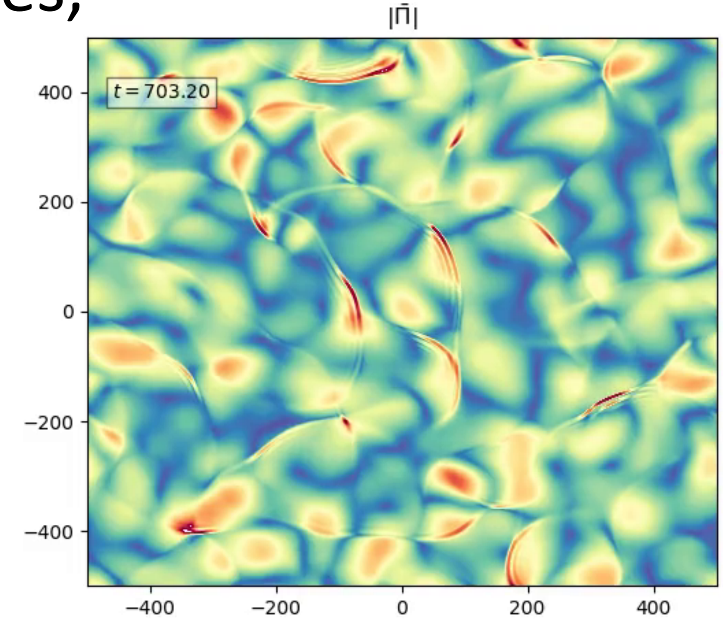
# Developments 3: non-linearities

- Fluid equations: non-linearities important after

$$\tau_{\mathrm{nl}} \sim L_f / \bar{U}_f$$

?

- Energy transport to small scales, where dissipated.
- Longitudinal v
  - Wave turbulence
  - Shocks  Pen, Turok 2015
- Transverse v
  - Vorticity production
  - Turbulence  Caprini, Durrer, Servant 2008  
    Gogoberidze, Kahniashvili, Kosowsky 2007



J Dahl, U Helsinki

# Non-linearities: dealing with uncertainty

- Non-linearities important after

$$\tau_{\rm nl} \sim L_f/\bar{U}_f$$

?

## Acoustic issues

- CWG 2016: Non-linear dissipation ignored
- source lifetime assumed to be $H_n^{-1}$

- Conservative fix: multiply PS by $\min(1, H_n\tau_{nl})$

## Turbulence issues

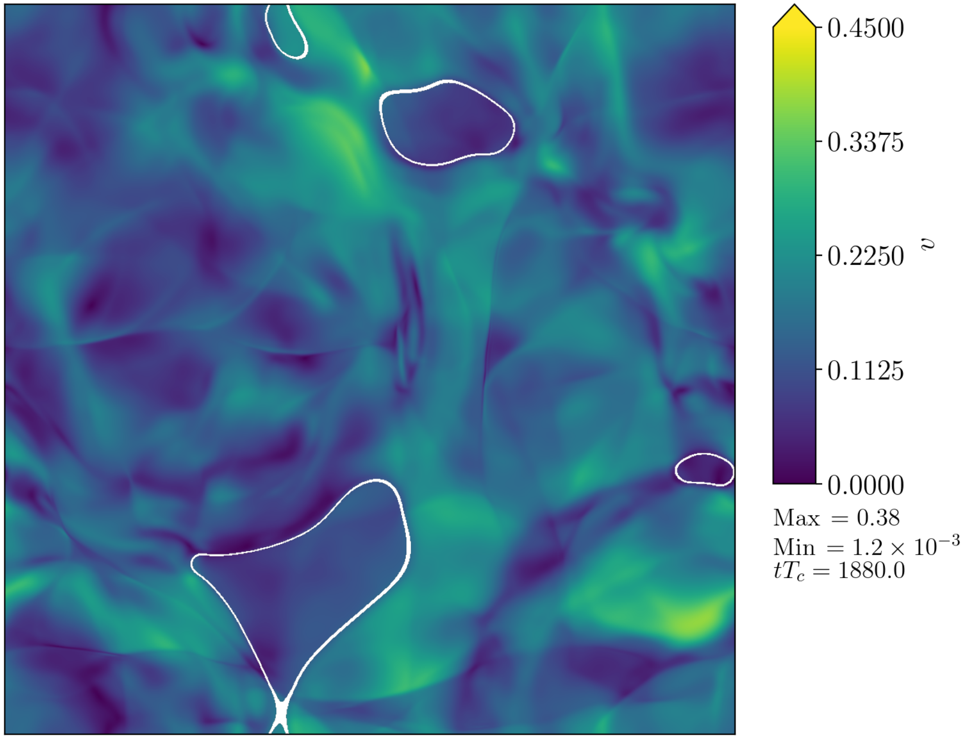
- CWG 2016: $K_{\rm tu} = 0.05K$
- but we don't really know $K_{\rm tu}/K$
- Disagreement on turbulence PS between CDS and GKK
- GW production from (longitudinal * transverse)
- Conservative fix: take $K_{\rm tu} = 0$

# ‘Strong’ phase transitions

- Recent simulations of strong transitions with $\alpha = 0.5$
- Relativistic velocity flows with maximum velocity $V > 0.6$
- During collision of bubble walls rotational velocity $V_\perp$ can be generated.
- Deflagrations visibly produce more $V_\perp$ than detonations.
- Suppression of kinetic energy and GW signal, especially deflagrations

Cutting, Hindmarsh, Weir (in prep.)

See D. Cutting’s talk at 14.40

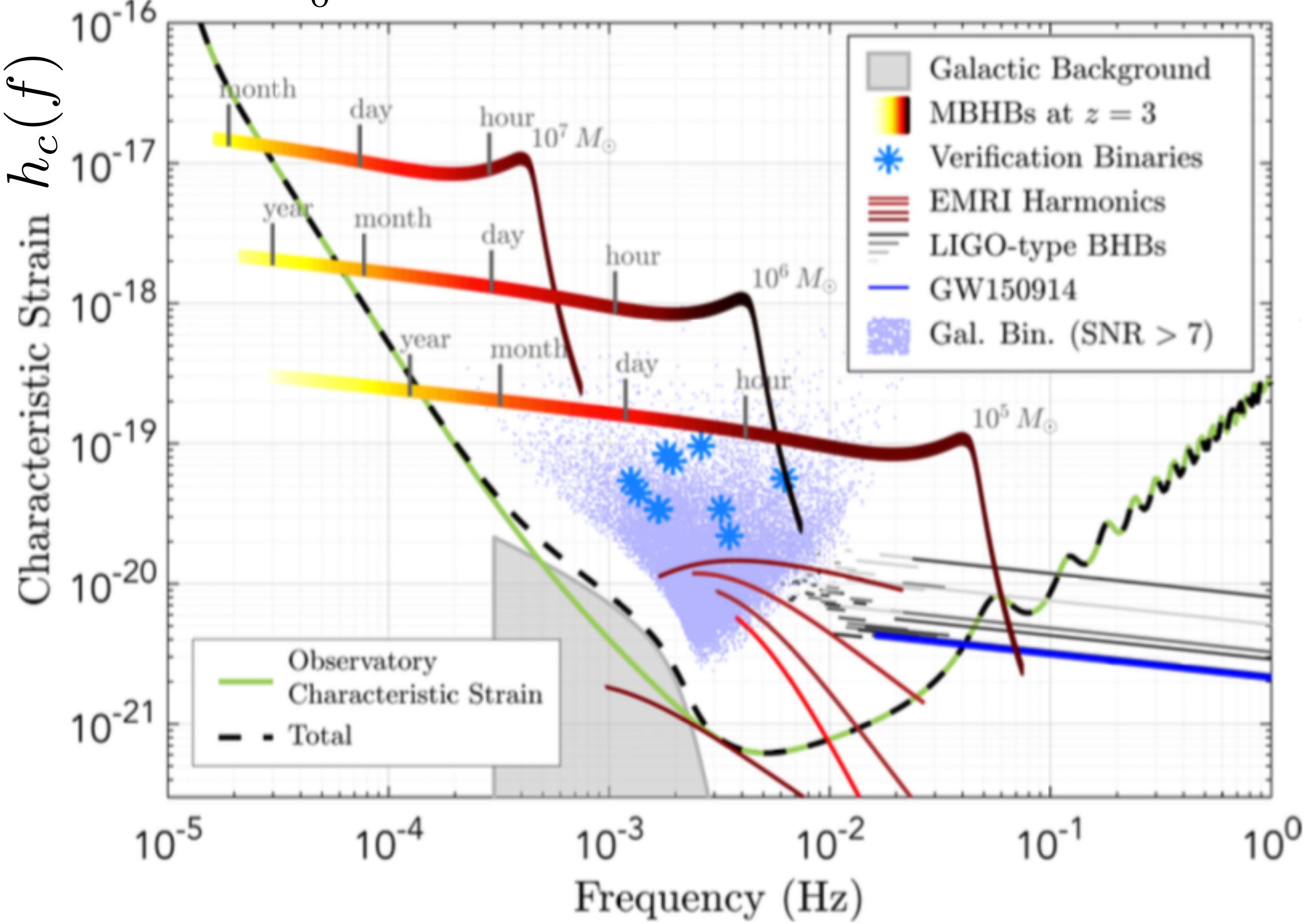


V fluid (slice)
White = bubble wall
$v_w$=0.44
Lattice $960^3$

# Laser Interferometer Space Antenna

$$\Omega_{\mathrm{gw},0}(f) = \frac{2\pi^2}{3}\frac{f^2}{H_0^2}h_c(f)^2$$

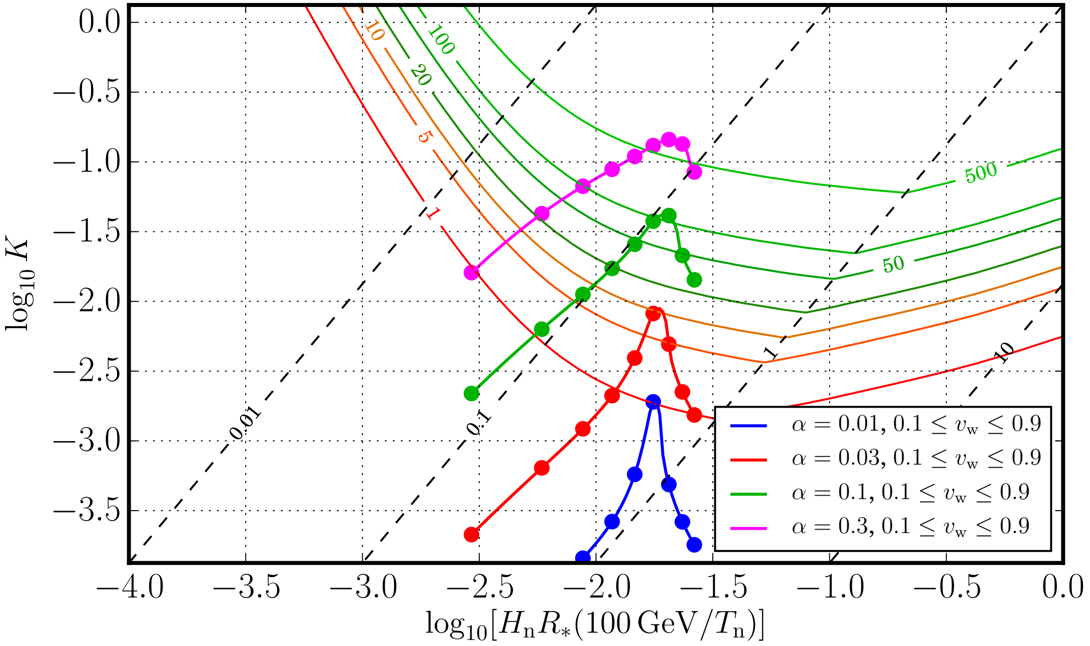
$$h_c(f) \simeq 0.87 \times 10^{-18}(f/\mathrm{Hz})^{-1}[\Omega_{\mathrm{gw}}(f)]^{1/2}$$



LISA proposal, Amaro-Seoane at al 2017

# Estimated LISA prospects

Kinetic energy fraction
$K = \overline{\Gamma U}^2$



- Estimate signal-to-noise ratio r

$$\rho^2 = T_{\rm obs} \int df \left( \frac{\Omega_{\rm gw}(f)}{\Omega_{\rm noise}(f)} \right)^2$$

  - Observation time 4 years
  - Neglect foregrounds
    - White Dwarf binaries (annual variation)
    - LIGO BHB precursors (negligible)

- E.g. (favourable cases):
  - $\beta/H_n = 100$ , $T_n = 100$ GeV
- NB $\alpha > 0.1$ highly uncertain

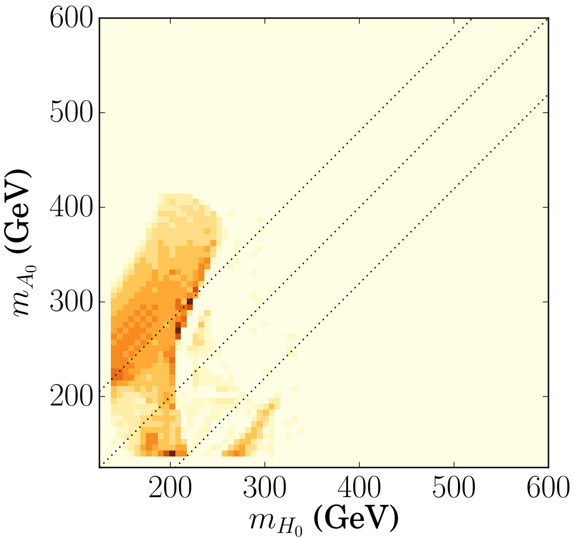
  Cutting, Hindmarsh, Weir (in prep.)
  - But important region for LISA

    Ellis, Lewicki, No (2018)

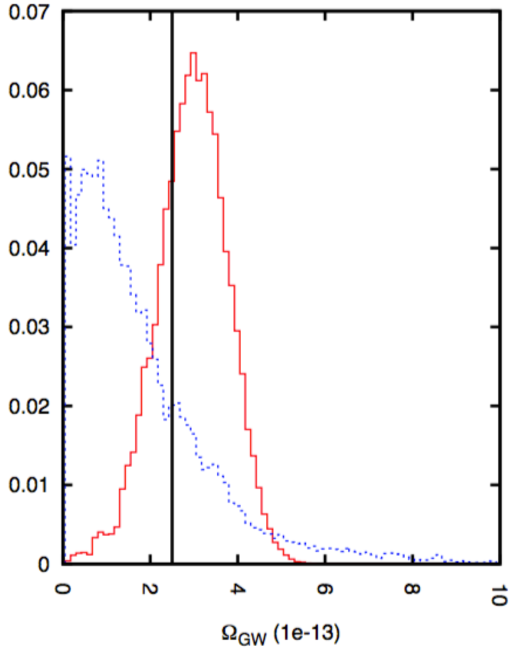
# Phase transitions: future challenges

- Full characterization of GW spectrum from phase transitions
  - Strong transitions Cutting, MH, Weir (in prep)
  - Magnetic fields? Pol et al (2019), Zhang et al (2019)
- Phase transition parameters from underlying particle physics models
  - E.g. transition strength in DR & non-perturbative 3D Moore, Rummukainen (2000)
    Laine, Nardini, Rummukainen (2012)
    Gould et al (2019)
    Kainulainen et al (2019)
- Connection to collider (LHC …) data
  - E.g. $\lambda_{hhh}$ Noble, Perelstein 2007; Patel, Ramsey-Musolf 2013
  - $A^0 \rightarrow H^0 Z$ Dorsch, Huber, Mimasu, No (2016)
    Andersen et al (2017)
- Distinguishing phase transitions from astrophysical GW foregrounds

  Adams, Cornish (2013), Hashino et al (2018)



Andersen et al 2017

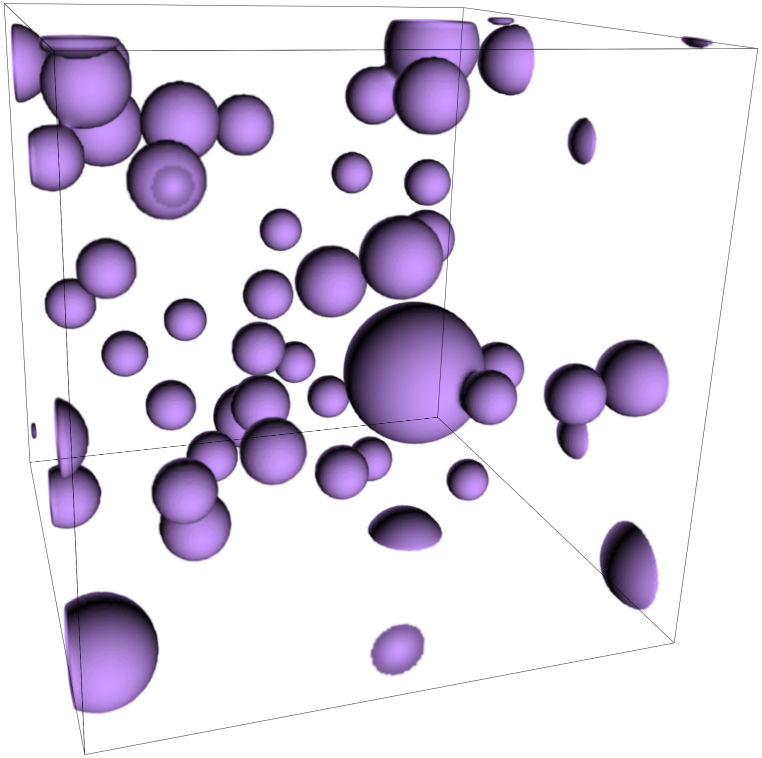


Adams, Cornish (2013)

# Summary

- GWs probe of physics of early universe at very high energy
- LISA will probe physics of Higgs phase transition from 2034
- Measure/constrain phase transition parameters
  - $T_n$ = nucleation temperature
  - $\alpha$ = (scalar potential)/(thermal energy)
  - $R_*$ = mean bubble centre separation
  - $v_\mathrm{w}$ = bubble wall speed
  - $g_\mathrm{eff}$ = effective d.o.f.

- Need 1: accurate calculations of GW power spectrum from parameters
- Need 2: accurate calculations of parameters from underlying particle physics models
- Need 3: reliable methods to extract parameters from real data
- Lots of work to do
  - only 16 years to go!



# Numerical simulation of phase transitions

Ignatius et al (1994), Kurki-Suonio, Laine (1996)

- Ingredients:
  - Higgs field $-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i\partial_i\phi)$
    - $\eta$ coupling to fluid (models energy transfer, friction)
    - $V_i$ velocity, $W$ γ-factor
  - Relativistic fluid equation

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi} W(\dot{\phi} + V^i\partial_i\phi) = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2.$$

$$\dot{Z}_i + \partial_j(Z_iV^j) + \partial_iP + \frac{\partial V}{\partial \phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi.$$

    - $E/W$ energy density, $P$ pressure, $Z_i$ momentum density,
  - Discretisation
  - Metric perturbation

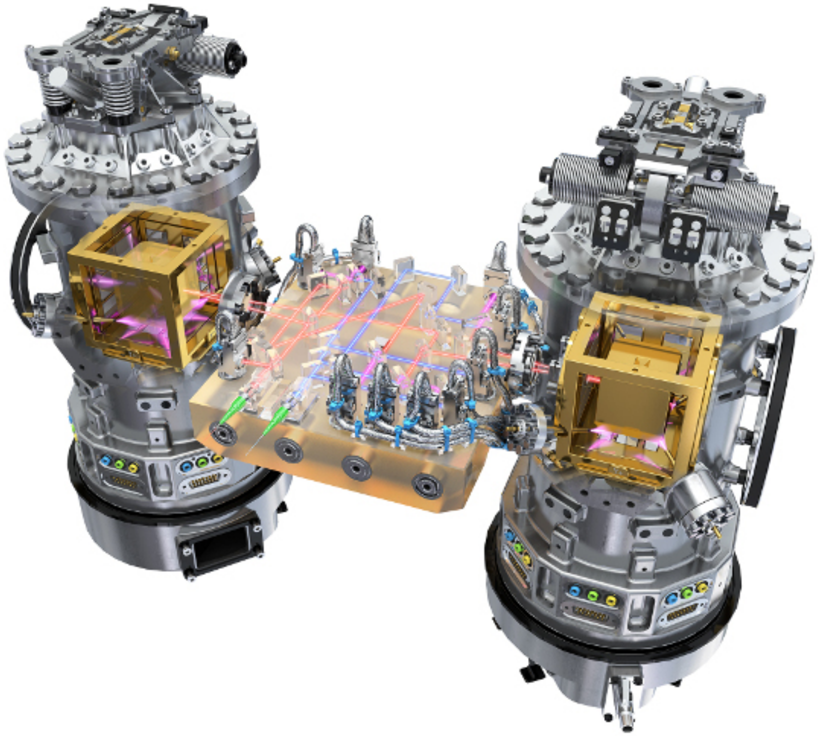
Wilson & Matthews (2003)
Different approach: Giblin, Mertens (2013)

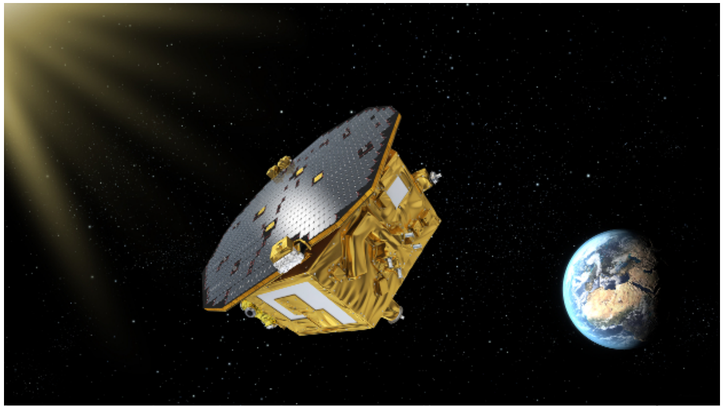
$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G T_{ij}$$
$$\dot{h}^{\rm tt}_{ij}(\mathbf{k}) = \Lambda^{\rm tt}_{ijkl}(\mathbf{k})\dot{u}_{ij}(\mathbf{k})$$

Garcia-Bellido, Figueroa, Sastre (2008)

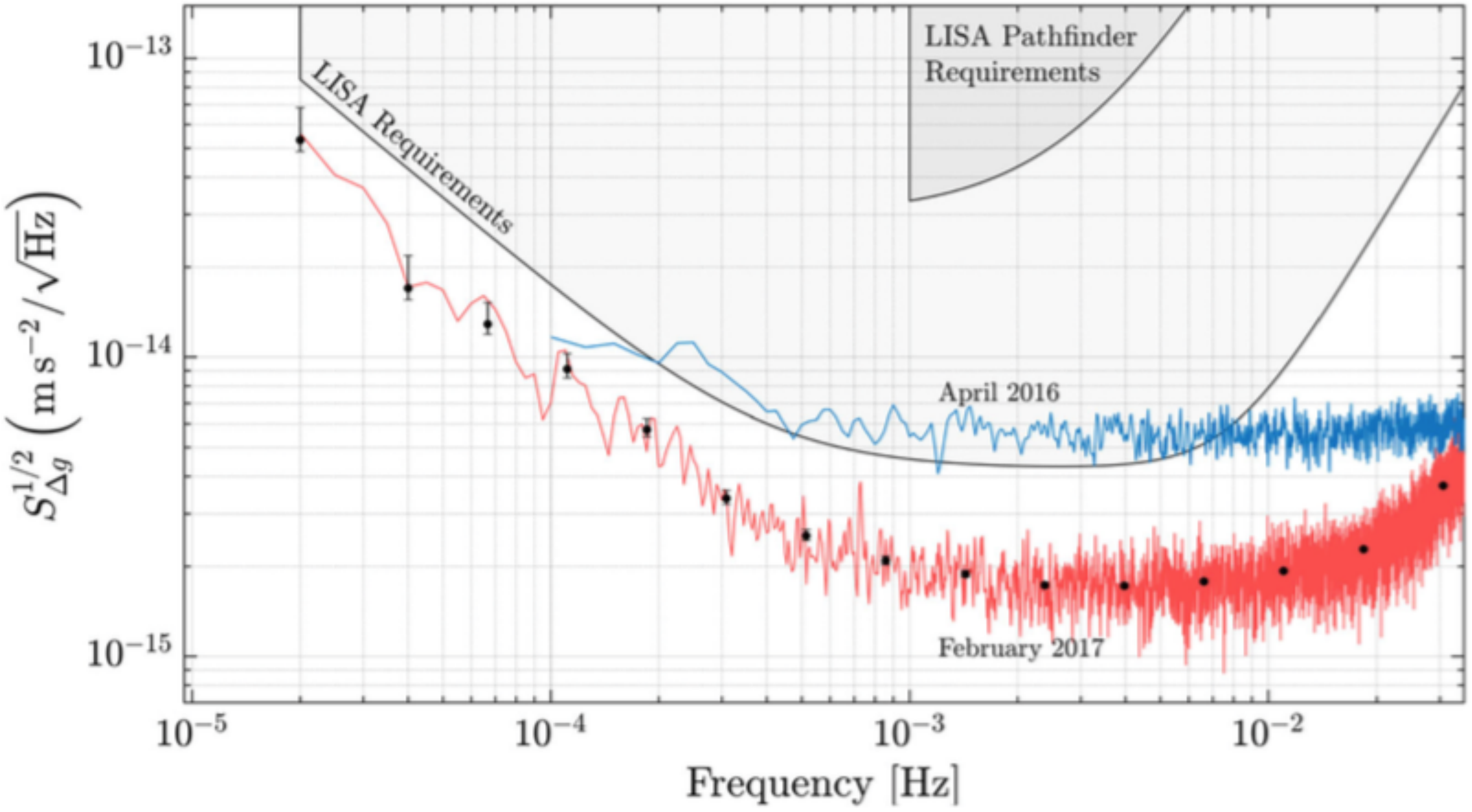
# LISA Pathfinder

- Technology tester for LISA
- Test masses in free-fall at L1
- Control and measure motion
- Launched 3/12/15
- Masses released 3/2/16
- Mission end 30/6/17
- Quantified acceleration noise – how good is the free fall





# LISA Pathfinder acceleration noise

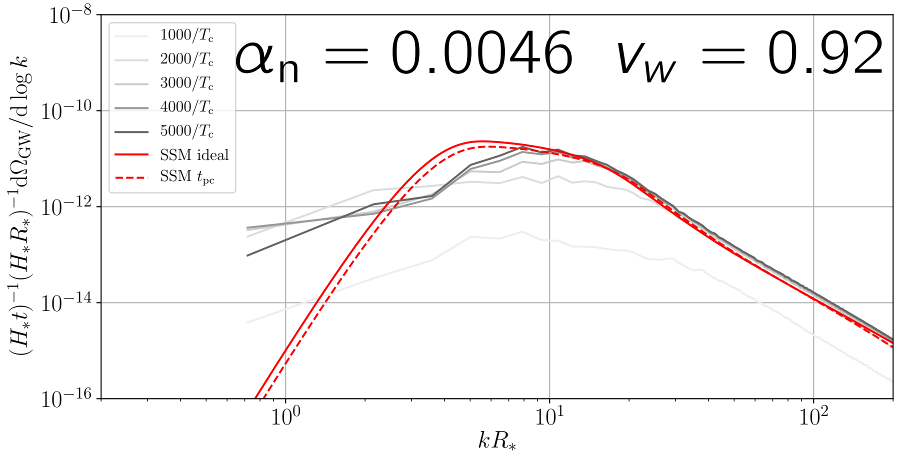


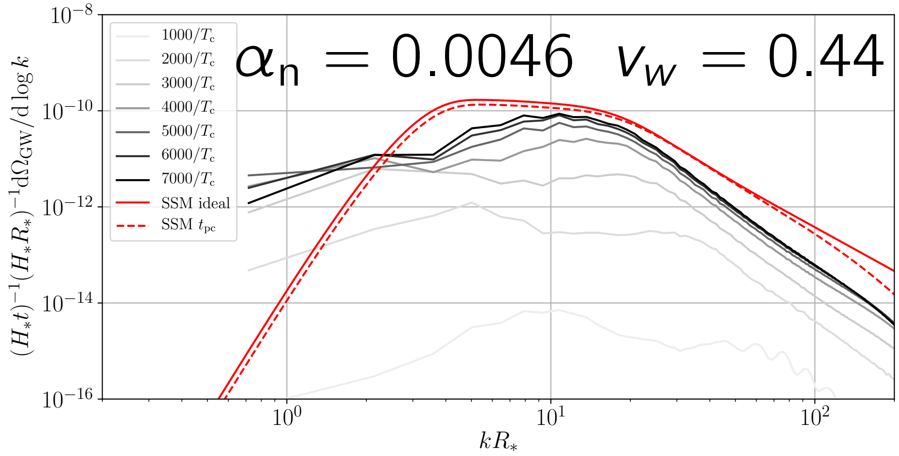
Armano (2018)

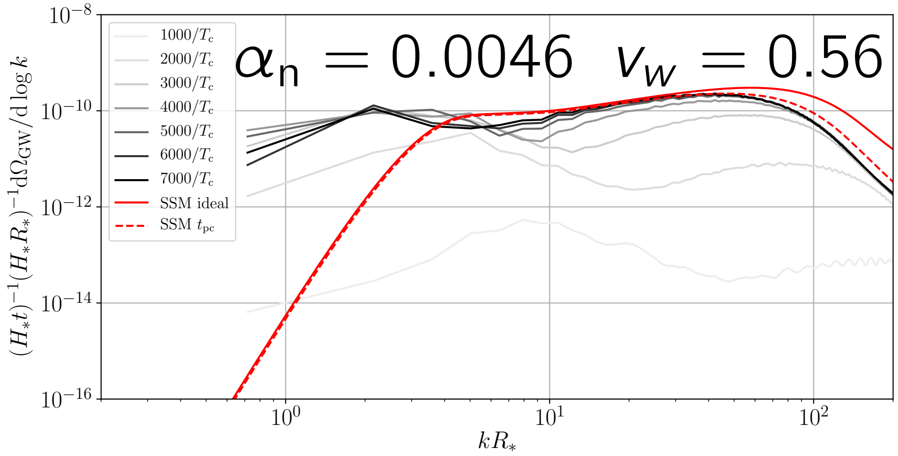
# Sound shell model vs. simulations $P_{\rm gw}$

- Solid: self-similar sound shell
- Dash: evolving sound shell at peak collision time
- Simultaneous nucleation

MH et al in prep 2019







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