

INCONSISTENCY OF AN INFLATIONARY SECTOR COUPLED ONLY TO EINSTEIN GRAVITY

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EPFL, Lausanne

(soon to be eating Paella & drinking Horchata)

TWO PAPERS / IDEAS*

(1811.04093)

Inconsistency of an inflationary sector....

(1905.11960)

Measuring the early Universe expansion rate...

*with Erwin H. Tanin, Master student @ EPFL

TWO PAPERS / IDEAS

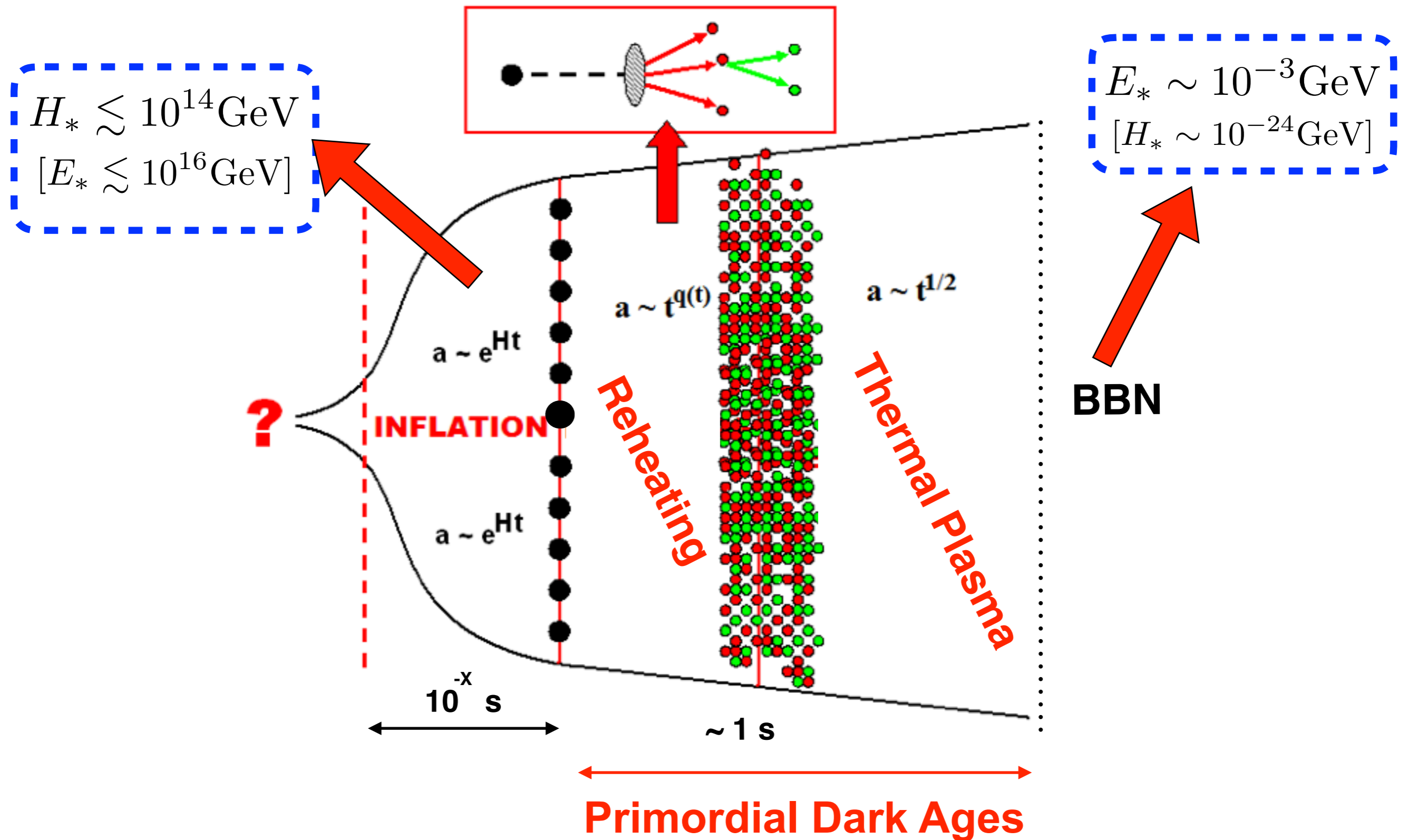
Inconsistency of an inflationary sector....

REHEATING / GRAVITATIONAL WAVES

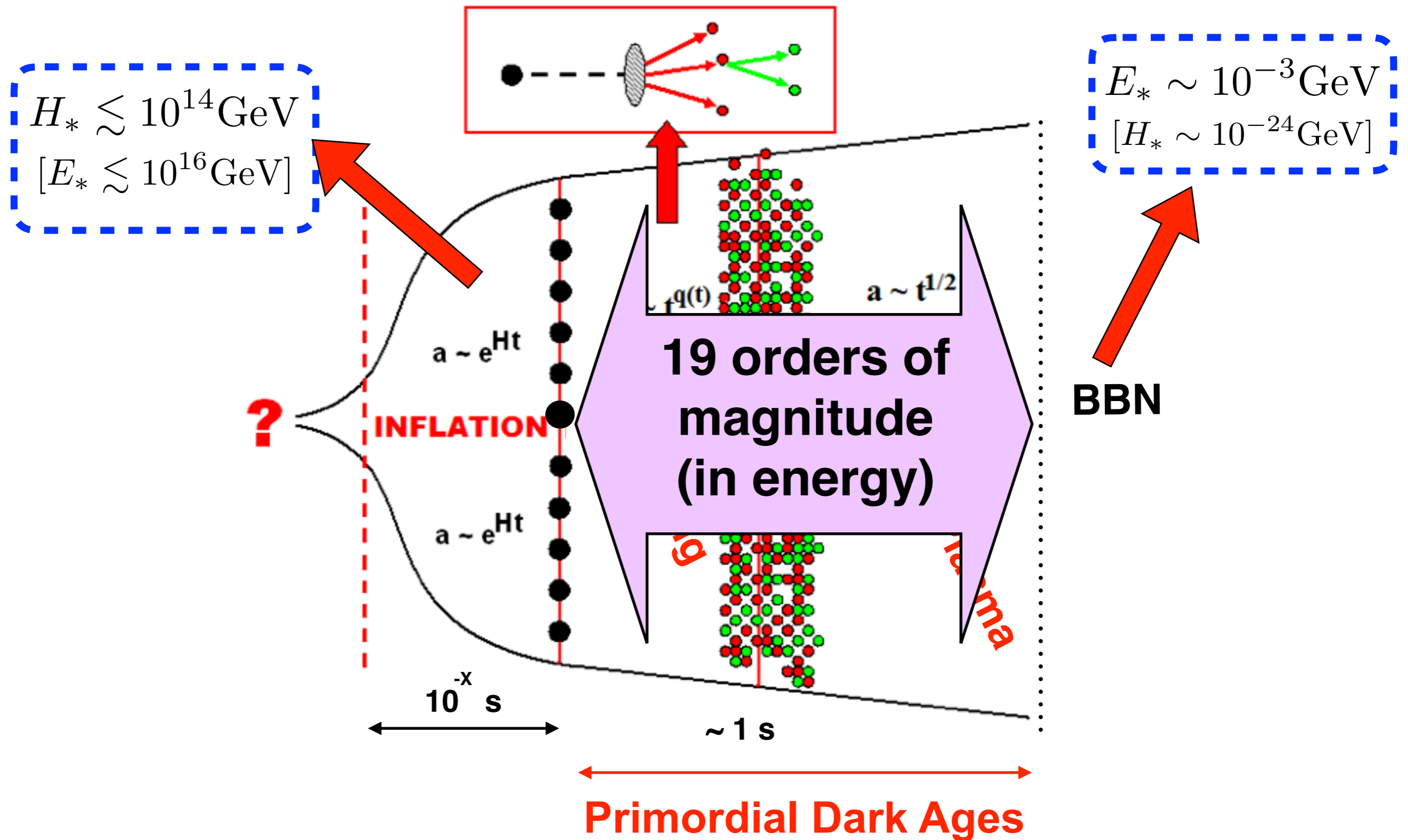
Measuring the early Universe expansion rate...



The context: The Early Universe



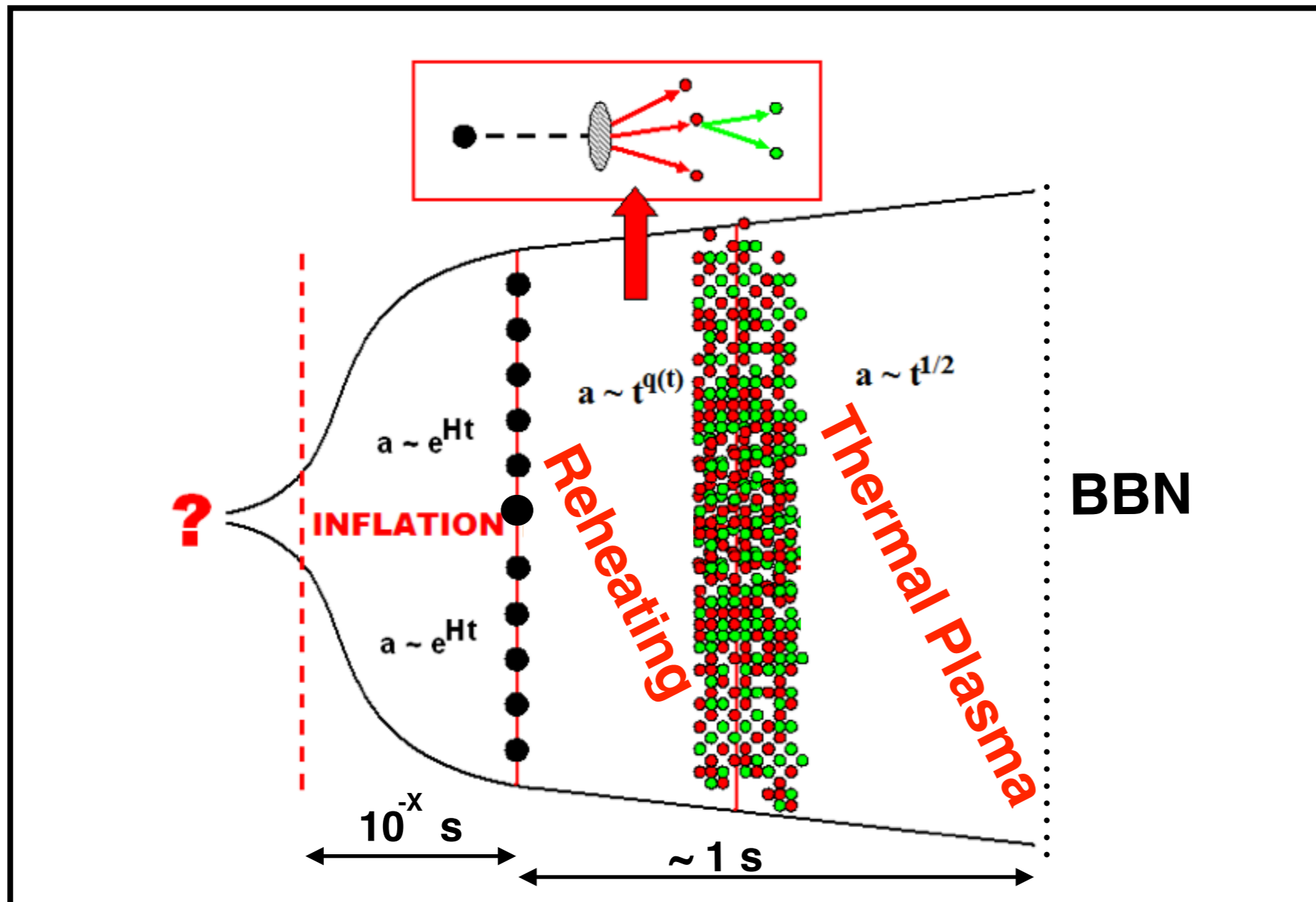
The context: The Early Universe



Successful Reheating



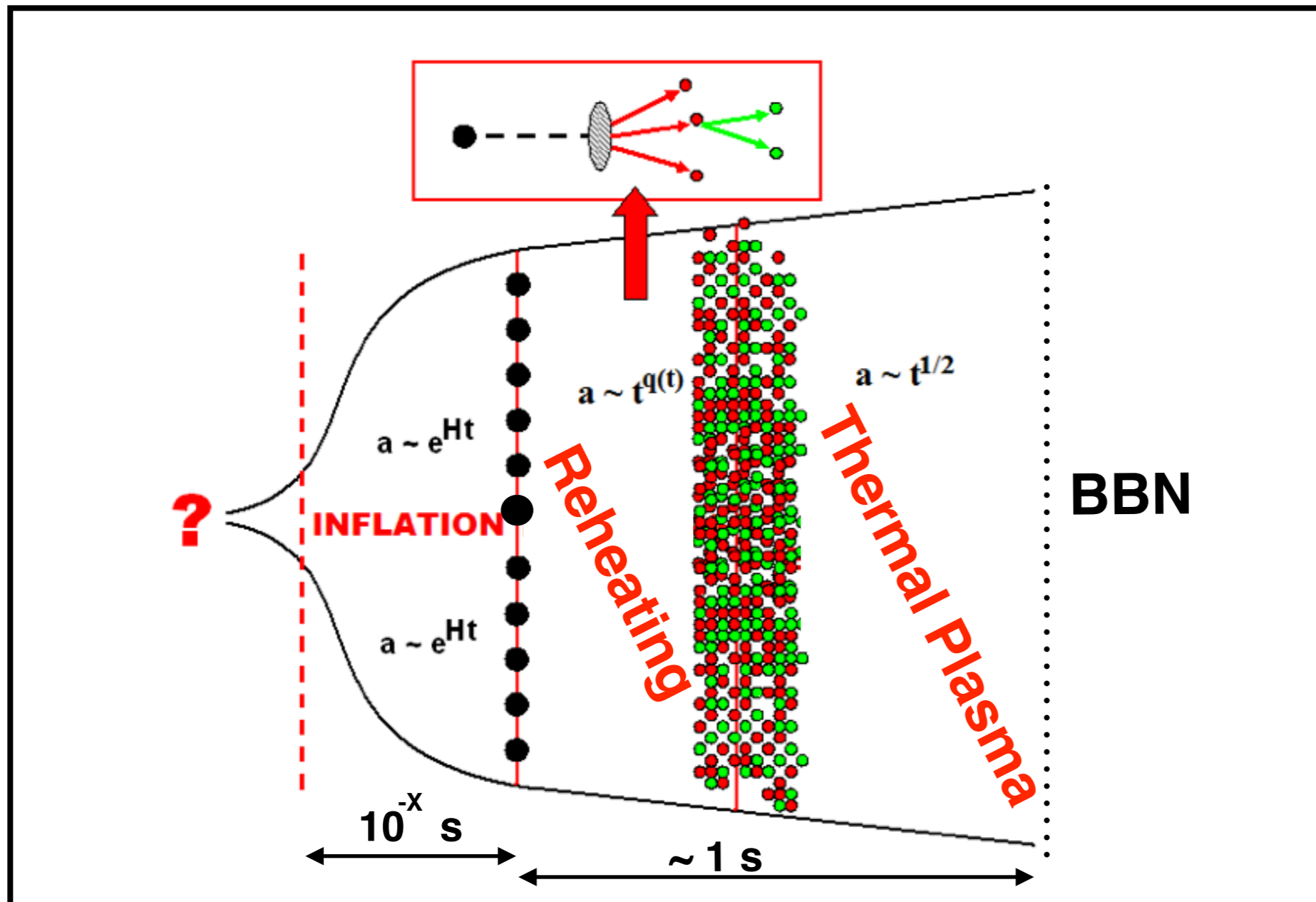
Thermal Ensemble of Relativistic Particle Species



Successful Reheating



Thermal Ensemble of Relativistic Particle Species



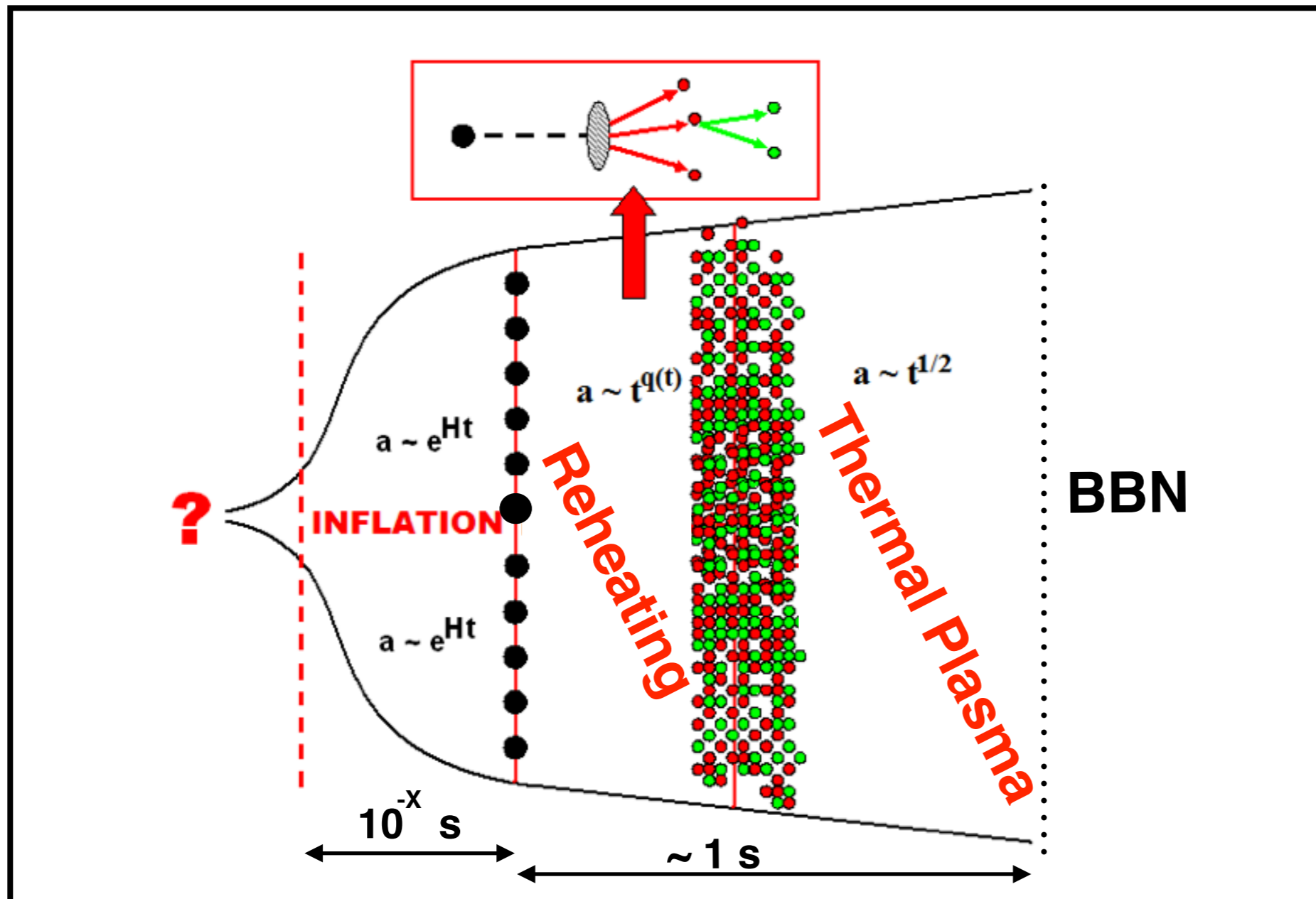
Connection between Particles and Inflationary Sector ?

- * **SM Portals ?**
- * **Mediator fields ?**
- * **No coupling ?**

Successful Reheating



Thermal Ensemble of Relativistic Particle Species



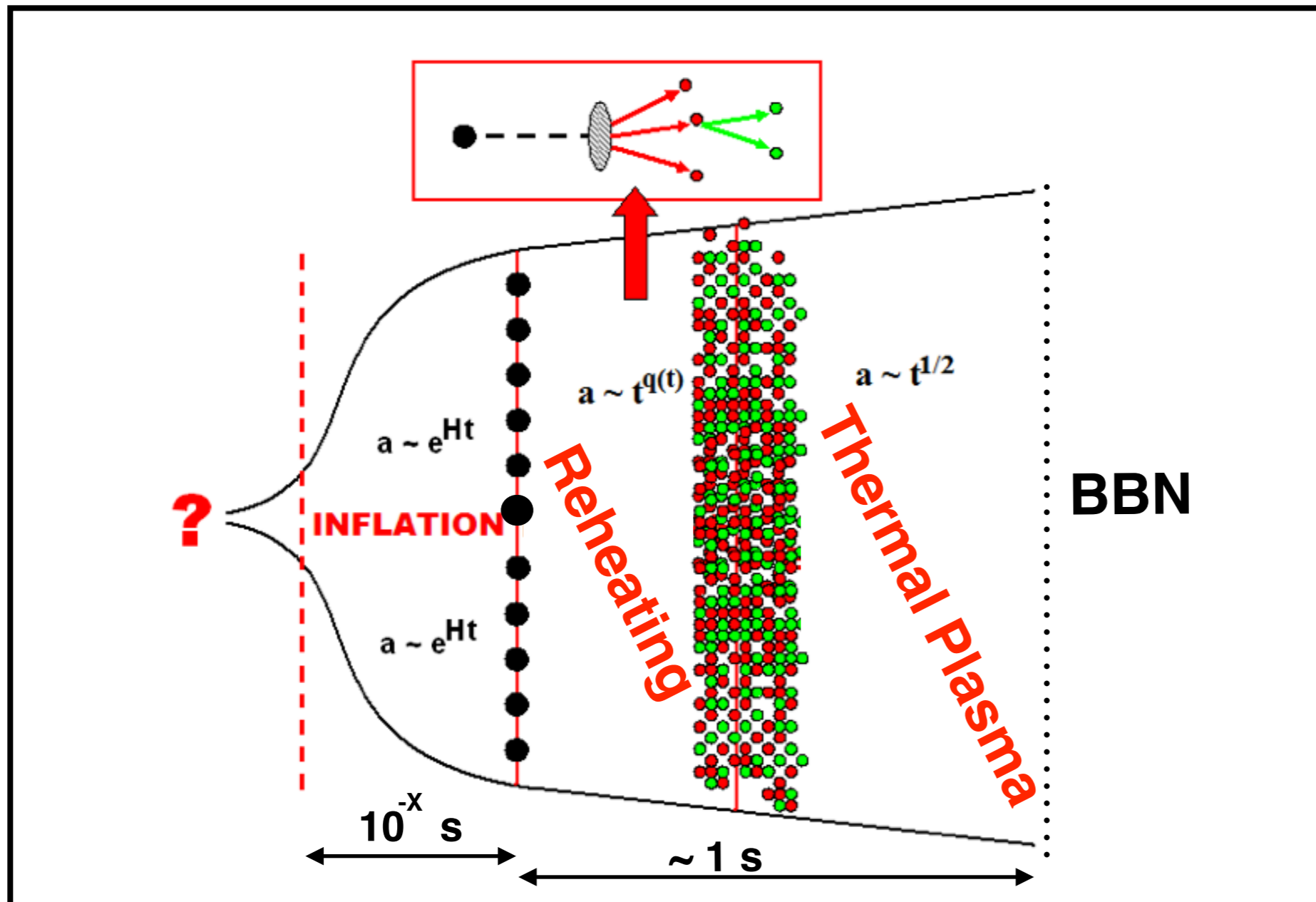
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Thermal Ensemble of Relativistic Particle Species



Connection between Particles and Inflationary Sector ?

- * **SM Portals ?**
- * **Mediator fields ?**
- * **Weak coupling !**

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}} \left\{ \underbrace{(\partial\phi)^2 - V_{\text{inf}}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2}m_{pl}^2 R}_{\text{gravity (GR)}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter}} - \cancel{g^2\chi^2\phi^2} \right\}$$

Need to excite matter
(to reheat the Universe)

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**Need to excite matter
(to reheat the Universe)**



$$\rho_{\text{rad}} = \delta \times 10^{-2} H_*^4$$

$$\delta \lesssim 1,$$

**Inflation does
the job !**

$$\delta \sim \begin{cases} \mathcal{O}(m^2/H_*^2) & , \text{ quantum - fluct. (light dof)} \\ \mathcal{O}(1) & , \text{ quantum - fluct. (self - interact.)} \\ \mathcal{O}(1)/\xi & , \text{ non - min grav, } \xi \gtrsim 1 \\ \mathcal{O}(|1 - 6\xi|^2) & , \text{ non - min grav, } |1 - 6\xi| \lesssim 1 \end{cases}$$

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

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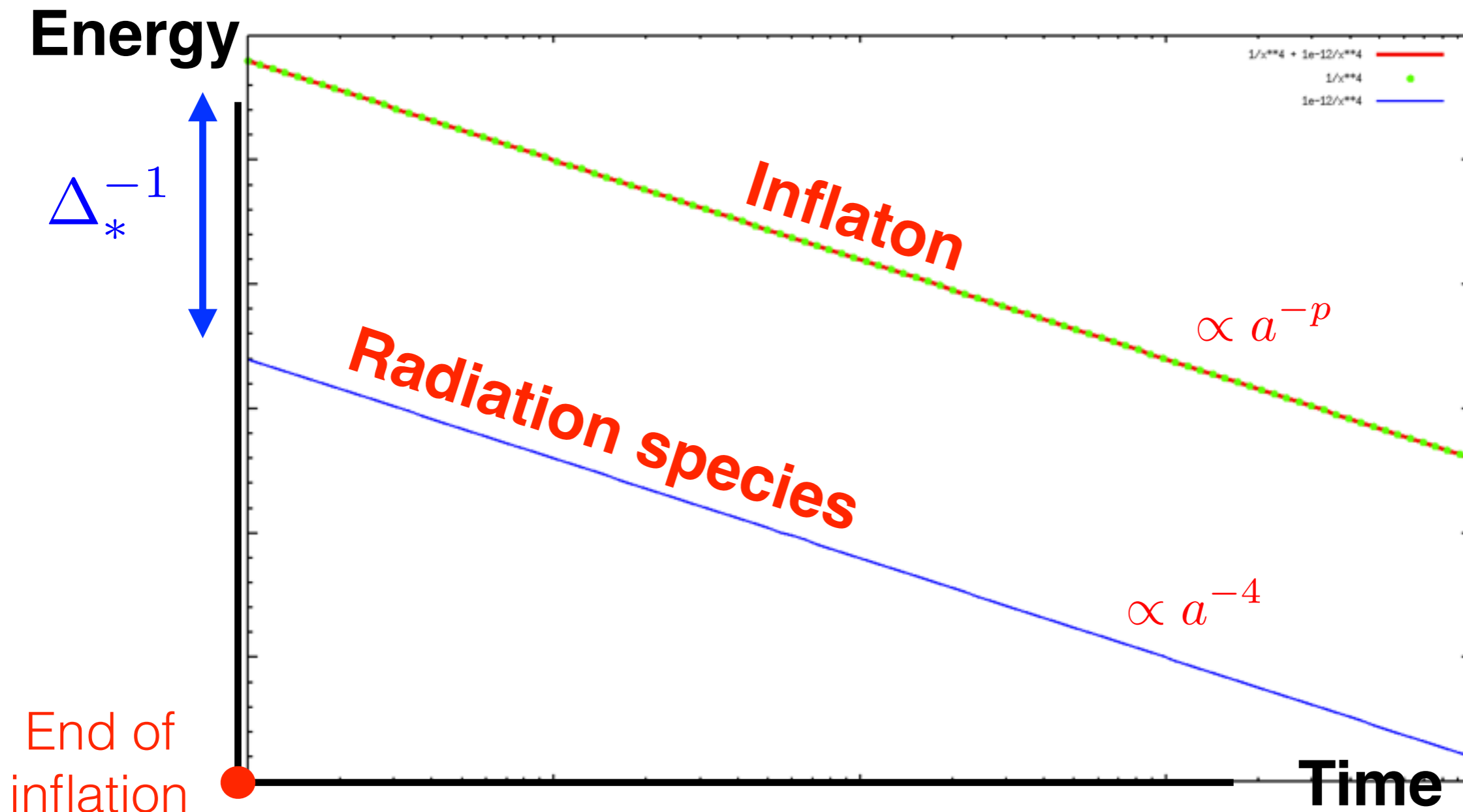
$$\Delta_* \equiv \frac{\rho_{\text{rad}}}{3m_p^2 H_*^2} = \frac{\delta}{300} \left(\frac{H_*}{m_p} \right)^2 \ll 1$$

Initial energy fraction radiation-to-total (end of inflation)

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\rho_{\text{rad}}^* \ll H_*^2 m_{\text{pl}}^2$$

Radiation excited but subdominant



Rad. Excited

$$\rho_{\text{rad}}^* \ll H_*^2 m_{pl}^2$$

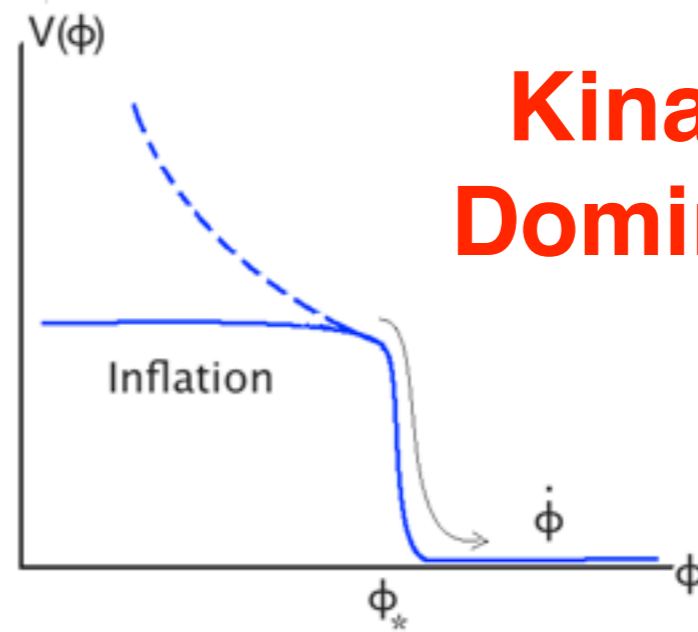
Rad. produced,
but subdominant



Rad. Excited

$$\rho_{\text{rad}}^* \ll H_*^2 m_{\text{pl}}^2$$

Rad. produced,
and dominant !



Kination-Domination

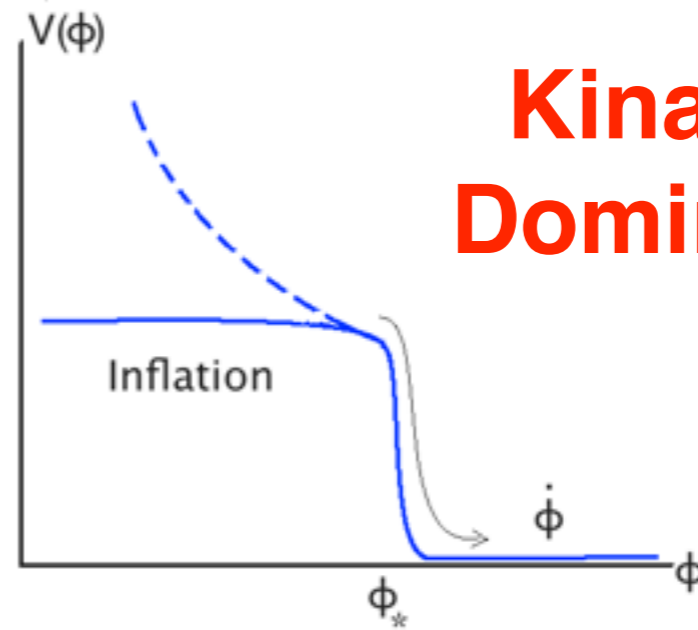
$$\text{Energy} \propto \frac{1}{a^6}$$

[Ford '86,
Spokoiny '93]

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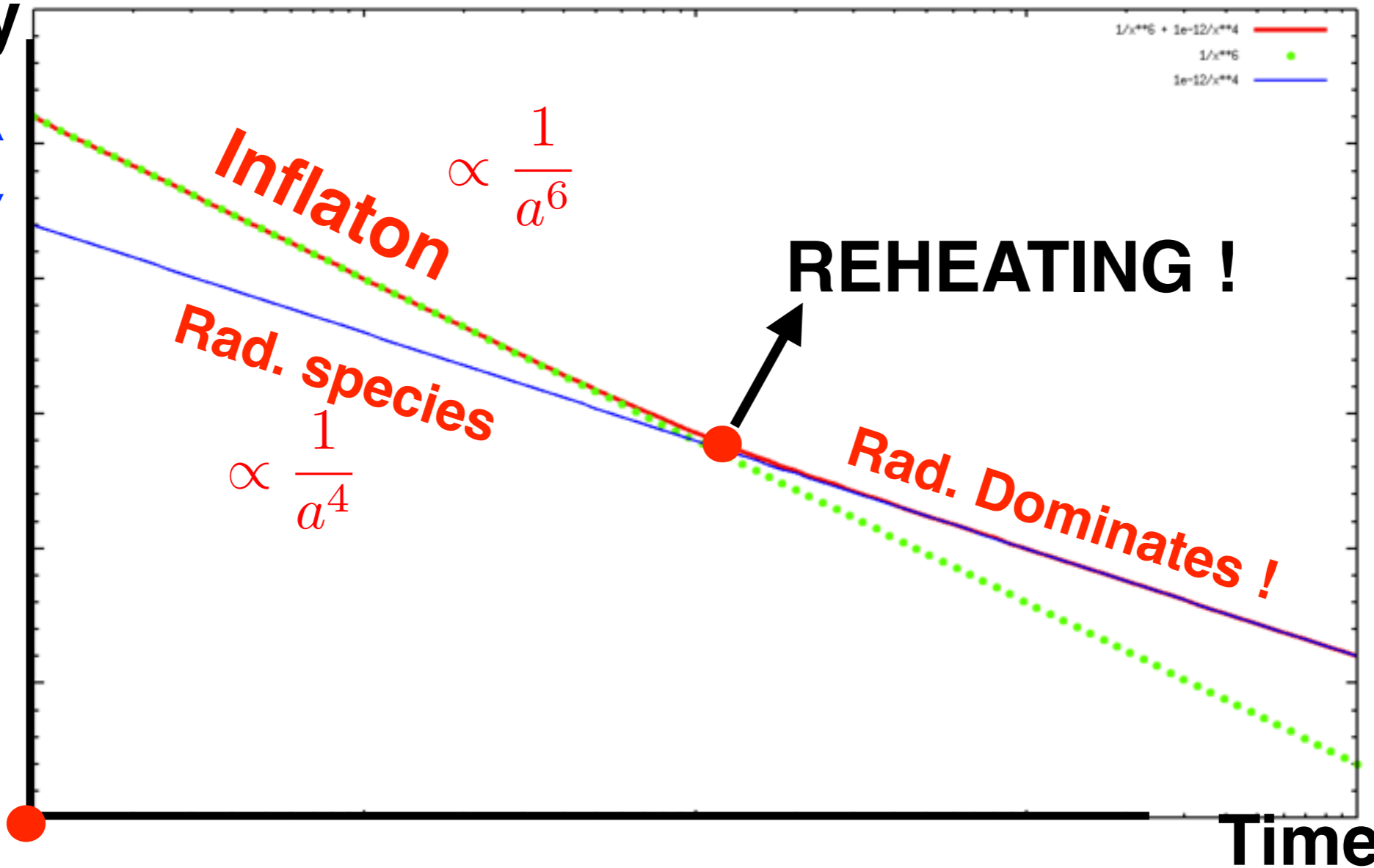
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Energy

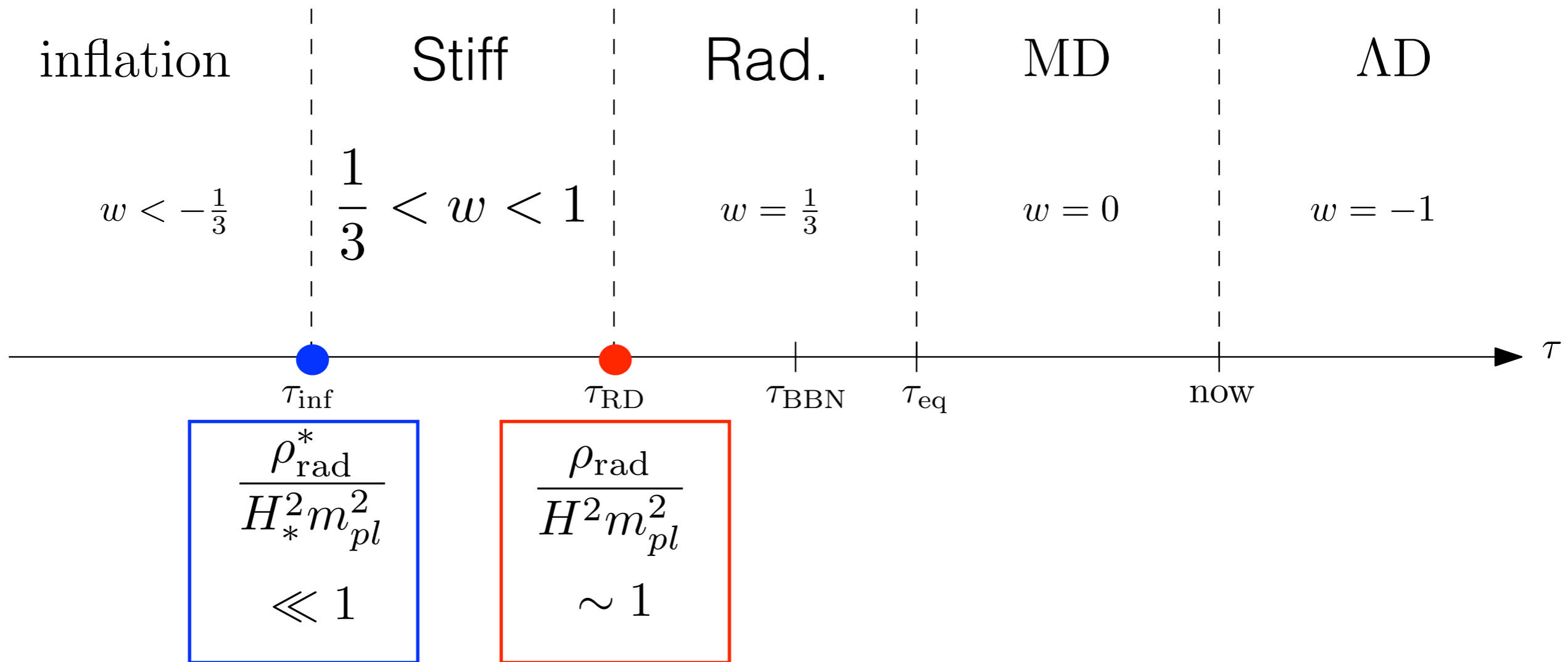
$$\Delta_*^{-1}$$



End of
inflation

GRAVITATIONAL REHEATING

Ford '86, Spokoiny '93, Joyce '97,
Giovannini '98/99, Peebles & Vilenkin '98,
[00' ... '18], DGF & Tanin '18/19

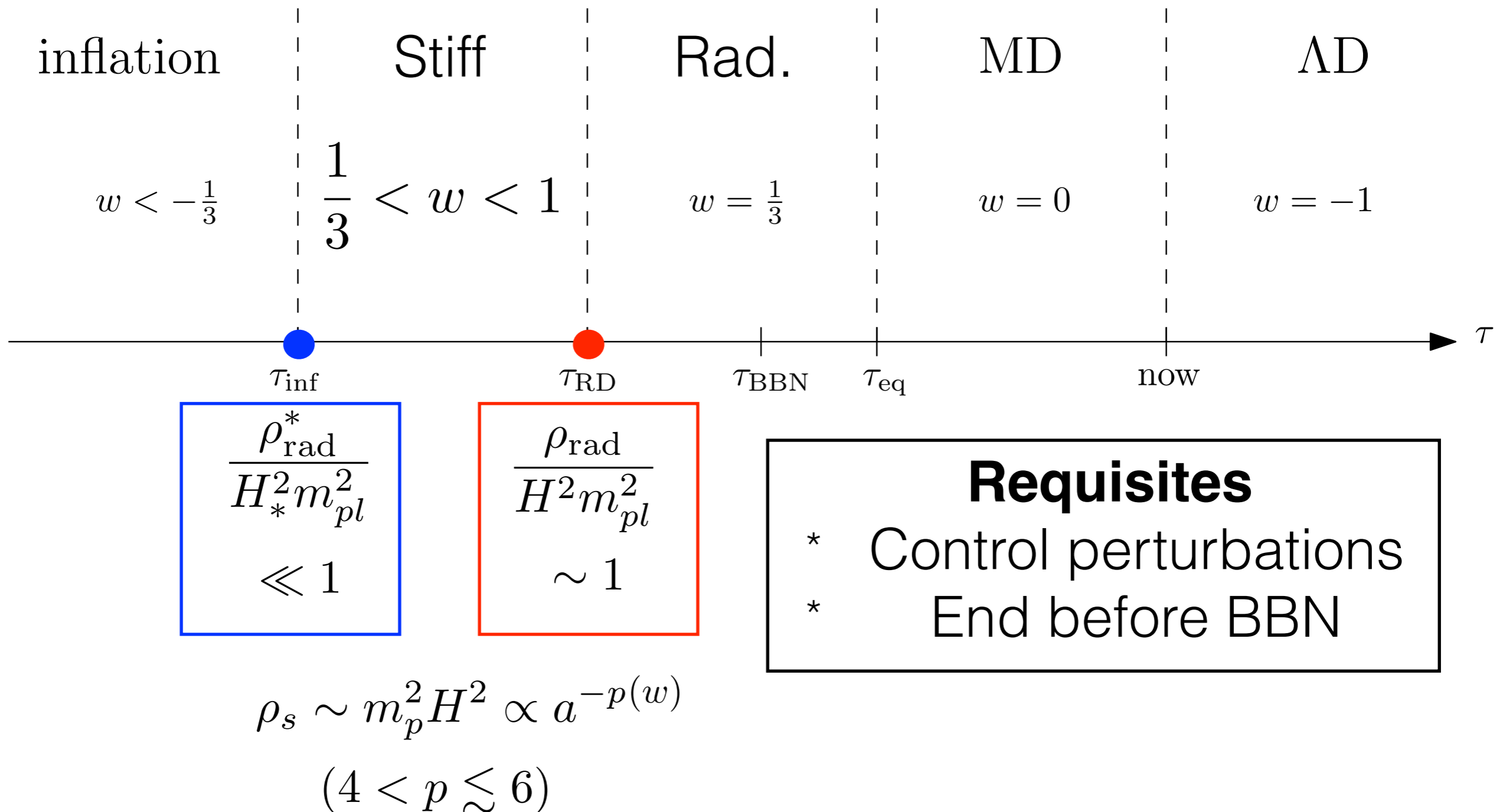


$$\rho_s \sim m_p^2 H^2 \propto a^{-p(w)}$$

$$(4 < p \lesssim 6)$$

GRAVITATIONAL REHEATING

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$$1/3 < w_s \lesssim 1$$

Stiff Eq. of State

Requisites

- * Control perturbations
- * End before BBN



We done ?

GRAVITATIONAL REHEATING

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Stiff Eq. of State

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We done ? Nope

Enhancement of inflationary Gravitational Waves (GW) !

[Giovannini '98/99, ..., Boyle & Buonanno '07, ..., DGF & Tanin '18]

Inflationary GW background

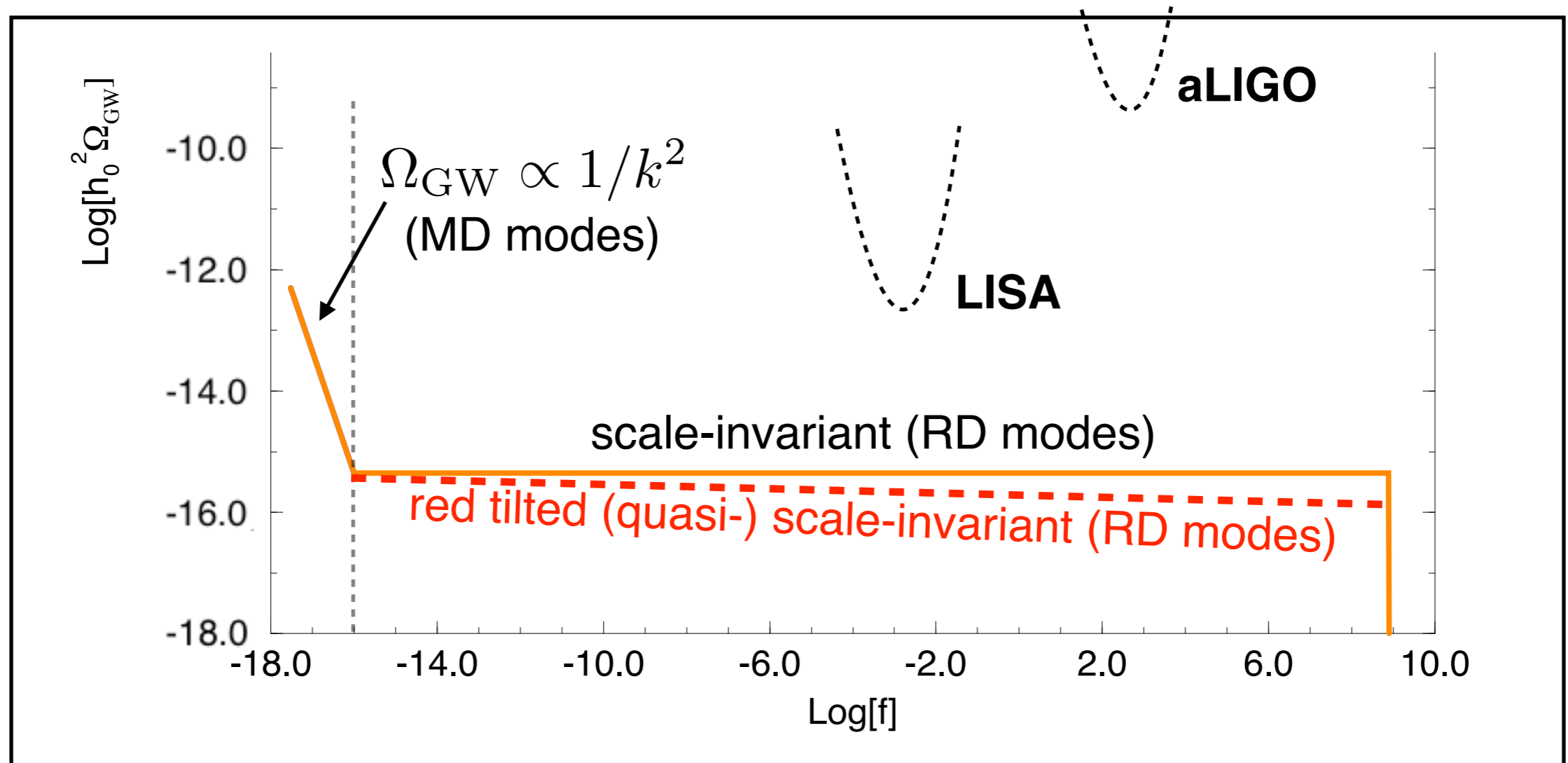
$$\Omega_{\text{GW}}^{(o)}(f) \equiv \frac{1}{\rho_c^{(o)}} \left(\frac{d \log \rho_{\text{GW}}}{d \log k} \right)_o = \underbrace{\frac{\Omega_{\text{Rad}}^{(o)}}{24}}_{\text{Transfer Funct.: } T(k) \propto k^0 \text{ (RD)}} \Delta_{h_*}^2(k)$$

$$\Delta_h^2(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p} \right)^2 \left(\frac{k}{aH} \right)^{n_t}$$

$$n_t \equiv -2\epsilon$$

energy scale

Transfer Funct.: $T(k) \propto k^0$ (RD)



Inflationary GW background

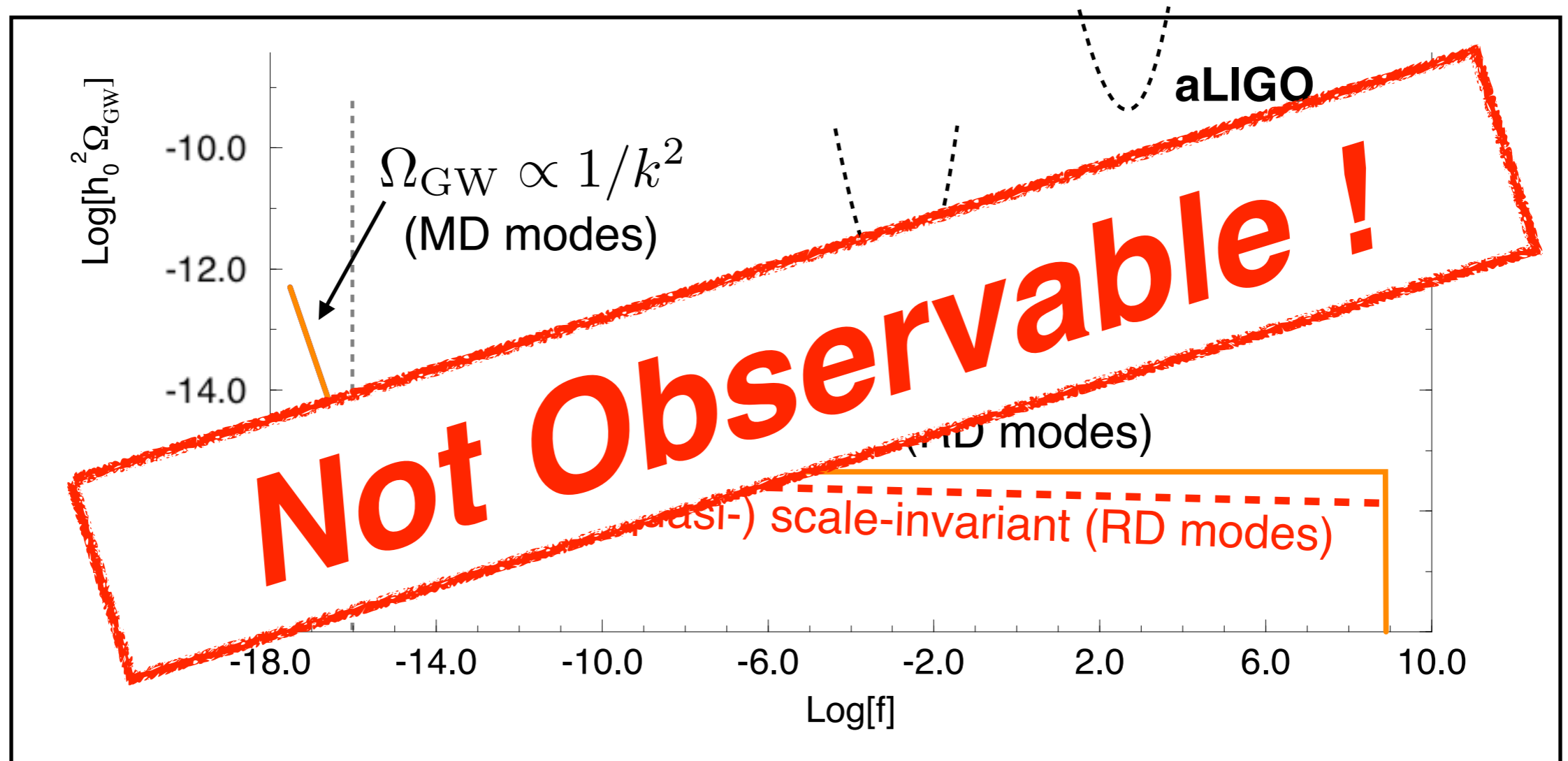
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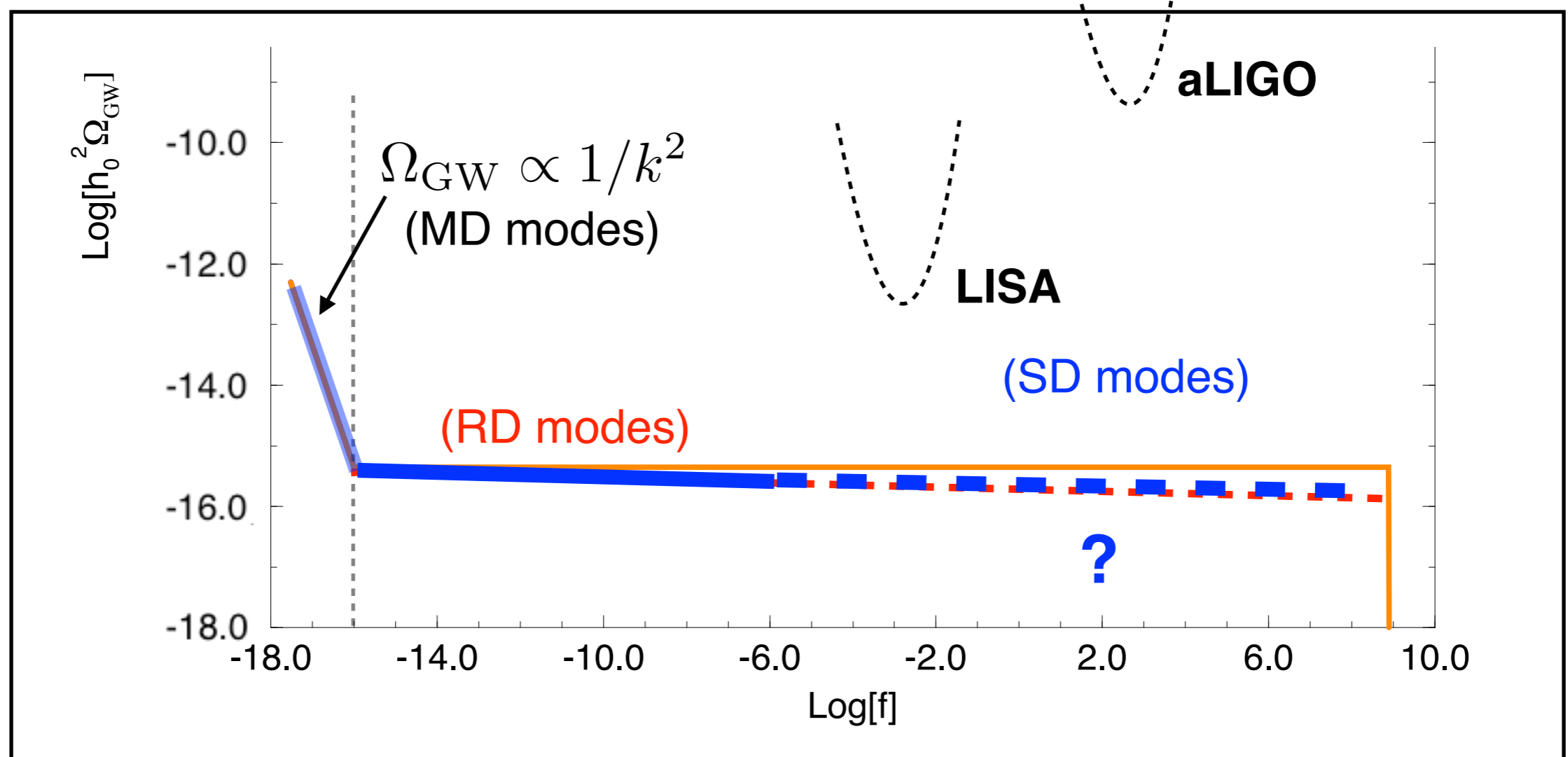
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Transfer Funct.: $T(k) \propto k^0$ (RD)

Stiff Period: $T(k) \propto k^{2 \frac{(w_s - 1/3)}{(w_s + 1/3)}}$



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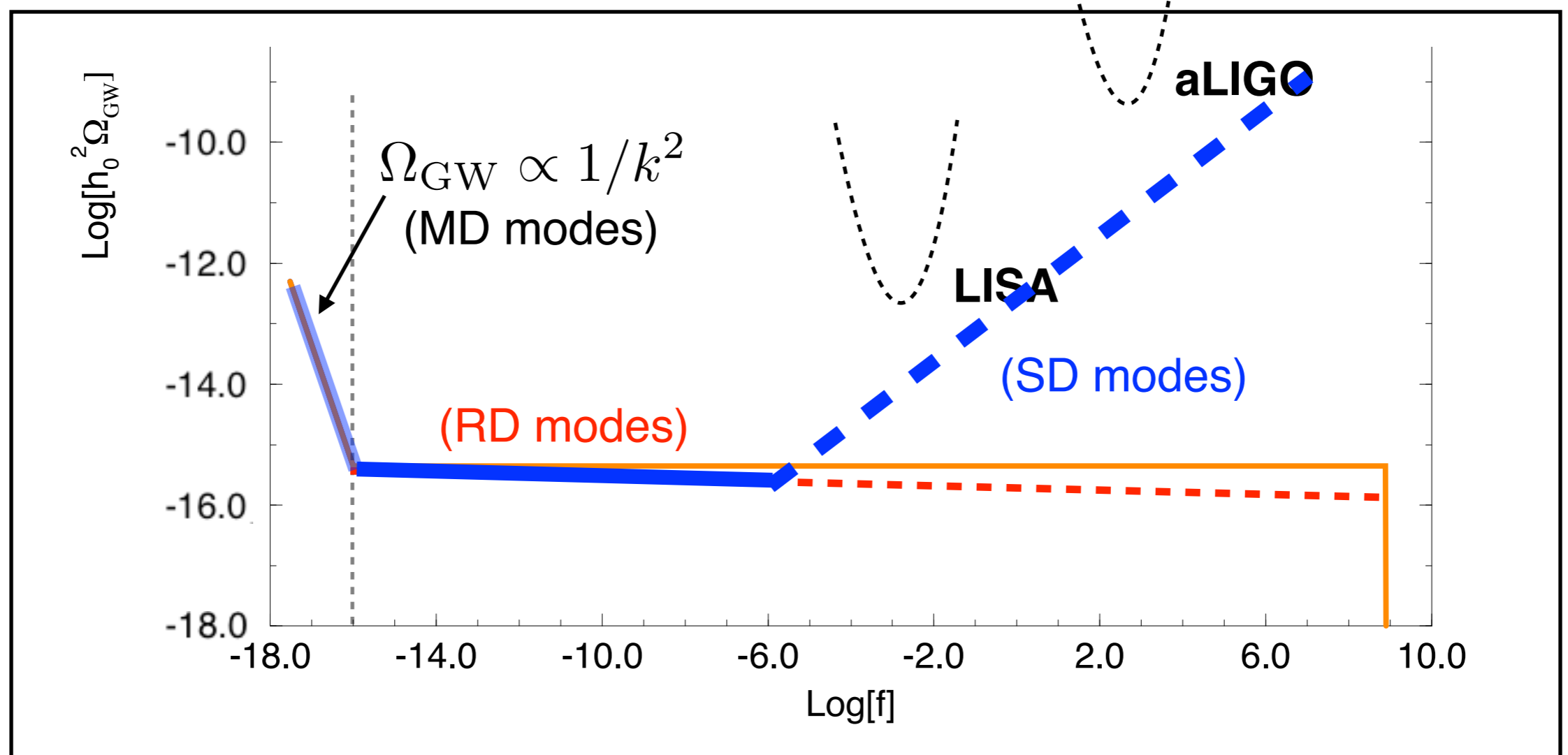
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Inflationary GW background

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**Rad.
Plateau**

Transfer Funct. Stiff Period
Window \times power-law

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

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**Not just Grav. RH !
Generic for Stiff Era**

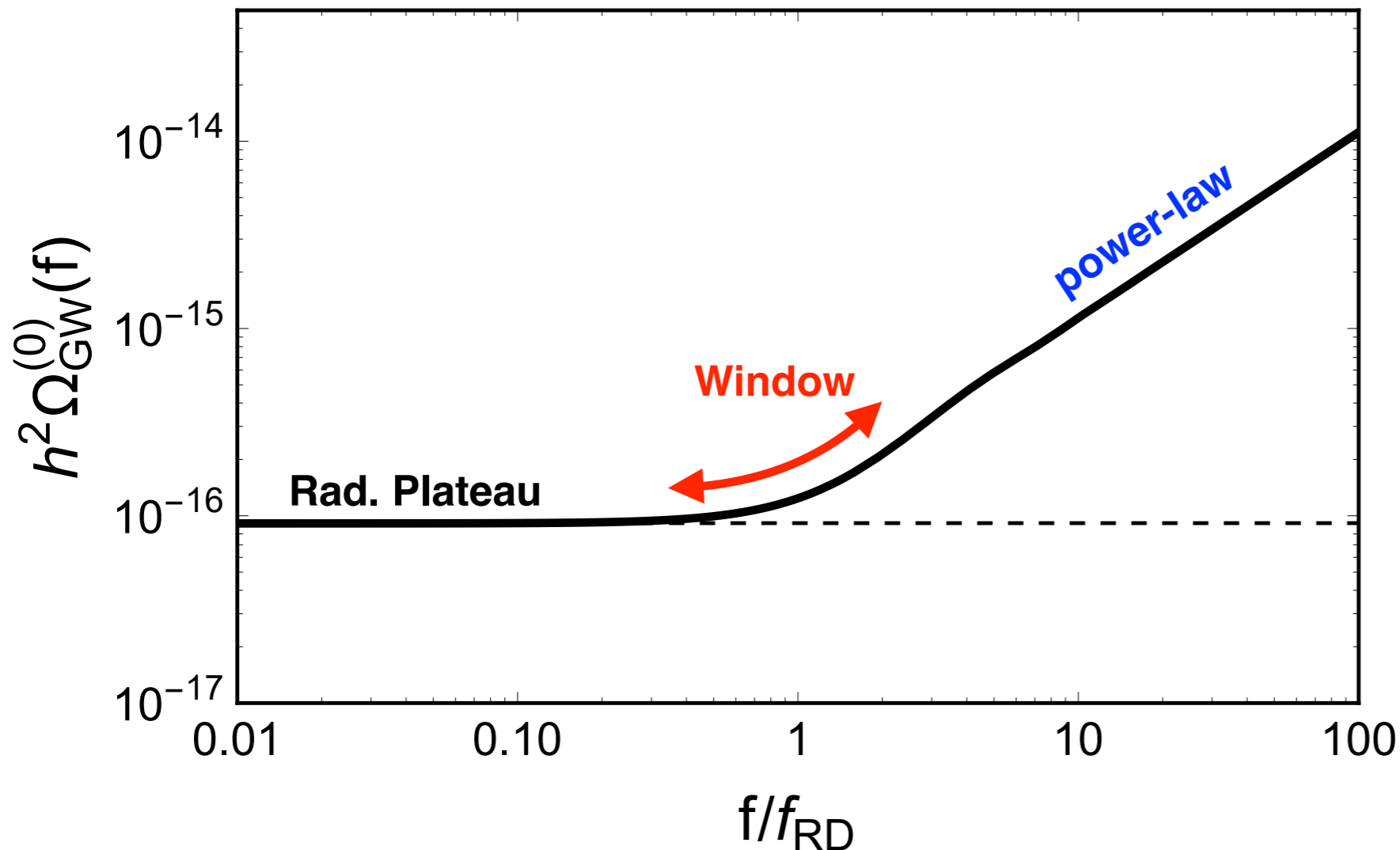
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**Blue-tilted GW
Background**

$$n_t \equiv \frac{d \log \Omega_{\text{GW}}^{(0)}}{d \log f} = 2 \left(\frac{3\omega_s - 1}{3\omega_s + 1} \right) > 0$$

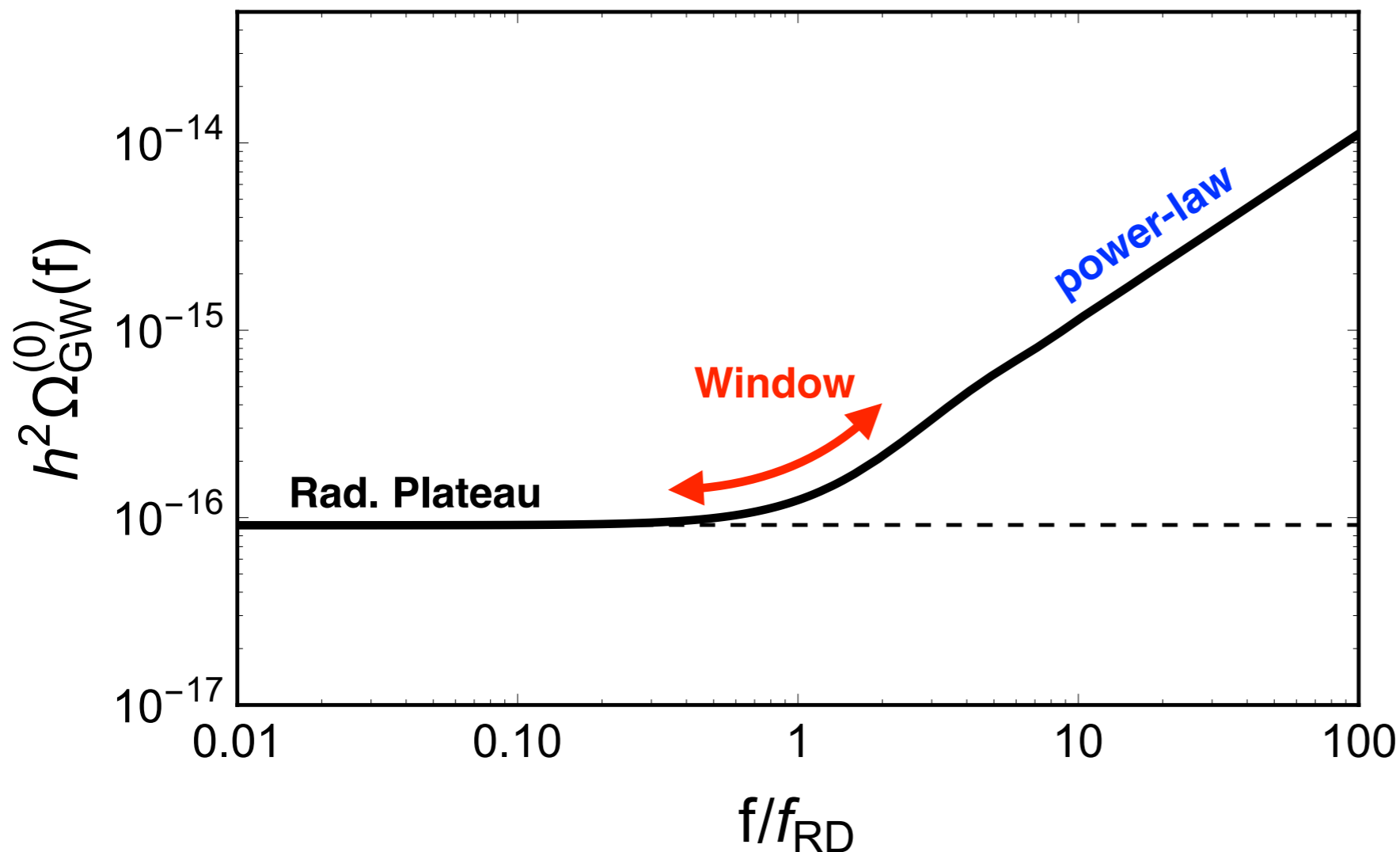
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Plateau

Transfer Funct. Stiff Period
Window \times power-law

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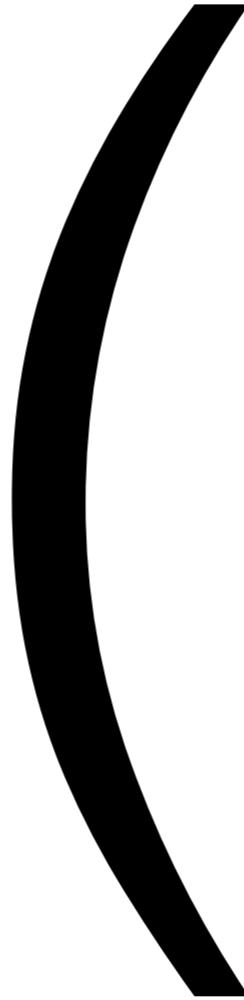
Why Blue-tilted?

$$\rho_{\text{GW}} \propto \frac{1}{a^4}$$

$$\rho_s \propto \frac{1}{a^{3(1+w_s)}}$$

$$\frac{\rho_{\text{GW}}}{\rho_s} \propto a^{(3w_s - 1)}$$

Growing funct.
for $w_s > 1/3$



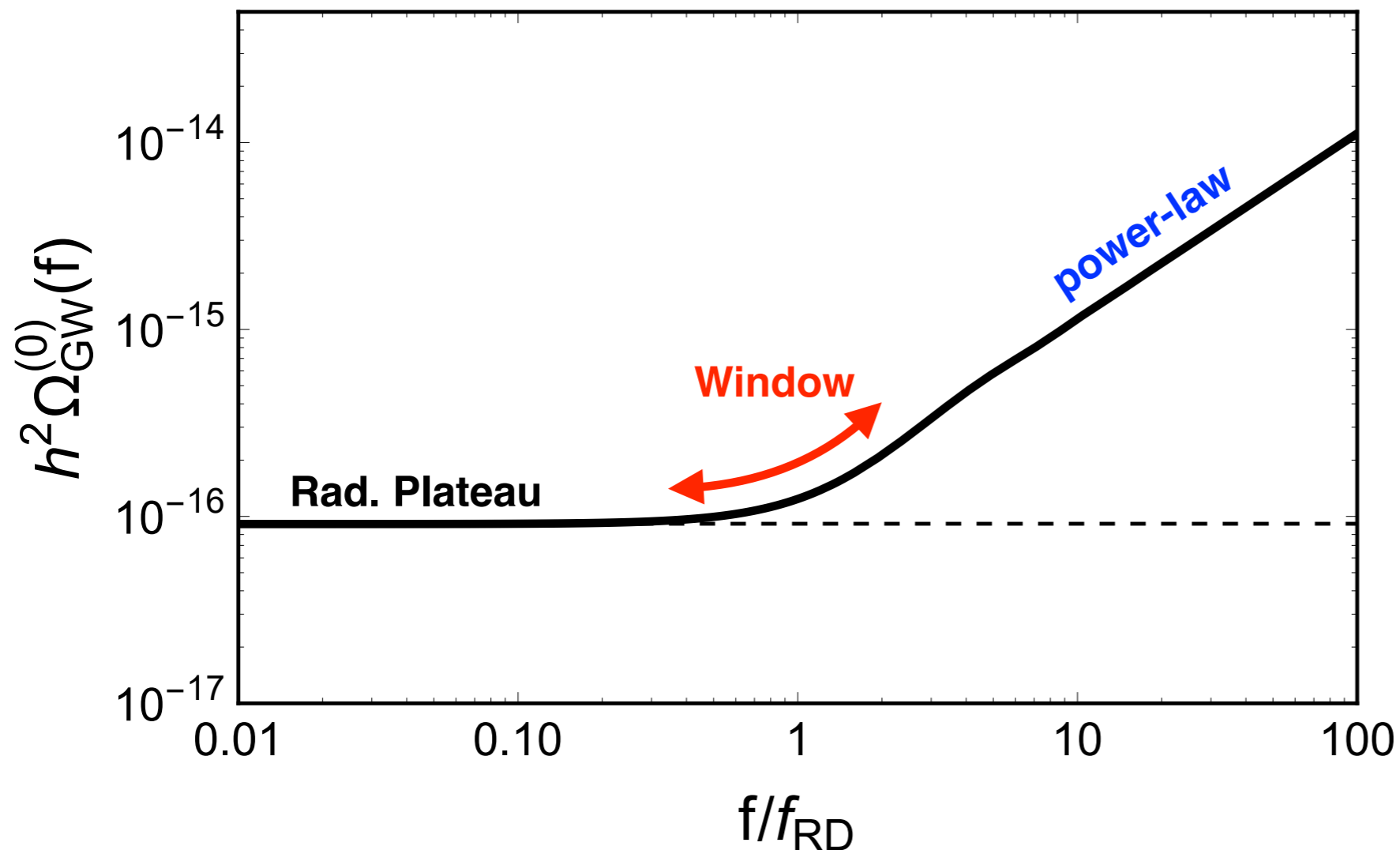
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Rad.
Plateau

Transfer Funct. Stiff Period
Window \times power-law



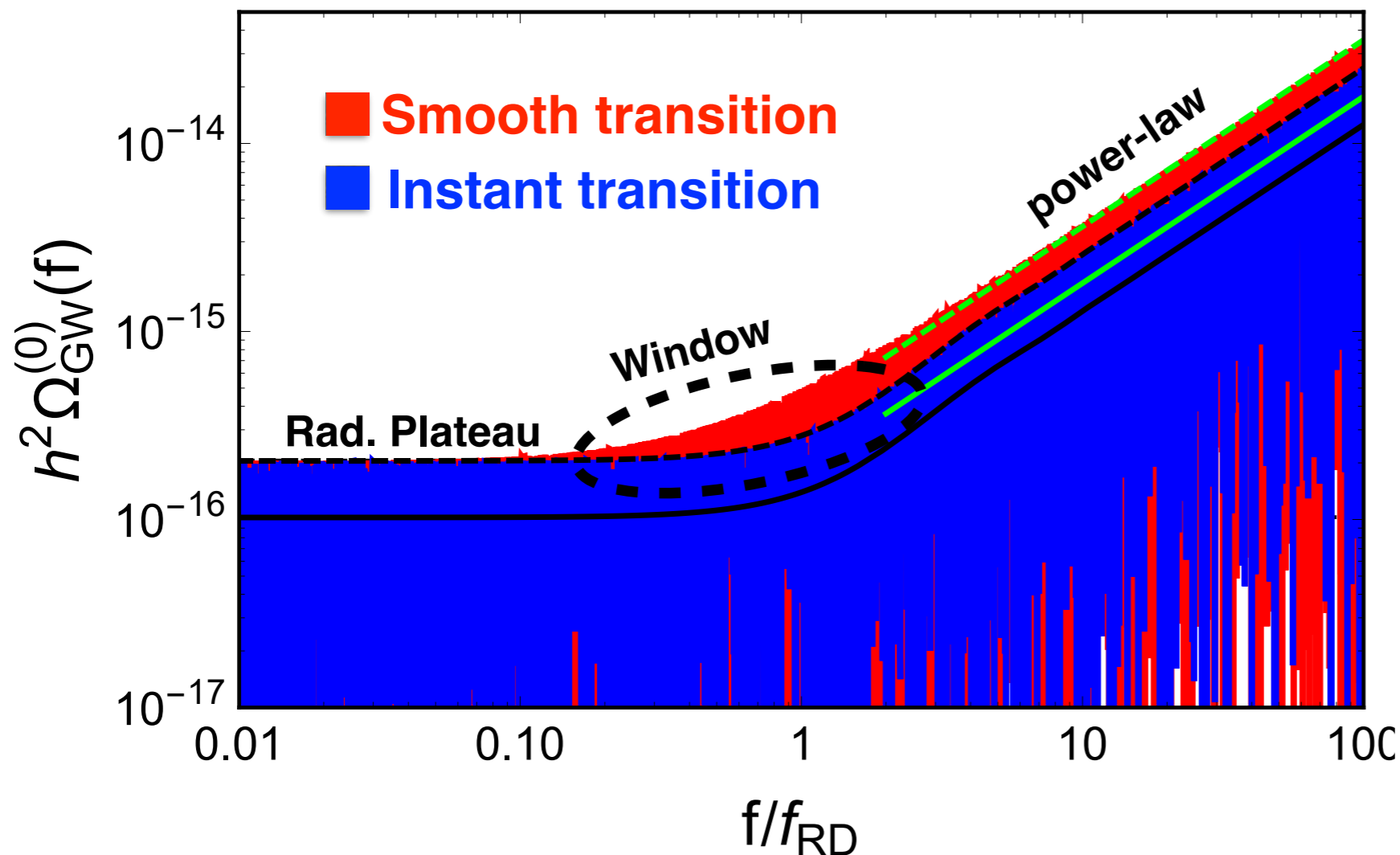
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Rad.
Plateau

Transfer Funct. Stiff Period
Window \times power-law

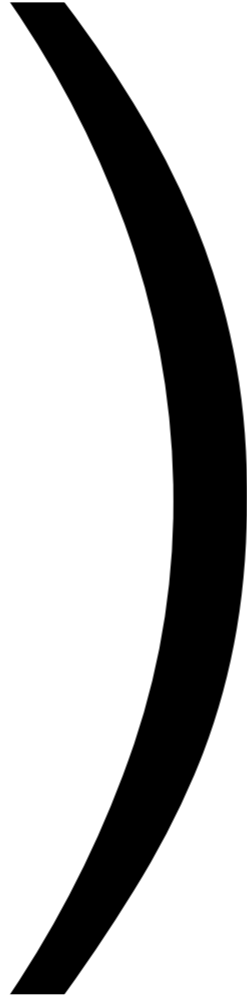
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**Real signal:
highly oscillatory**

**Stochastic Signal:
average measurement**

$$\langle \dot{h}_{ij}(f) \dot{h}_{ij}(f) \rangle = \mathcal{P}_h(f)$$



Inflationary GW background

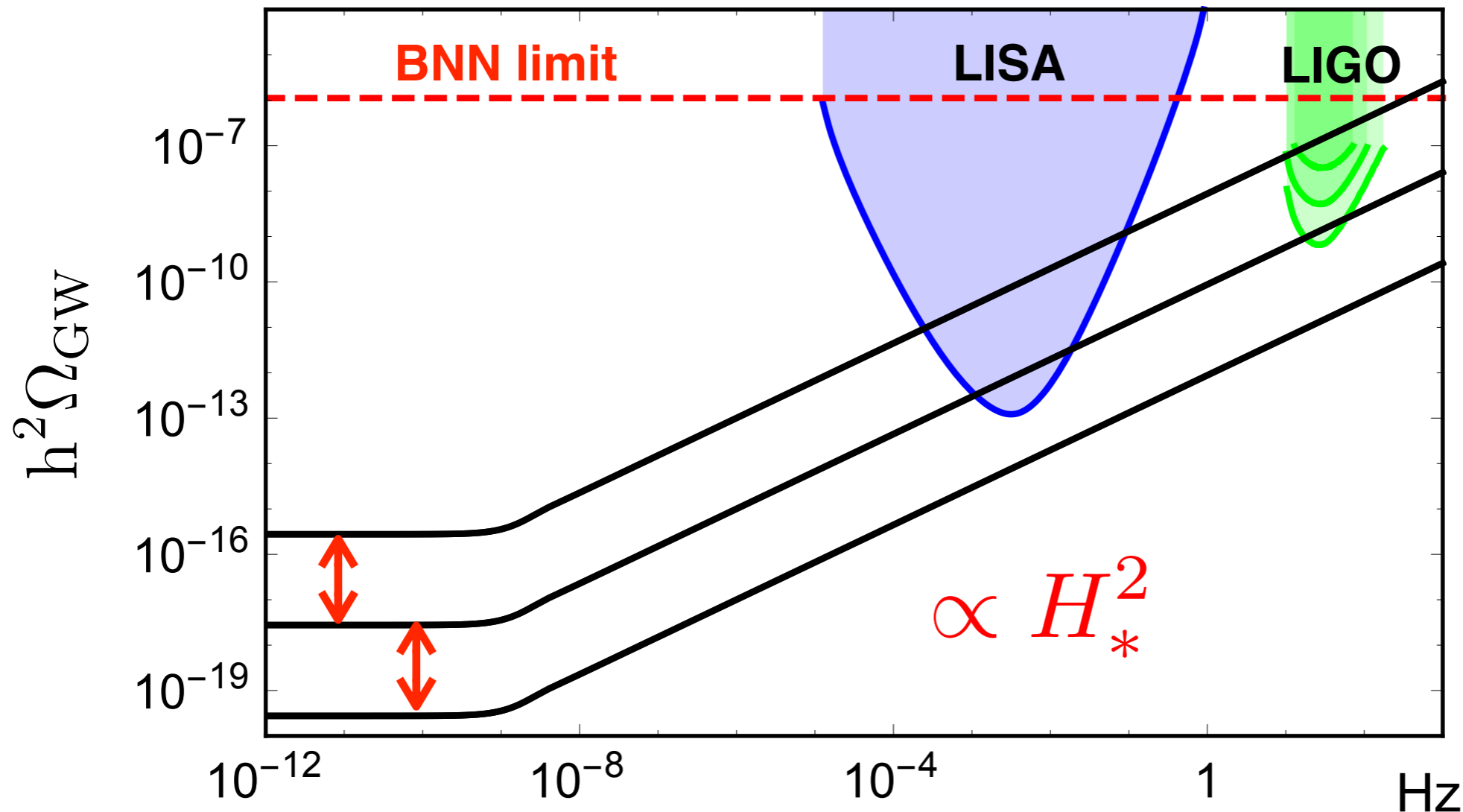
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Rad.
Plateau

Transfer Funct. Window
Stiff Period
Window x power-law

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

$$n_t \equiv -2\epsilon$$



Overall Amplitude
(Energy Scale Inflation)

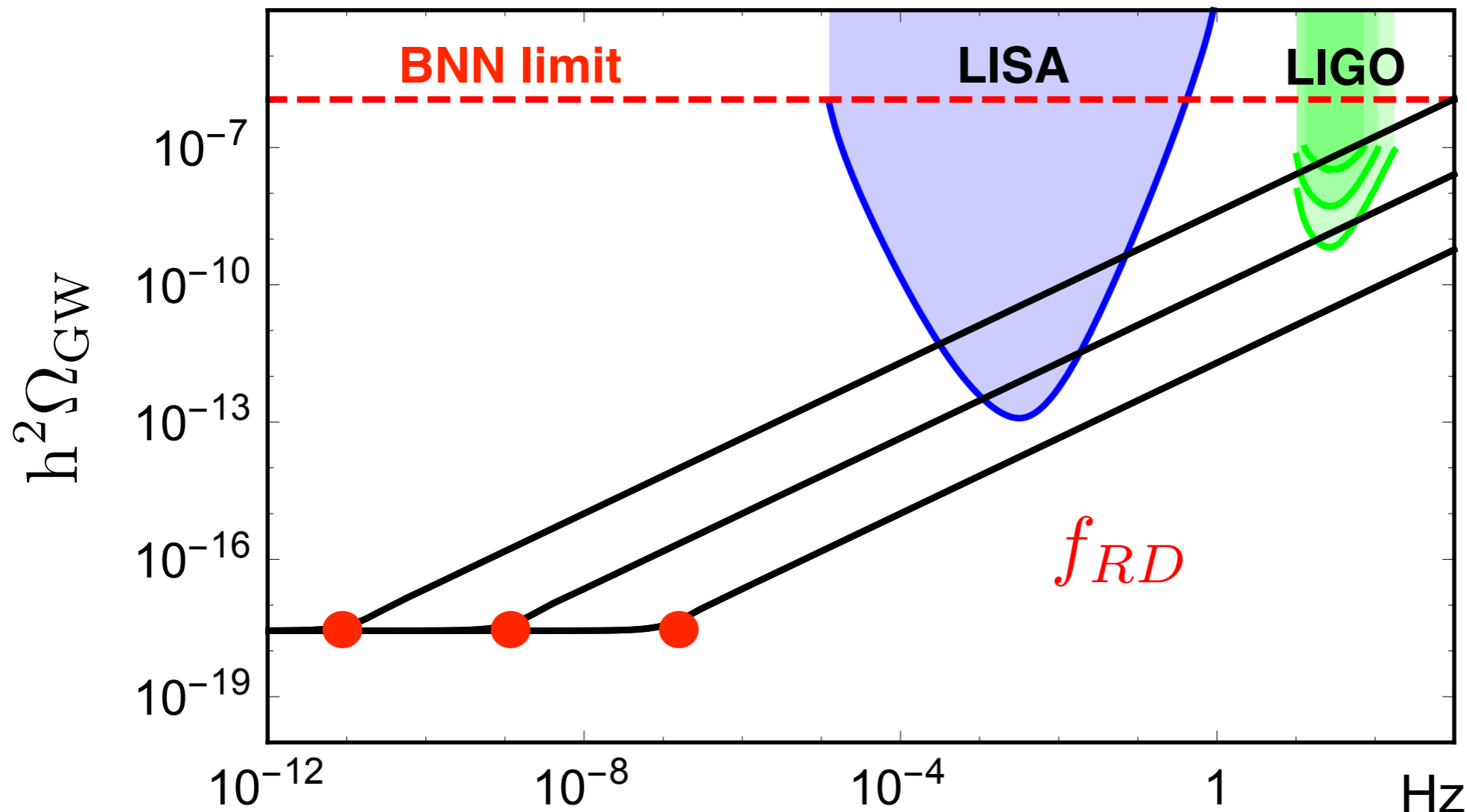
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Rad. Plateau
Transfer Funct. Window
Stiff Period
Window x power-law

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Freq. RD

$$k_{\text{RD}} = a_{\text{RD}} H_{\text{RD}}$$

$$f_{\text{RD}} \equiv k_{\text{RD}} / (2\pi a_0)$$

SD-to-RD transition

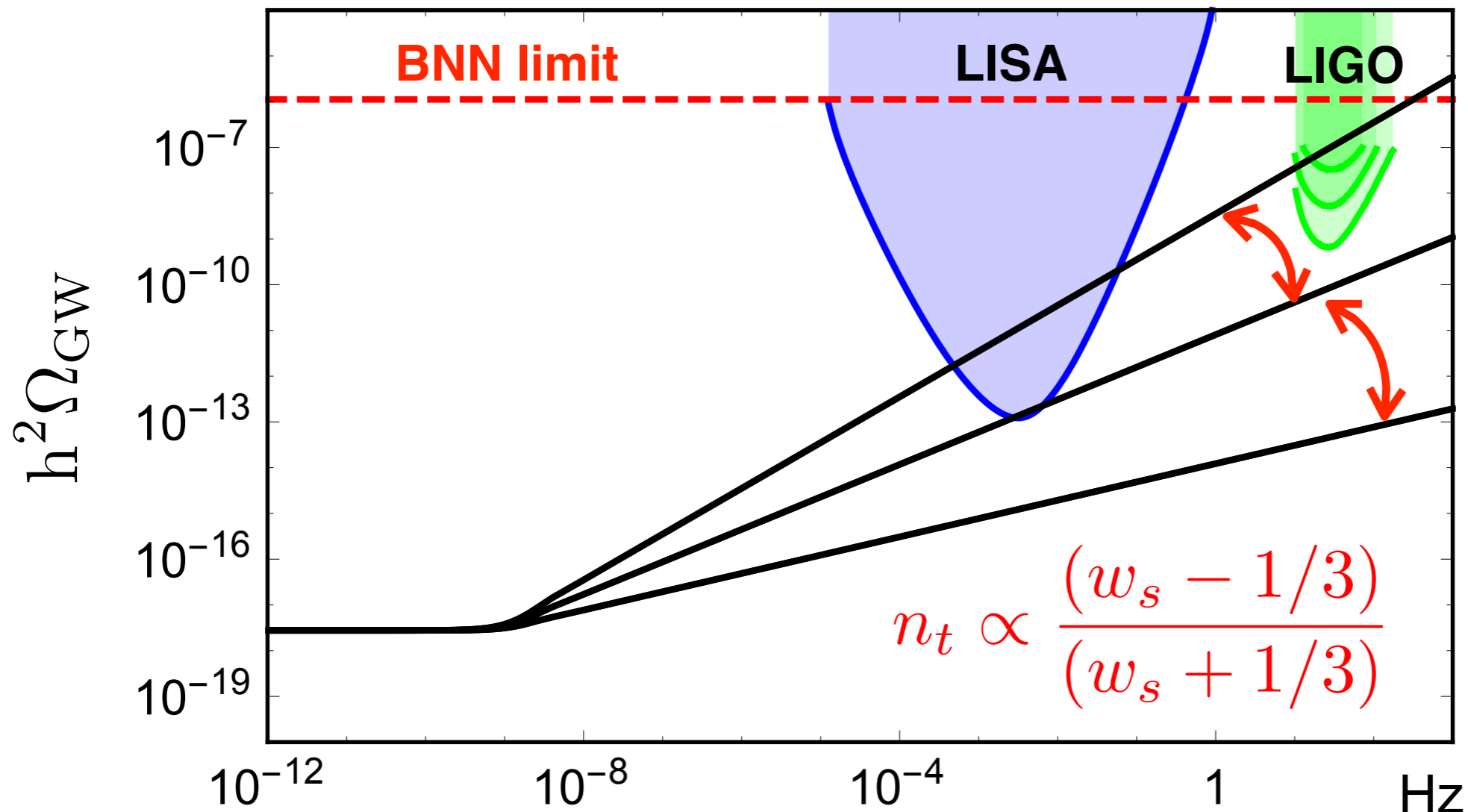
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Rad. Plateau
Transfer Funct. Window
Stiff Period
Window \times *power-law*

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**Rad.
Plateau**

Transfer Funct. Stiff Period
Window \times *power-law*

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$$\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{\text{RD}}})$$

**Energy
Scale
Inflation**

**EoS
Stiff
Period**

**Duration
Stiff
Period**

GW background $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$
Observability @ LISA (~ 2034) Energy Scale EoS Stiff Duration Stiff

GW background

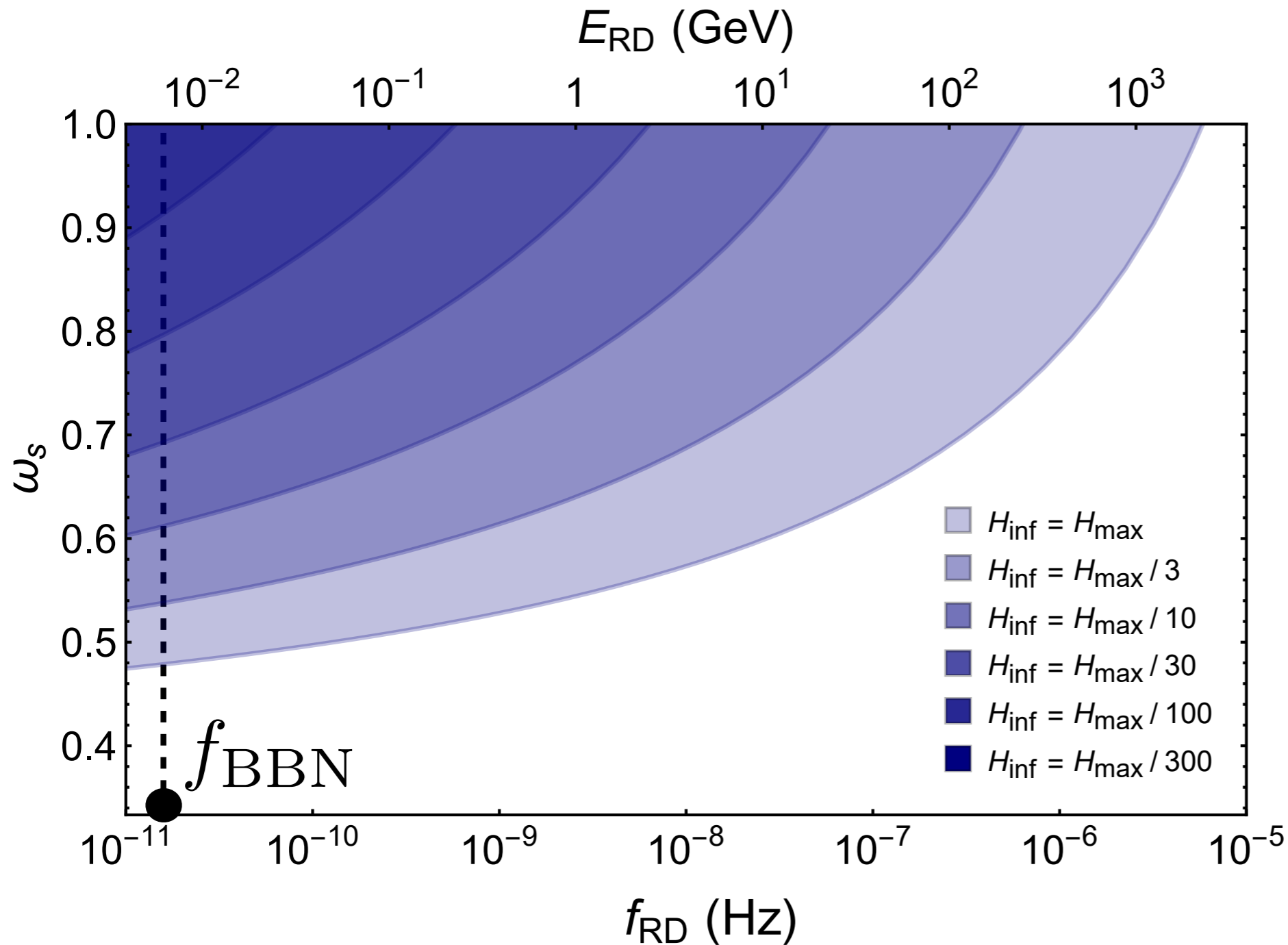
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Observability @ LISA (~ 2034)

Energy
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EoS
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GW background

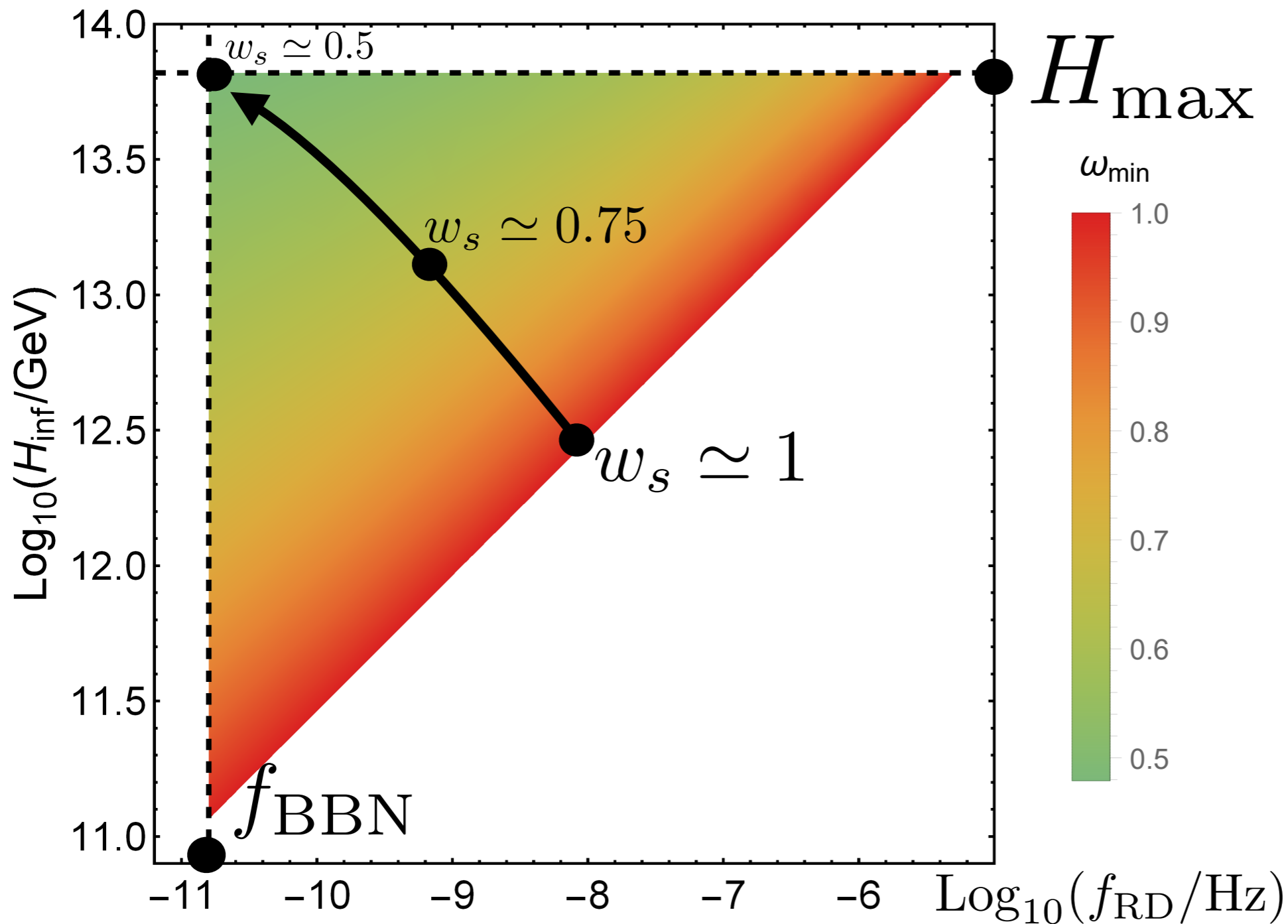
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Observability @ LISA (~ 2034)

Energy
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GW background $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_S}, \underline{f_{RD}})$

Observability @ LISA (~ 2034) Energy Scale EoS Stiff Duration Stiff

$$9.1 \times 10^{10} \text{ GeV} < H_{\text{inf}} < 6.6 \times 10^{13} \text{ GeV}$$

$$0.47 < w_S < 1$$

$$10^{-11} \text{ Hz} \lesssim f_{\text{RD}} < 4.6 \times 10^{-6} \text{ Hz}$$

$$10^{-3} \text{ GeV} \lesssim E_{\text{RD}} < 5.91 \times 10^3 \text{ GeV}$$

Significant fraction of param. space observable !

GW background $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$
Observability @ LISA (~ 2034) Energy Scale EoS Stiff Duration Stiff

But ...

GW background $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$
Observability @ LIGO (today) Energy Scale EoS Stiff Duration Stiff

GW background

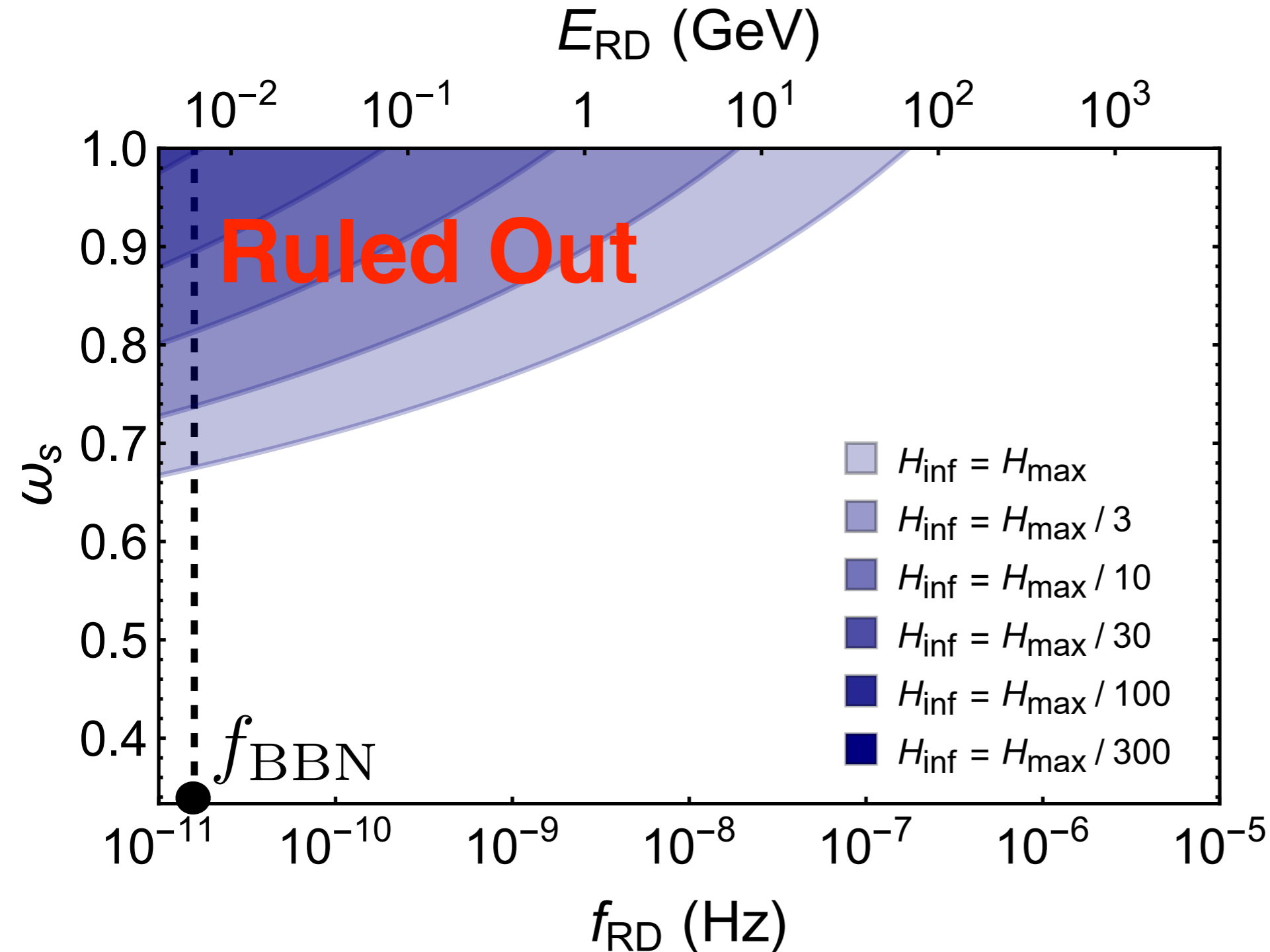
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Observability @ LIGO (today)

Energy
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GW background

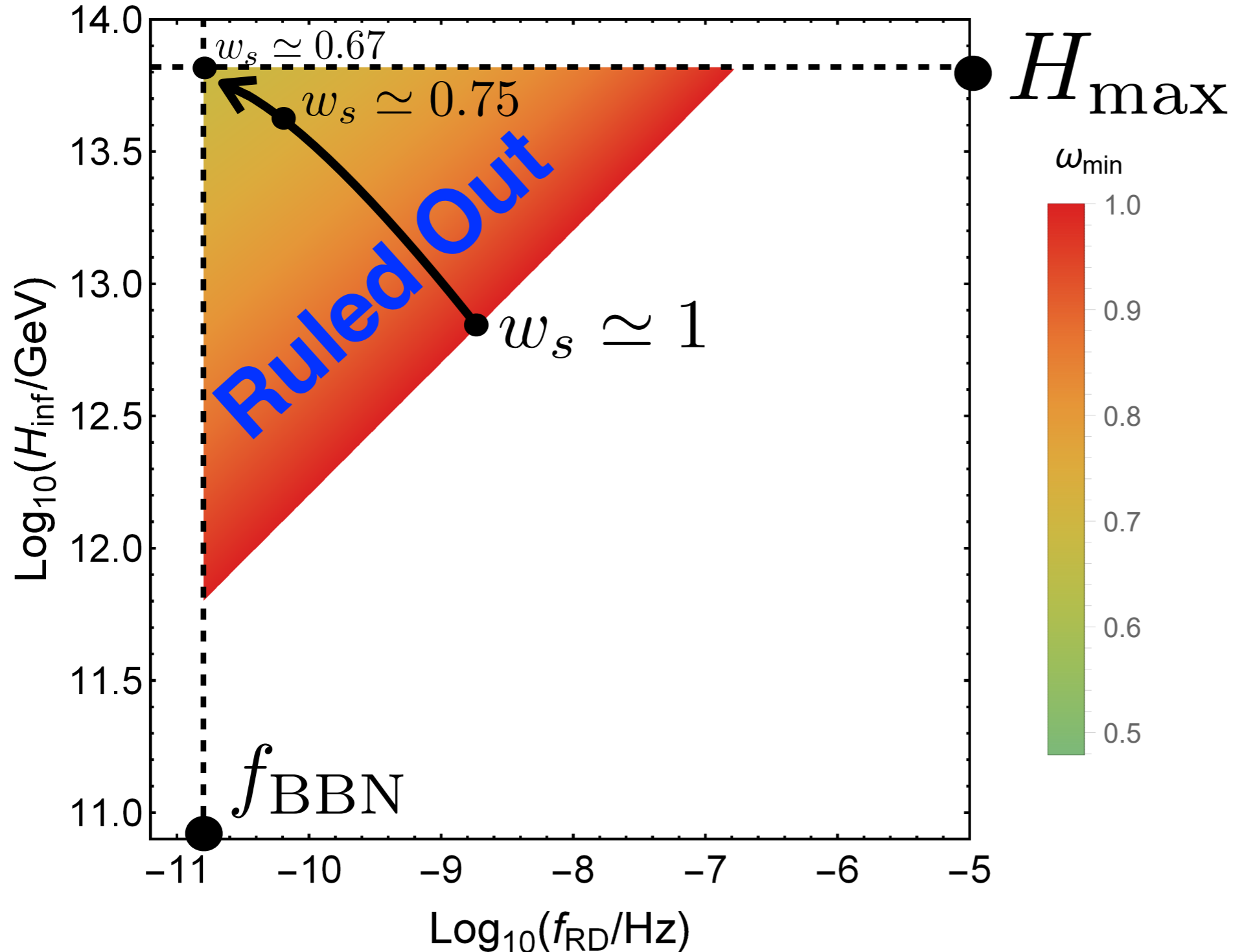
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Observability @ LIGO (today)

Energy
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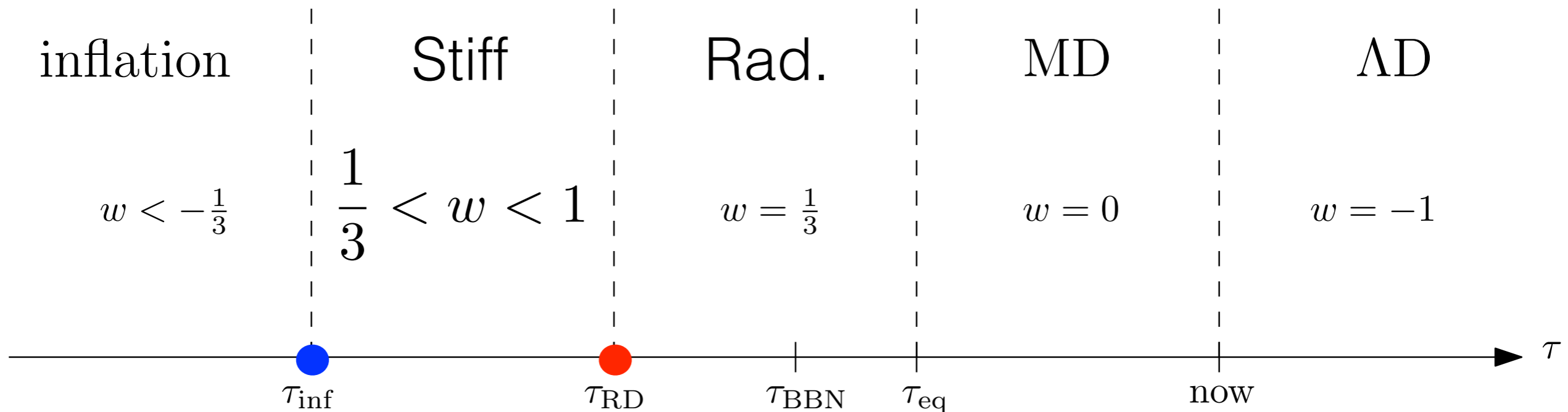
Duration
Stiff



LIGO
reduces
parameter
space
probe-able
by LISA !

Let's look at
consistency
of scenarios
before

BACK to ... GRAVITATIONAL REHEATING



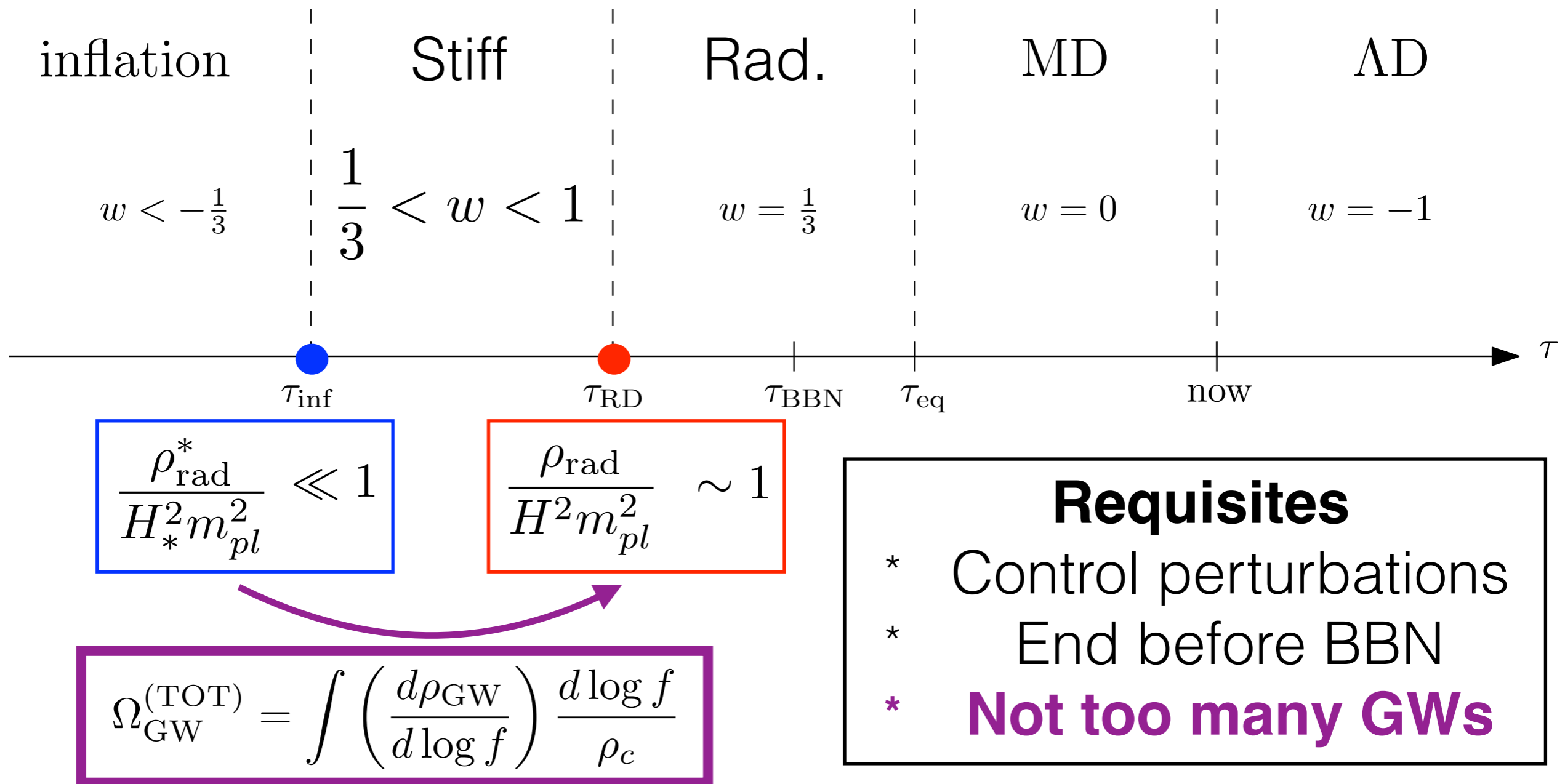
$$\frac{\rho_{\text{rad}}^*}{H_*^2 m_{\text{pl}}^2} \ll 1$$

$$\frac{\rho_{\text{rad}}}{H^2 m_{\text{pl}}^2} \sim 1$$

- Requisites**
- * Control perturbations
 - * End before BBN

$$\Omega_{\text{GW}}^{(\text{TOT})} = \int \left(\frac{d\rho_{\text{GW}}}{d \log f} \right) \frac{d \log f}{\rho_c}$$

BACK to ... GRAVITATIONAL REHEATING



BIG BANG NUCLEOSYNTHESIS

Expansion rate (Rad. Dom): ~ Extra relativistic species

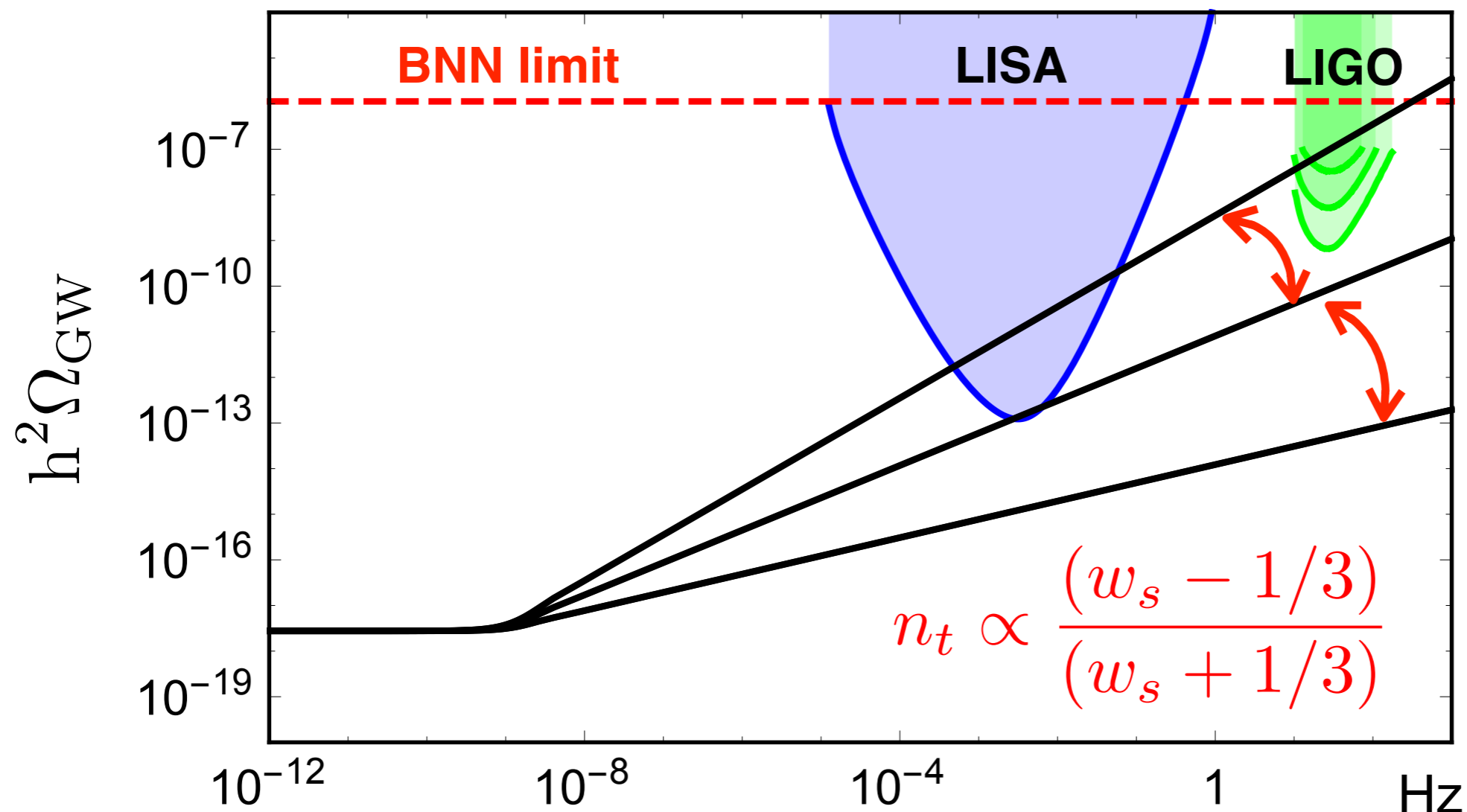
$$\int \frac{df}{f} h^2 \Omega_{\text{GW}}(f) \leq 1.12 \times 10^{-6}$$

$$\Delta N_\nu = 0.2 \text{ (95\% C.L.) [latest CMB]}$$

BIG BANG NUCLEOSYNTHESIS

Expansion rate (Rad. Dom): \sim Extra relativistic species

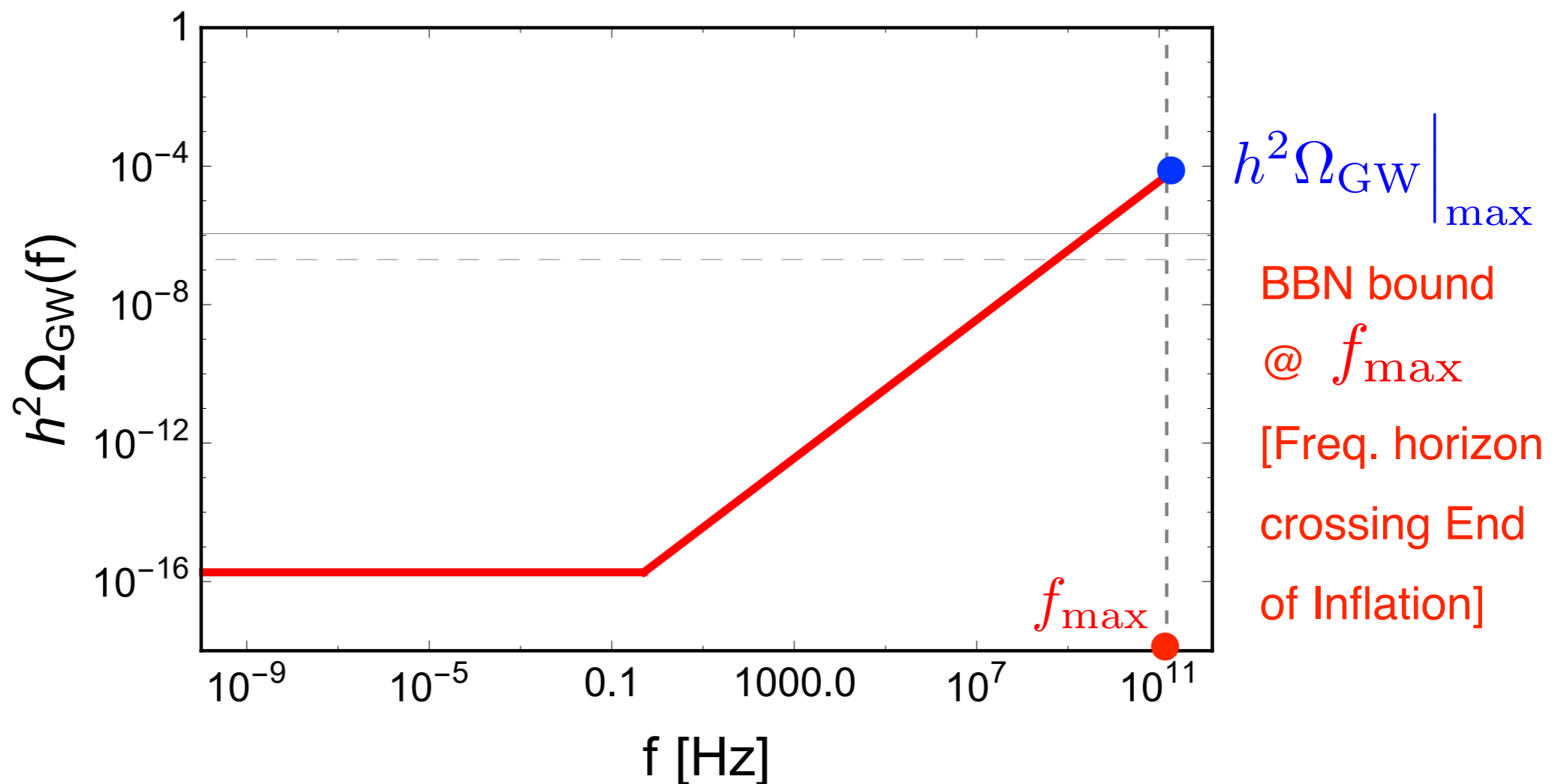
$$\int \frac{df}{f} h^2 \Omega_{\text{GW}}(f) \leq 1.12 \times 10^{-6}$$



BBN: $\int \frac{df}{f} h^2 \Omega_{\text{GW}}(f) \leq 1.12 \times 10^{-6}$

Grav. Reheating: $\Omega_{\text{GW}}(f) \propto (f/f_{\text{RD}})^2 \left(\frac{w_s - 1/3}{w_s + 1/3} \right)$

Monotonically growing signal !



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Monotonically growing signal !

BBN bound @ f_{max} [Freq. horizon crossing End of Inflation]

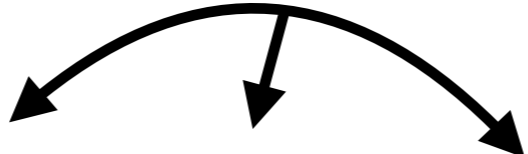
$$h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{\text{RD}}) \lesssim 10^{-6}$$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

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Grav. Reheating: $\Delta_* \equiv \frac{\rho_{\text{rad}}}{3m_p^2 H_*^2} = \frac{\delta}{300} \left(\frac{H_*}{m_p} \right)^2, \quad \delta \lesssim 1,$

$$f_{\text{RD}} = f_{\text{RD}}(H_*, w_s, \Delta_*) = f_{\text{RD}}(H_*, w_s, \delta)$$



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Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1,$

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

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↓

$$2 \leq f(w_s) \leq 2.54$$

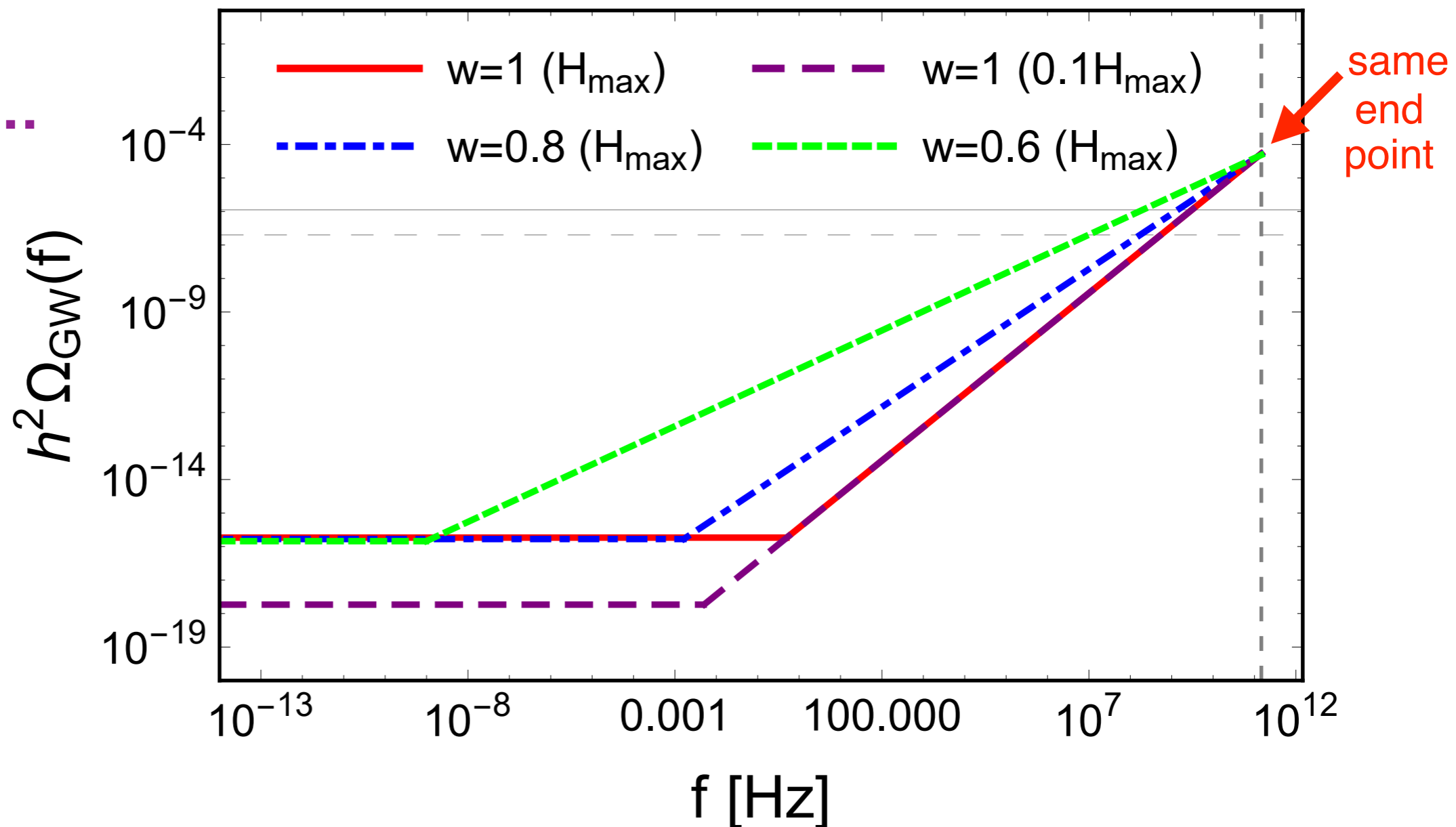
($w_s = 1/3$) ($w_s = 1$)

$$f(w_s) \equiv \frac{2^{\frac{3(1-w_s)}{1+3w_s}} \Gamma^2 \left(\frac{5+3w_s}{2+6w_s} \right)}{\left(\frac{2}{1+3w_s} \right)^{\frac{4}{1+3w_s}} \Gamma^2 \left(\frac{3}{2} \right)}$$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1,$

However ...



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Why ?

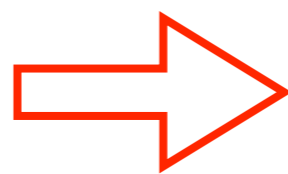
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Why ?

$$\left. \begin{aligned} \rho_{\text{rad}} &\propto H_*^4 a^{-4} \\ \rho_{\text{GW}} &\propto H_*^4 a^{-4} \end{aligned} \right\}$$



$$\frac{\rho_{\text{GW}}}{\rho_{\text{rad}}} \sim \text{const.}$$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

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$$\Delta_* \equiv \frac{\rho_{\text{rad}}}{3m_p^2 H_*^2} = \frac{\delta}{300} \left(\frac{H_*}{m_p} \right)^2$$

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So ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} \simeq \frac{\text{const.}}{\delta} \lesssim 10^{-6} \quad \Leftrightarrow \quad \delta \gtrsim 50$

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Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}$, $\delta \lesssim 1$,

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (\cancel{H_*}, \cancel{w_s}; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

So ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} \simeq \frac{\text{const.}}{\delta} \lesssim 10^{-6} \iff \delta \gtrsim 50$ (!)

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$\frac{\rho_{\text{GW}}}{\rho_{\text{rad}}} \sim \text{const.} \gg 1$ **Universe dominated by GWs!**

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

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So ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} \simeq \frac{\text{const.}}{\delta} \lesssim 10^{-6} \iff \delta \gtrsim 50$ (!)

CMB: $\delta \gtrsim 200$ (!!)

(standard) Grav. Reheating incompatible with BBN !

Therefore...

- 1) Either we modify Grav. Reheating**
- 2) We use modified gravity in Inflationary Sector**
- 3) We couple the inflaton and reheat via such couplings**

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Standard (P)reheating

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I'm very happy with General Relativity !

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Therefore...

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But if you are not ...

Y. Watanabe and E. Komatsu, Phys. Rev. **D75**, 061301 (2007), gr-qc/0612120.

Y. Watanabe, Phys. Rev. **D83**, 043511 (2011), 1011.3348.

A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980), [,771(1980)].

A. De Felice and S. Tsujikawa, Living Rev. Rel. **13**, 3 (2010), 1002.4928.

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$$\Delta_* \equiv \frac{\rho_{\text{rad}}}{3m_p^2 H_*^2} = \frac{\delta}{300} \left(\frac{H_*}{m_p} \right)^2 \longrightarrow \mathcal{N}_f \Delta_*$$

All \mathcal{N}_f fields
same properties !

$$\begin{aligned} \delta &= \delta_1 \times \mathcal{N}_f, \\ \mathcal{N}_f &\gtrsim \mathcal{O}(10^3) \end{aligned}$$

**Ad hoc
tuning !**

(standard) Grav. Reheating incompatible with BBN !

Therefore...

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Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

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Standard Grav. RH ?



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$m_\chi^2 < 0$ @ Stiff Period,

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Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

Standard Grav. RH wrong !
 $m_\chi^2 < 0$ @ Stiff Period, but
self-interactions regularize

(standard) Grav. Reheating incompatible with BBN !

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Corrected in DGF & Byrnes '16
Phys.Lett. B767 (2017) 272-277
Arxiv: 1604.03905

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1) Either we modify Grav. Reheating

Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

Standard Grav. RH wrong !

$$\delta \sim \mathcal{O}(10^3) \frac{\xi^2}{\lambda} \gg 1$$

$$\lambda > 0 \text{ (stability)}, \quad \xi \gtrsim 1$$

Grav.
Reheating
OK !

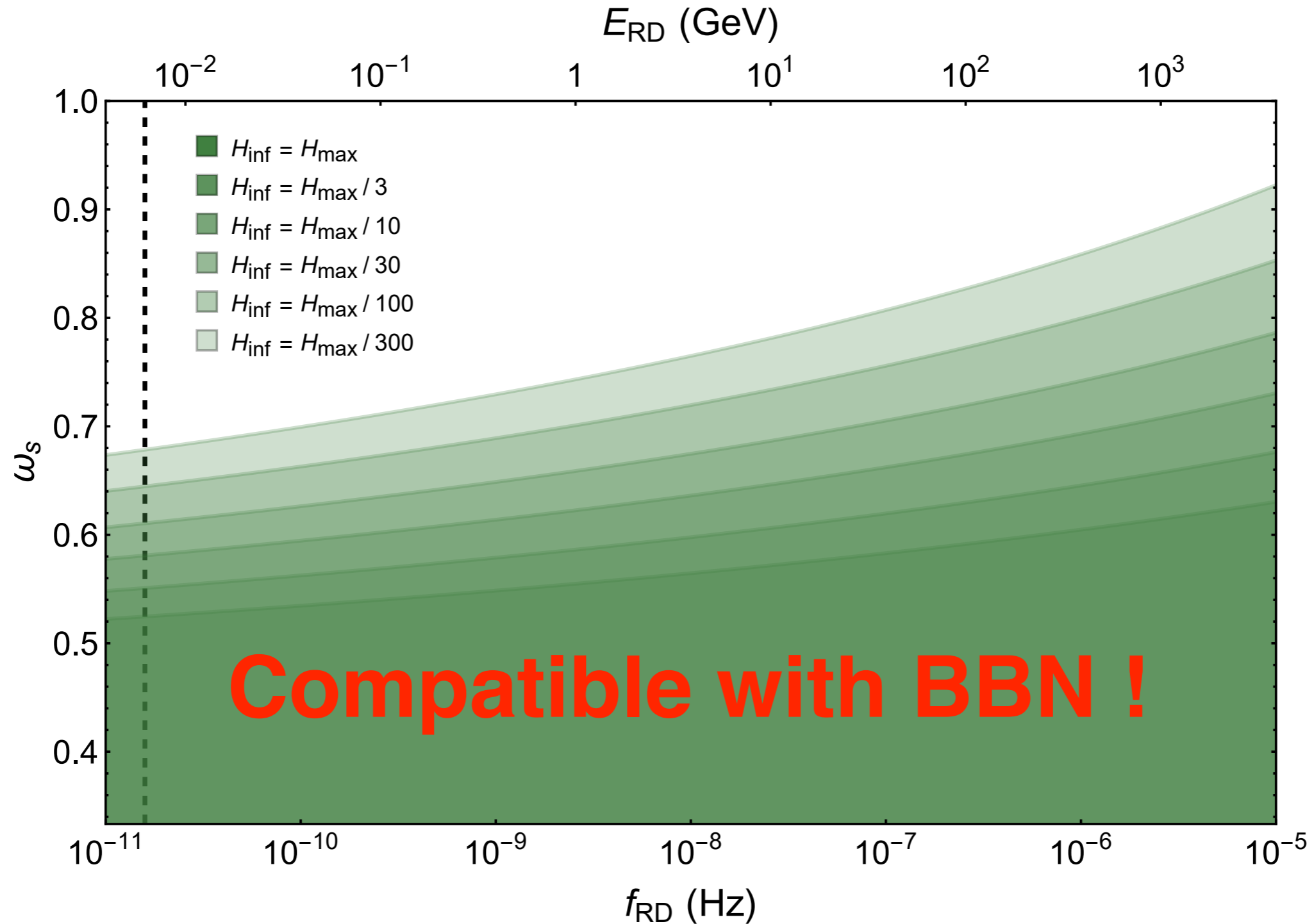
Corrected in DGF & Byrnes '16
Phys.Lett. B767 (2017) 272-277
Arxiv: 1604.03905

See also [1803.07399](#)
[1905.06823](#) for generic $\lambda\chi^4$

BBN: further implications

BBN Bound

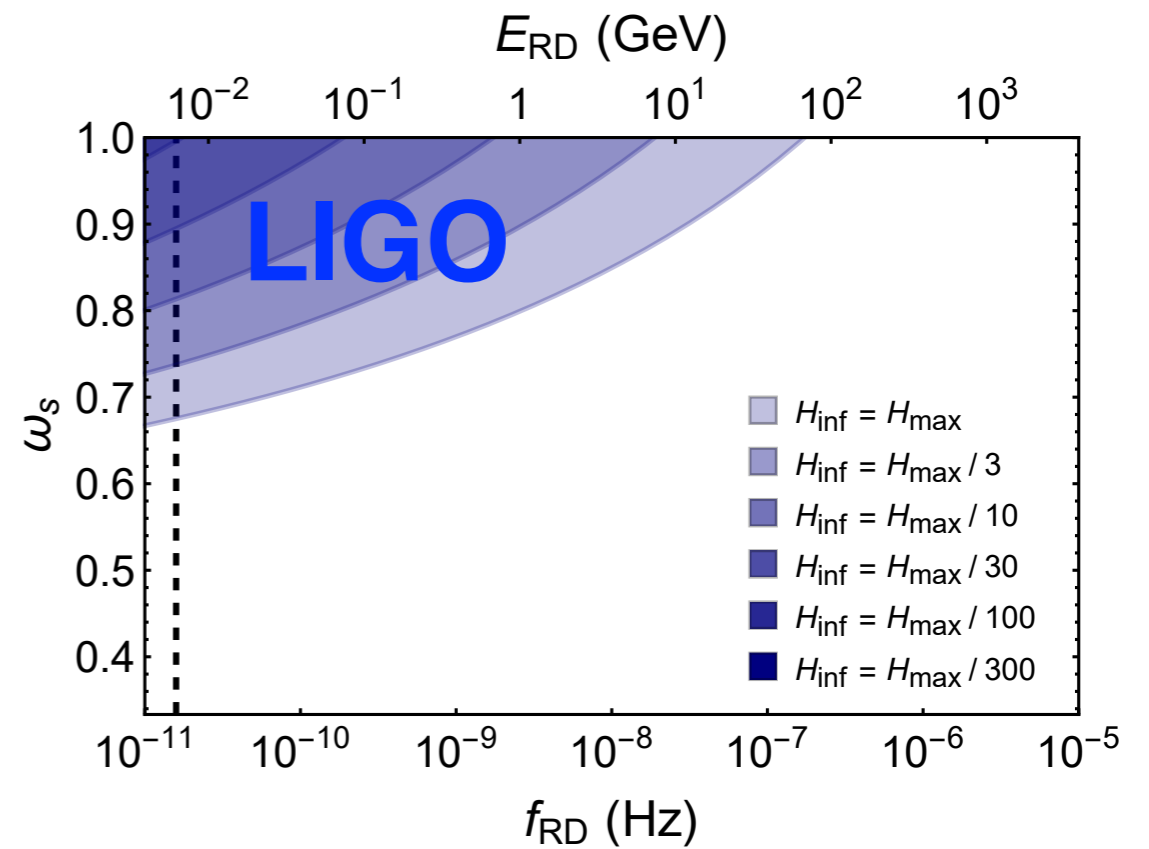
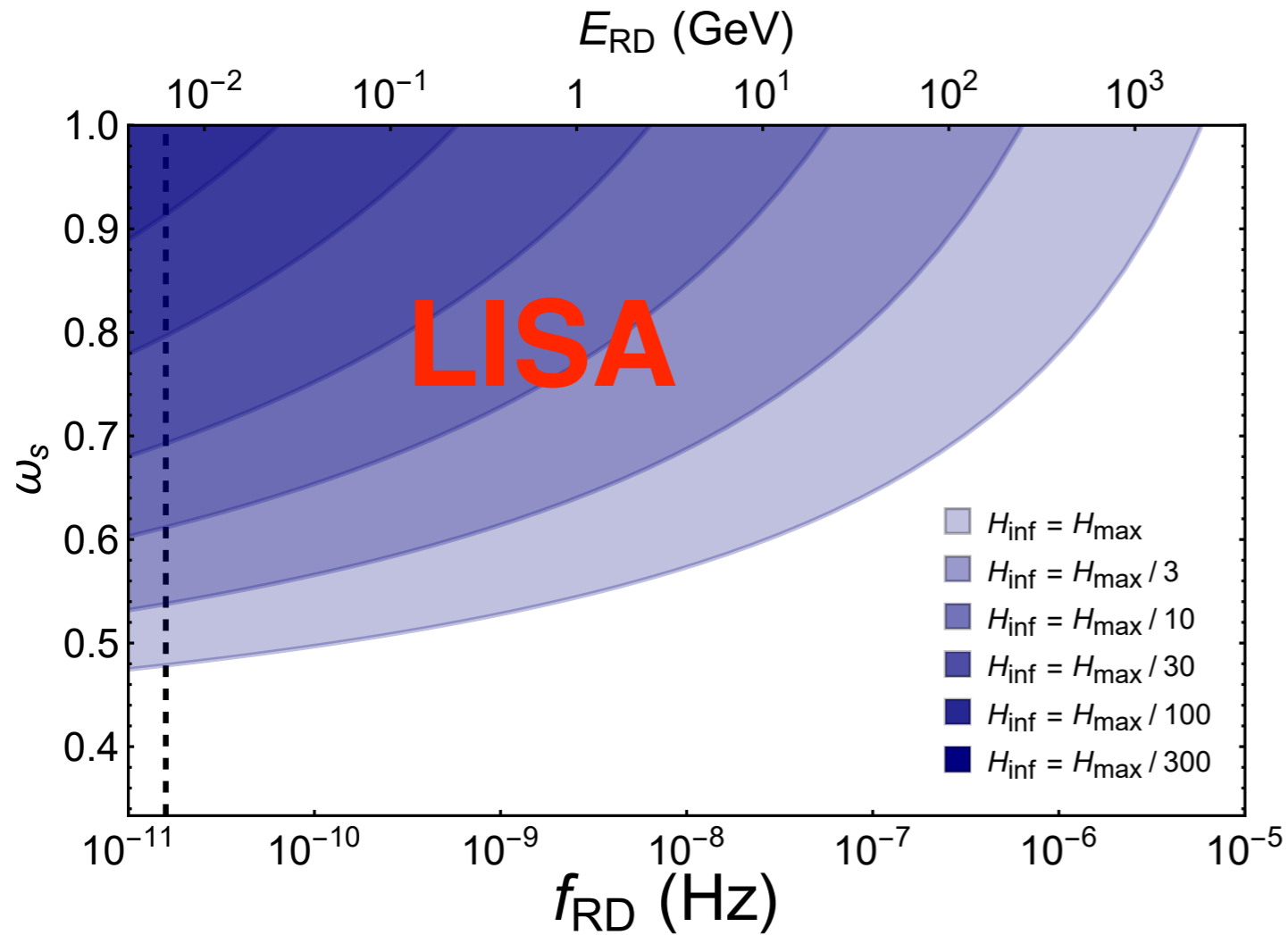
$$\Omega_{\text{GW}}^{(0)}(f; \underbrace{H_*}_{\text{Energy Scale}}, \underbrace{w_s}_{\text{EoS Stiff}}, \underbrace{f_{\text{RD}}}_{\text{Duration Stiff}}) \lesssim 10^{-6}$$



BBN Bound

$$\Omega_{\text{GW}}^{(0)}(f; \underbrace{H_*}_{\text{Energy Scale}}, \underbrace{w_s}_{\text{EoS Stiff}}, \underbrace{f_{\text{RD}}}_{\text{Duration Stiff}}) \lesssim 10^{-6}$$

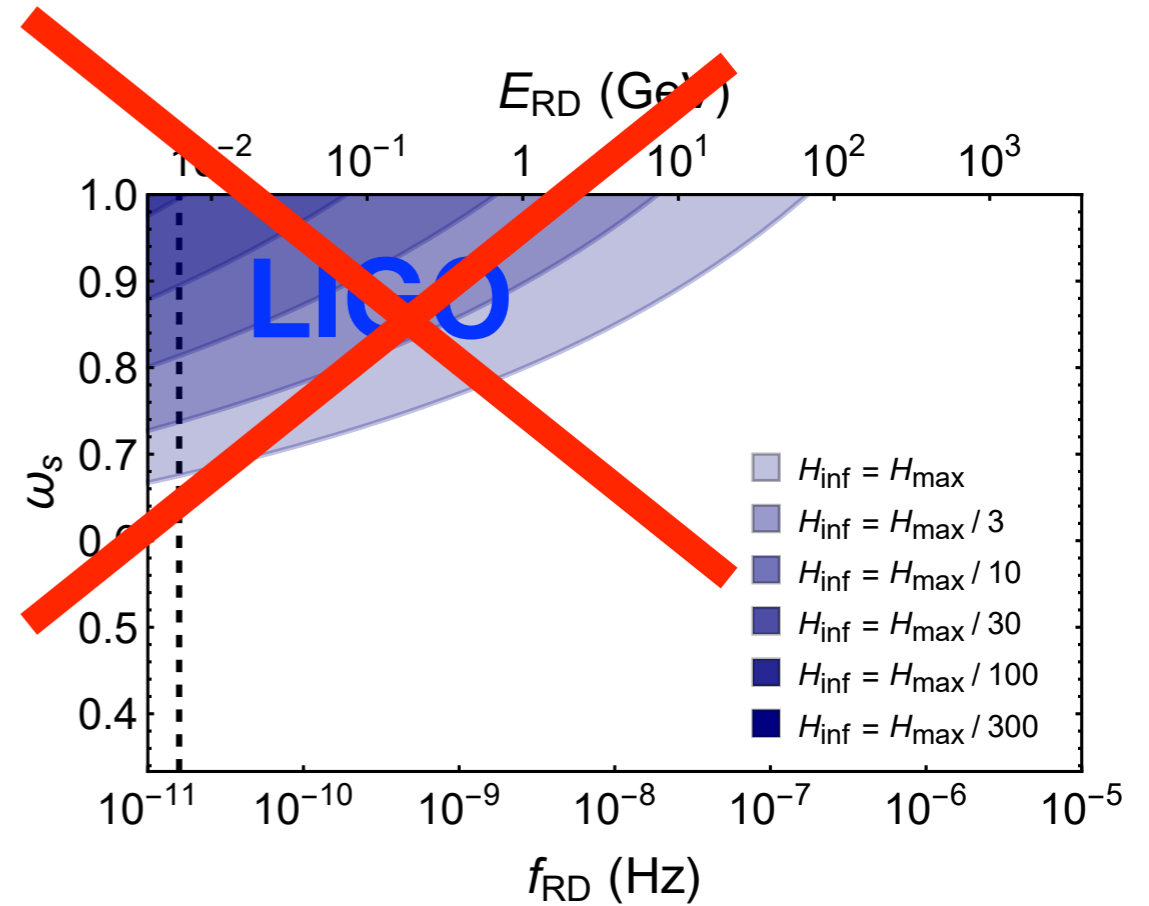
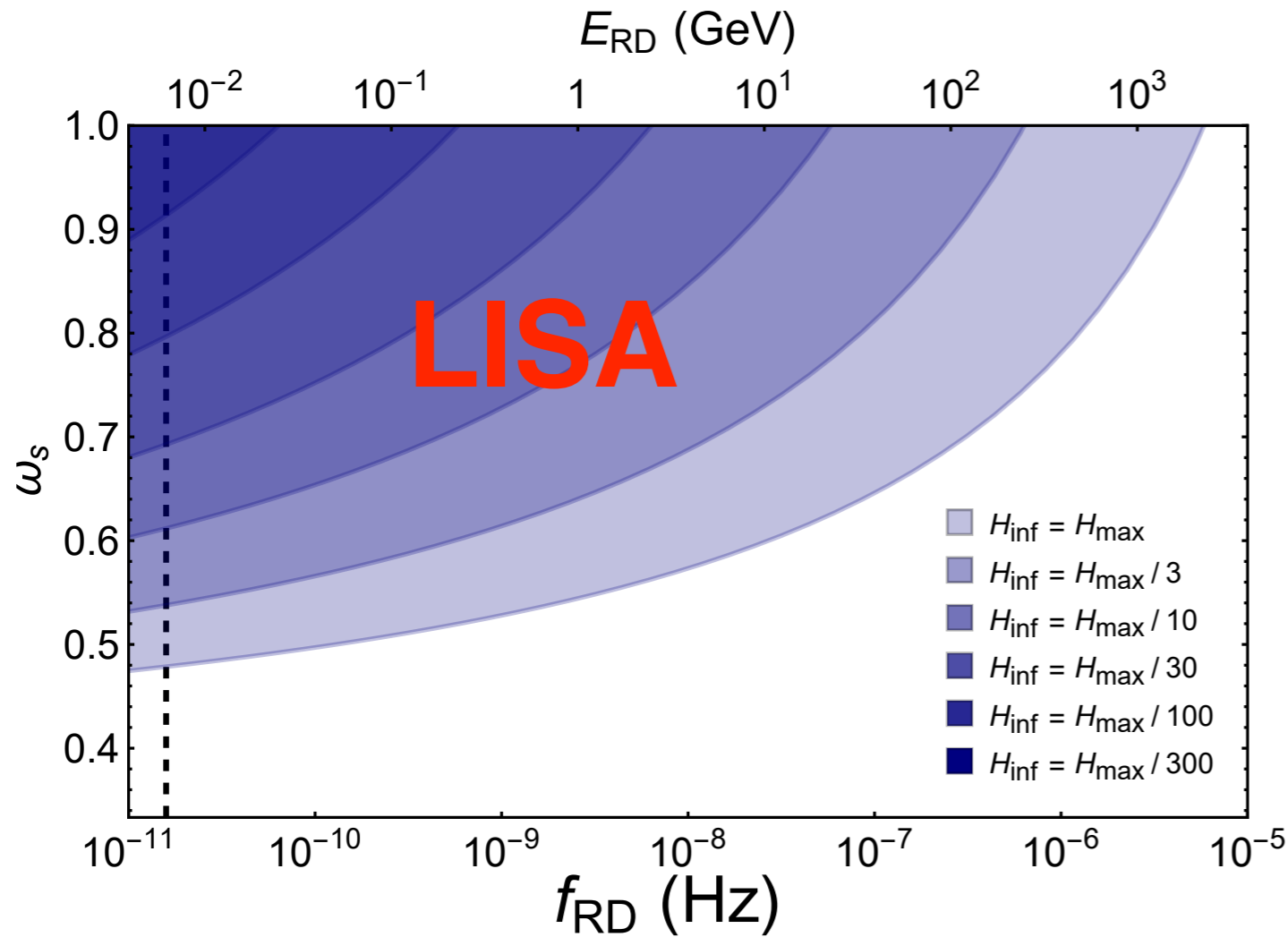
Energy Scale EoS Stiff Duration Stiff



BBN Bound

$$\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}}) \lesssim 10^{-6}$$

Energy Scale EoS Stiff Duration Stiff



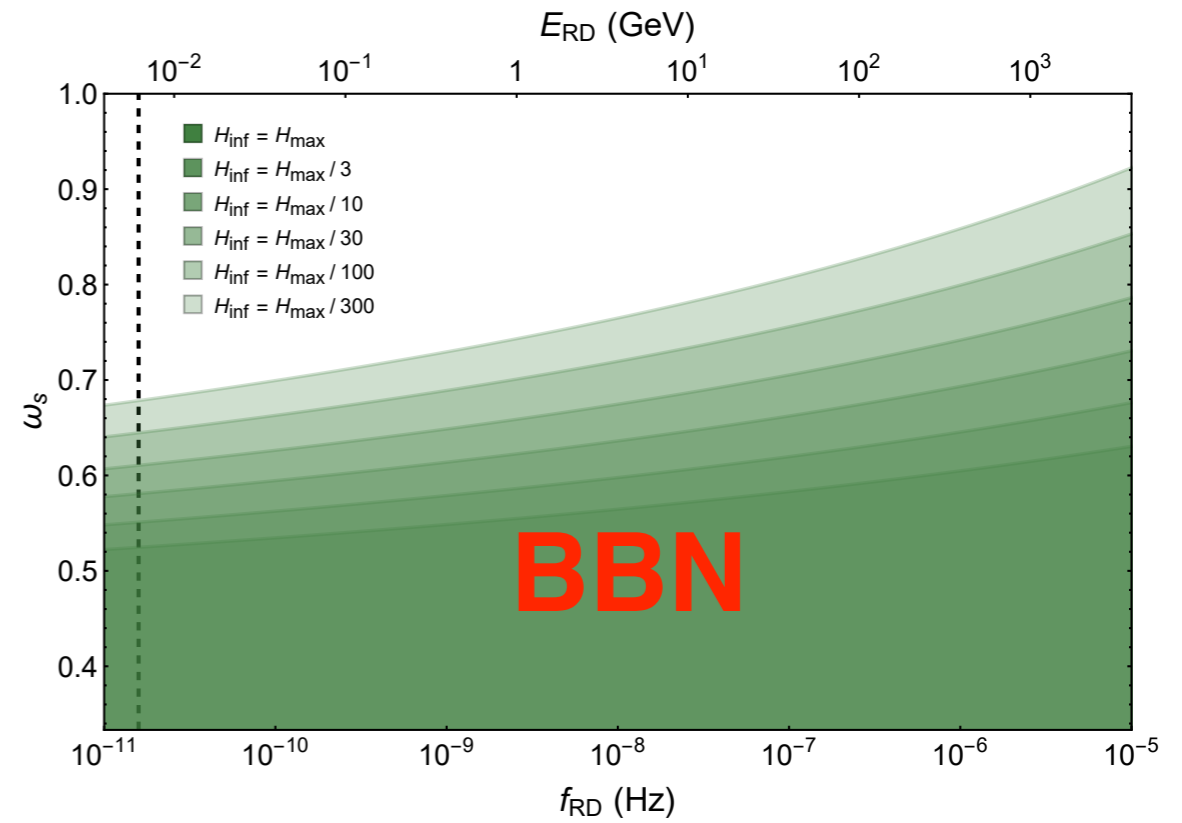
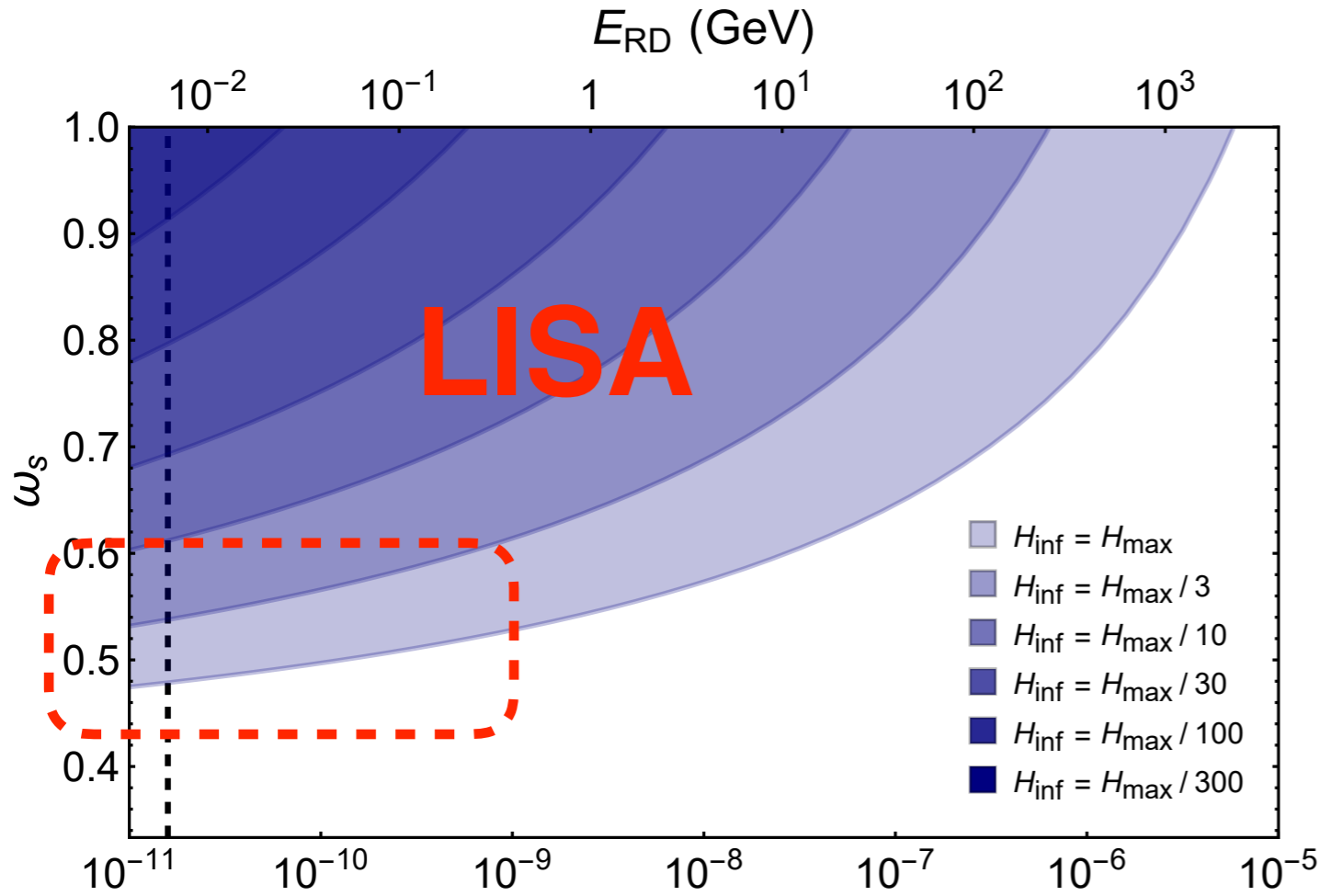
LIGO cannot probe parameter space compatible with BBN !

BBN Bound

LISA ?

$$\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}}) \lesssim 10^{-6}$$

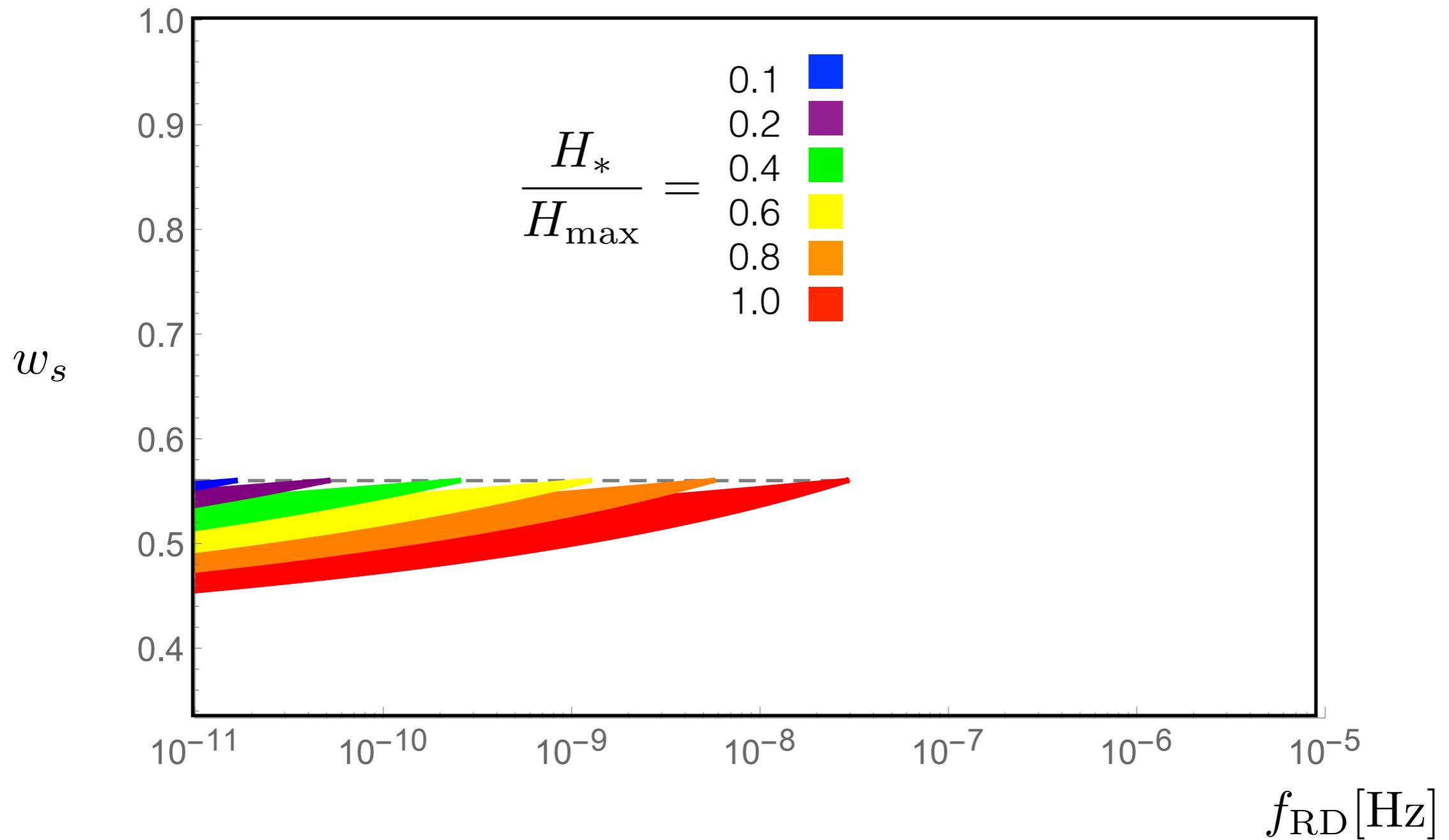
Energy Scale EoS Stiff Duration Stiff



BBN Bound $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}}) \lesssim 10^{-6}$

LISA ✓

Energy Scale EoS Stiff Duration Stiff



OUTLOOK

0) Reheating w/o couplings requires imagination:

Grav. Reheating or Modified Gravity

1) (Standard) Grav. Reheating is inconsistent

Too many GWs (violates BBN/CMB bounds)

2) Inf. sector only (minimally) coupled

to gravity inconsistent unless:

i) Inflation ~ Modify gravity: I don't want to

ii) $O(1000)$ spectator fields identical: ad hoc tuning

iii) SM Higgs + Non-Min coupling: works (not observable)

3) Stiff Era (in general): not observable @ LIGO, barely @ LISA

**Kiitos
huomiostanne !**