General Relativity (FYSS7320), 2022 **Excercise 7.**

Return by 12.00, Wednesday 2.3.2022 (electronically to pyry.m.rahkila@jyu.fi).

1. **Schwarzschild solution:** Modify and extend the Mathematica code found in the course home page (or write your own using your preferred program) to compute the non-vanishing components of the Riemann tensor for the Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ .$$

Modify the code to compute the invariant $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$. Are there any other non-trivial curvature invariants that need to be considered to show the non-singularity of the Schwartzhild metric at $r = 2GM$ (given what we know about the Ricci tensor already)?

2. **Radar signal in Scwarzschild:** Consider a lightlike $ds^2 = 0$ radar signal in Schwarzschild spacetime. The signal is sent from (r_2, θ_0, ϕ_0) to (r_1, θ_0, ϕ_0), immediately reflected and travels back again. Here $r_2 > r_1$ and the motion is radial, i.e. θ_0 and ϕ_0 are constants. Find the roundtrip time $\Delta\tau$ measured by an observer at $(r_2, \theta_0\phi_0)$.

3. **Radar time delay:** Let us continue on the previous problem. The distance between the two points is

$$\Delta R = \int_{r_1}^{r_2} \frac{dr}{\sqrt{1 - \frac{2GM}{r}}} = \sqrt{r_2(r_2 - 2GM)} - \sqrt{r_1(r_1 - 2GM)} + 2GM \ln\left(\frac{\sqrt{r_2} + \sqrt{r_2 - 2GM}}{\sqrt{r_1} + \sqrt{r_1 - 2GM}}\right) \ ,$$

so that one might expect the round trip time to be

$$\Delta\tilde{\tau} = 2\Delta R \ .$$

This is not the case, however, and the difference $\Delta\tau - \Delta\tilde{\tau}$ is called radar delay. Show that for $r_1 \gg 2GM$ the radar delay is

$$\Delta\tau - \Delta\tilde{\tau} \simeq 2GM\left(\ln\frac{r_2}{r_1} - \frac{r_2 - r_1}{r_2}\right) \ .$$

Explain the cause of this delay.

4. **Delayed lecture starts:** Occasional delays in lecture starts might be caused by creation of black holes in the acceleration lab. Assume the spacetime is described by the Schwarzschild metric and the r coordinate values of the lecturers office and the lecture room are $r_1 = 20$ m and $r_2 = 40$ m, respectively. Compute the mass M of a black hole required to generate a lag of 2 minutes between the clocks under a coordinate time interval $\Delta t = 15$ min (flat space mean propagation time from r_1 to r_2). Does the office lie outside the Schwarzschild radius, $r_1 > 2GM$, as required by consistency of the model?